RESEARCH PAPER

CHARACTERIZATION OF $F_{(i,j)}-$ OPEN SETS AND $F_{(i,j)}-$ CONTINEOUS MAPS ON FbTS

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ABSTRACT: The fuzzy bi-topological space offers suitable methods for solving problems which involve uncertainty as well as any area that lacks clear structure. To do so, bi-topological structures come in the form of fuzzy logic. The characterization of fuzzy open sets ($F_{(i,j)}-OS'S$) and maps ($F_{(i,j)}-CM'S$) in FbTS is the object of this study and we shall emphasize both their properties and relationships. The approach extends classical topology by enabling multi-dimensional evaluation of intricate systems--all these elements make it an attractive entry point into contemporary systems.

KEYWORDS: $F_{(i,j)}-OS'S$, $F_{(i,j)}-CS'S$, $F_{(i,j)}-SOS'S$, $F_{(i,j)}-POS'S$, FbTS, $F_{(i,j)}-CM'S$

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INTRODUCTION

Fuzzy bi-topological spaces are a useful tool to apply multi-dimensional analysis in complex problems to increase our capability to approach such problems. Unlike classical topological spaces, they allow for a fuller exploration of sets and their connection to multiple continuity types via dual structures including fuzzy logic and bi-topology. General topology is one of the earliest areas of mathematics to systematically utilise fuzzy set theory. In these three chapters we present the ideas, concepts, and methods of fuzzy set theory which together with general topology are combined to introduce fuzzy topology as a major development of classical topology. Fuzzy topology expands our understanding of elementary mathematical structures yet gives us greater flexibility in handling unknowns. Zadeh [1] introduced the idea of fuzzy sets, laying an influential groundwork for fuzzy mathematics. Levine [6] expanded on this notion by implementing the concept of semi-open sets and continuity. Kandil [11] incorporated fuzzy bi-topological spaces into the work in 1989. Later work expanded the theory. Abd El-Monsef et al. [8] presented β-open sets and β-continuous mappings, and related developments contributed to generalized fuzzy open structures.

The concept of fuzzy semi pre-open sets [10] was introduced by Thakur. Strongly α-continuous functions and their related continuity concepts have been studied in generalized settings by Császár [2], establishing broader forms of continuity. Császár [1], [3], [4] introduced generalized topological spaces and generalized continuity along with related operators, and Palani Chetty [15] developed generalized fuzzy topology and its properties. These developments are particularly significant in the real-world domain because many real-world phenomena—such as artificial intelligence and biology—all show vagueness and uncertainty. Fuzzy bi-topological structures have also been used to analyze complex systems, such as in complex algebraic structure interactions, classification in matroid theory, and modeling of molecular interactions. So, the bi-topological spaces of fuzzy ideas provide a crucial link between theoretical results of fuzzy frameworks and practical application results.

2. PRELIMINARIES

Definition 2.1: Let (X,τ_1,τ_2) consisting of a universal set X with the fuzzy topologies "τ_1" and "τ_2" on X is called fuzzy bi-topological space

Definition 2.2: A fuzzy set A of fuzzy bi-topological space (X,τ_1,τ_2) is called $\text{fuzzy}\,(i,j)-\text{semi}-\text{open set}$ if $A\subseteq\tau_j-cl(\tau_i-int(A))$. In a fuzzy bi-topological space (X,τ_1,τ_2), every fuzzy $\tau_i-\text{open set}$ $(i=1,2)$ is $\text{fuzzy}\,(i,j)-\text{semi}-\text{open set}$ but not converse. A fuzzy set A of fuzzy bi-topological space (X,τ_1,τ_2) is called $\text{fuzzy}\,(i,j)-\text{semi}-\text{closed set}$ if $A^c=1-A$ is $\text{fuzzy}\,(i,j)-\text{semi}-\text{open set}$

Definition 2.3: A fuzzy set A of fuzzy bi-topological space (X,τ_1,τ_2) is called $\text{fuzzy}\,(i,j)-\text{pre}-\text{open set}$ if $A\subseteq\tau_i-int(\tau_j-cl(A))$. A fuzzy set A of fuzzy bi-topological space (X,τ_1,τ_2) is called fuzzy $(i,j)-$

pre − closed set if $A^c = 1 - A$ is fuzzy (i,j) − pre − open set.

Definition 2.4: A fuzzy set A of fuzzy bi-topological space (X, τ_1, τ_2) is called fuzzy $(i,j) - \beta -$ open set if $A \subseteq \tau_j - cl(\tau_i - int(\tau_j - cl(A)))$. A fuzzy set A of fuzzy bi-topological space (X, τ_1, τ_2) is called fuzzy $(i,j) - \beta -$ closed set if $A^c = 1 - A$ isfuzzy $(i,j) - \beta -$ open set.

Definition 2.5: A fuzzy set A of fuzzy bi-topological space (X, τ_1, τ_2) is called fuzzy $(i,j) - \alpha -$ open set if $A \subseteq \tau_j - int(\tau_i - cl(\tau_j - int(A)))$. A fuzzy set A of fuzzy bi-topological space (X, τ_1, τ_2) is called fuzzy $(i,j) - \alpha -$ closed set if $A^c = 1 - A$ is fuzzy $(i,j) - \alpha -$ open set.

Definition 2.6: A fuzzy set A of fuzzy bi-topological space (X, τ_1, τ_2) is called

- Fuzzy (i,j) − regular − open set if $\tau_j - int(\tau_i - cl(A)) = A$
- Fuzzy (i,j) − regular − closed set if $t_j - cl(t_i - int(A)) = A$

3. $F_{(i,j)}$ − OS on FbTS

Definition 3.1: A Fuzzy bi-topological space (**FbTS**) is an ordered triplet $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$, whereas $\mathcal{A}$ is universal set and $\mathcal{T}_1$ & $\mathcal{T}_1$ are two fuzzy topologies defined on $\mathcal{A}$.

Example 3.1: Let $\mathcal{A} = \{a_1, a_2, a_3\}$ and α and β be FSs on $\mathcal{A}$ given as, $\alpha = \left\{\left(\frac{a_1}{0.9}\right), \left(\frac{a_2}{0.8}\right), \left(\frac{a_3}{1}\right)\right\}, \beta = \left\{\left(\frac{a_1}{0.7}\right), \left(\frac{a_2}{0.8}\right), \left(\frac{a_3}{0.5}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$ and $\mathcal{T}_2 = \{0, \beta, 1\}$. Hence $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is FbTS

Definition 3.2: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \mathbb{SOS}$ if $\alpha \subseteq \mathcal{T}_j - \mathbb{CL}(\mathcal{T}_i - \mathbb{I}(\alpha))$

Example 3.2: Let $\mathcal{A} = \{a_1, a_2\}$ and α, β, $\mathbb{C}$ and $\mathbb{D}$ be FSs on $\mathcal{A}$ given as, $\alpha = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.7}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.6}\right)\right\}$, $\gamma = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.4}\right)\right\}$ and $\theta = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.5}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, \beta, 1\}$ and $\mathcal{T}_2 = \{0, \gamma, \theta, 1\}$ be FTS on $\mathcal{A}$. Now consider, $ZETA = \left\{\left(\frac{a_1}{0.7}\right), \left(\frac{a_2}{0.8}\right)\right\}$. Clearly $ZETA \subseteq \mathcal{T}_2 - \mathbb{CL}(\mathcal{T}_1 - \mathbb{I}(ZETA))$. Hence, ZETA is $\mathbb{F}_{(1,2)} - \mathbb{SOS}$. Again $ZETA \subseteq \mathcal{T}_1 - \mathbb{CL}(\mathcal{T}_2 - \mathbb{I}(ZETA))$. This implies ZETA is also $\mathbb{F}_{(2,1)} - \mathbb{SOS}$

Remark 3.1: In FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$, for $(i = 1,2)$, $\mathbb{F}_i - \mathbb{OS} \Rightarrow\not\Leftarrow \mathbb{F}_{(i,j)} - \mathbb{SOS}$. In Example 2.2, ZETA is $\mathbb{F}_{(i,j)} - \mathbb{SOS}$ but not $\mathbb{F}_i - \mathbb{OS}$ $(i = 1,2)$, i.e. $\mathbb{F}_{(i,j)} - \mathbb{SOS} \nRightarrow \mathbb{F}_i - \mathbb{OS}$

Definition 3.3: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \mathbb{SCS}$ if $\alpha^c \subseteq 1 - \alpha$ is $\mathbb{F}_{(i,j)} - \mathbb{SOS}$

Definition 3.4: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \mathbb{POS}$ if $\alpha \subseteq \mathcal{T}_i - \mathbb{I}(\mathcal{T}_j - \mathbb{CL}(\alpha))$

Example 3.3: Let $\mathcal{A} = \{a_1, a_2\}$ and α, β, γ and θ be FSs on $\mathcal{A}$ given as $\alpha = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.5}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.3}\right), \left(\frac{a_2}{0.5}\right)\right\}$, $\gamma = \left\{\left(\frac{a_1}{0.4}\right), \left(\frac{a_2}{0.4}\right)\right\}$ and $\theta = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.5}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, \gamma, 1\}$ and $\mathcal{T}_2 = \{0, \beta, 1\}$ be FTS on $\mathcal{A}$. Clearly $\theta \subseteq \mathcal{T}_1 - \mathbb{CL}(\mathcal{T}_2 - \mathbb{I}(\theta))$. Hence θ is also $\mathbb{F}_{(1,2)} - \mathbb{POS}$

Definition 3.5: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \mathbb{PCS}$ if $\alpha^c \subseteq 1 - \alpha$ is $\mathbb{F}_{(i,j)} - \mathbb{POS}$**.** In Example 3.3, FS $ZETA = \left\{\left(\frac{a_1}{0.4}\right), \left(\frac{a_2}{0.5}\right)\right\}$ is $\mathbb{F}_{(1,2)} - \mathbb{PCS}$ in $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$

Definition 3.6: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \beta\mathbb{OS}$ if $\alpha \subseteq \mathcal{T}_j - \mathbb{CL}(\mathcal{T}_i - \mathbb{I}(\mathcal{T}_j - \mathbb{CL}(\alpha)))$**.** In Example 3.3, fuzzy set "θ" is $\mathbb{F}_{(1,2)} - \beta\mathbb{OS}$ in $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$

Definition 3.7: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \beta\mathbb{CS}$ if $\alpha^c \subseteq 1 - \alpha$ is $\mathbb{F}_{(i,j)} - \beta\mathbb{OS}$. In Example 3.3, FS $ZETA = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.5}\right)\right\}$ is $\mathbb{F}_{(1,2)} - \beta\mathbb{CS}$ in $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$

Remark 3.2: In FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$,$\mathbb{F}_{(i,j)} - \mathbb{POS} \Rightarrow\not\Leftarrow \mathbb{F}_{(i,j)} - \beta\mathbb{OS}$. This can be shown in Example 3.4

Example 3.4: Let $\mathcal{A} = \{a_1, a_2\}$ and α, β, γ and θ be FSs on $\mathcal{A}$ defined as $\alpha = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.3}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.3}\right), \left(\frac{a_2}{0.2}\right)\right\}$, $\gamma = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.7}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$ and $\mathcal{T}_2 = \{0, \beta, 1\}$ be FTS on $\mathcal{A}$. Now Clearly $\gamma \subseteq \mathcal{T}_2 - \mathbb{CL}(\mathcal{T}_1 - \mathbb{I}(\mathcal{T}_2 - \mathbb{CL}(\gamma)))$. This implies $\mathbb{F}_{(i,j)} - \beta\mathbb{OS} \nRightarrow \mathbb{F}_{(i,j)} - \mathbb{POS}$

Definition 3.8: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \alpha\mathbb{OS}$ if $\alpha \subseteq \mathcal{T}_j - \mathbb{I}(\mathcal{T}_i - \mathbb{CL}(\mathcal{T}_j - \mathbb{I}(\alpha)))$

Example 3.5: Let $\mathcal{A} = \{a_1, a_2\}$ and α, β, γ and θ be FSs on $\mathcal{A}$ defined as $\alpha = \left\{\left(\frac{a_1}{0.2}\right), \left(\frac{a_2}{0.3}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.5}\right)\right\}$, $\gamma = \left\{\left(\frac{a_1}{0.2}\right), \left(\frac{a_2}{0.4}\right)\right\}$ and $\theta = \left\{\left(\frac{a_1}{0.3}\right), \left(\frac{a_2}{0.4}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, \gamma, 1\}$ and $\mathcal{T}_2 = \{0, \beta, 1\}$ be FTS on . Clearly $\theta \subseteq \mathcal{T}_1 - \mathbb{I}(\mathcal{T}_2 - \mathbb{CL}(\mathcal{T}_1 - \mathbb{I}(\alpha)))$. Hence, θ is also $\mathbb{F}_{(1,2)} - \alpha\mathbb{OS}$

Definition 3.9: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \alpha\mathbb{CS}$ if $\alpha^c \subseteq 1 - \alpha$ is $\mathbb{F}_{(i,j)} - \alpha\mathbb{OS}$. In Example 3.5, FS, $ZETA = \left\{\left(\frac{a_1}{0.7}\right), \left(\frac{a_2}{0.6}\right)\right\}$ is $\mathbb{F}_{(1,2)} - \alpha\mathbb{CS}$ in

$(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$

Definition 3.10: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(i,j)} - \mathbb{ROS}$ if $\mathcal{T}_j - \mathbb{I}(\mathcal{T}_i - \mathbb{CL}(\alpha)) = \alpha$

Definition 3.11: FS "α" of FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$ is called $\mathbb{F}_{(1,2)} - \mathbb{RCS}$ if $\mathcal{T}_j - \mathbb{CL}(\mathcal{T}_i - \mathbb{I}(\alpha)) = \alpha$

Remark 3.3: In FbTS $(\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2)$, $\mathbb{F}_{(i,j)} - \mathbb{ROS} \Rightarrow\not\Leftarrow \mathbb{F}_i - \mathbb{OS}$ and $\mathbb{F}_{(i,j)} - \mathbb{RCS} \Rightarrow\not\Leftarrow \mathbb{F}_i - \mathbb{OS}$

4. $F_{(i,j)}$ − CM's on FbTS

Definition 4.1: A mapping $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is said to be pairwise fuzzy continuous map $(\mathcal{PwFCM})$ if $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1) \to (\mathcal{B}, \mathcal{T}_1')$ and $\mathcal{F}: (\mathcal{A}, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_2')$ are fuzzy continuous maps $(\mathcal{FCM})$

Example 4.1: Let $\mathcal{A} = \{a_1, a_2\}$ and $\mathcal{B} = \{b_1, b_2\}$. Suppose $\alpha = \left\{\left(\frac{a_1}{0.2}\right), \left(\frac{a_2}{0.1}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.7}\right)\right\}$, $\gamma = \left\{\left(\frac{b_1}{0.2}\right), \left(\frac{b_2}{0.1}\right)\right\}$, $\theta = \left\{\left(\frac{b_1}{0.6}\right), \left(\frac{b_2}{0.7}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$, $\mathcal{T}_2 = \{0, \beta, 1\}$, $\mathcal{T}_1' = \{0, \gamma, 1\}$ and $\mathcal{T}_2' = \{0, \theta, 1\}$ be two FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ Such that, $\mathcal{F}(a_1) = b_1$ and $\mathcal{F}(a_2) = b_2$ is $\mathcal{PwFCM}$

Definition 4.2: Mapping $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is said to be $\mathcal{PwFSCM}$ if inverse image of everb $\mathcal{T}_i' - \mathcal{FOS}$ in $\mathcal{B}$ is $\mathbb{F}_{(i,j)} - \mathbb{SOS}$ in $\mathcal{A}$

Example 4.2: Let $\mathcal{A} = \{a_1, a_2\}$ and $\mathcal{B} = \{b_1, b_2\}$. Suppose $= \left\{\left(\frac{a_1}{0.2}\right), \left(\frac{a_2}{0.3}\right)\right\}$, $\beta = \left\{\left(\frac{b_1}{0.3}\right), \left(\frac{b_2}{0.4}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$, $\mathcal{T}_2 = \{0,1\}$, $\mathcal{T}_1' = \{0, \beta, 1\}$ and $\mathcal{T}_2' = \{0,1\}$ be two FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$. Such that, $\mathcal{F}(a_1) = b_1$ and $\mathcal{F}(a_2) = b_2$. Clearly $\mathcal{F}^{-1}(\beta) = \left\{\left(\frac{a_1}{0.3}\right), \left(\frac{a_2}{0.4}\right)\right\}$ and $\alpha \subseteq \mathcal{F}^{-1}(\alpha) \subseteq \mathcal{T}_2 - \mathbb{CL}(\alpha) = 1$, implies $\mathcal{F}^{-1}(\beta)$ is $\mathbb{F}_{(1,2)} - \mathbb{SOS}$ in $\mathcal{A}$. Again, $\mathcal{F}^{-1}(0)$ and $\mathcal{F}^{-1}(1)$ are $\mathbb{F}_{(1,2)} - \mathbb{SOS}$ in $\mathcal{A}$. Thus the mapping, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is $\mathcal{PwFSCM}$

Definition 4.3: A mapping $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is said to be $\mathcal{PwFPCM}$ if inverse image of every $\mathcal{T}_i' - \mathcal{FOS}$ in $\mathcal{B}$ is $\mathbb{F}_{(i,j)} - \mathbb{POS}$ in $\mathcal{A}$

Example 4.3: Let $\mathcal{A} = \{a_1, a_2, a_3\}$ and $\mathcal{B} = \{b_1, b_2, b_3\}$. Suppose $= \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.5}\right), \left(\frac{a_3}{0.5}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.4}\right), \left(\frac{a_2}{0.4}\right), \left(\frac{a_3}{0.4}\right)\right\}$, $\gamma = \left\{\left(\frac{b_1}{0.1}\right), \left(\frac{b_2}{0.1}\right), \left(\frac{b_3}{0.1}\right)\right\}$ and $\theta = \left\{\left(\frac{b_1}{0.2}\right), \left(\frac{b_2}{0.2}\right), \left(\frac{b_3}{0.2}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$, $\mathcal{T}_2 = \{0, \beta, 1\}$, $\mathcal{T}_1' = \{0, \gamma, 1\}$ and $\mathcal{T}_2' = \{0, \theta, 1\}$ be FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ Such that, $\mathcal{F}(a_1) = b_1$, $\mathcal{F}(a_2) = b_2$, $\mathcal{F}(a_3) = b_3$. Clearly, $\mathcal{F}^{-1}(\gamma) = \left\{\left(\frac{a_1}{0.1}\right), \left(\frac{a_2}{0.1}\right), \left(\frac{a_3}{0.1}\right)\right\}$, $\mathcal{F}^{-1}(\theta) = \left\{\left(\frac{a_1}{0.1}\right), \left(\frac{a_2}{0.1}\right), \left(\frac{a_3}{0.1}\right)\right\}$, $\mathcal{F}^{-1}(0) = 0$, $\mathcal{F}^{-1}(1) = 1$ are $\mathbb{F}_{(1,2)} - \mathbb{POS}$ in $\mathcal{A}$. Thus, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is $\mathcal{PwFPCM}$

Definition 4.4: A mapping $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is said to be $\mathcal{PwFIM}$ if inverse image of everb $\mathbb{F}_{(i,j)} - \mathbb{POS}$ in $\mathcal{B}$ is $\mathbb{F}_{(i,j)} - \mathbb{POS}$ in $\mathcal{A}$

Example 4.4: Let $\mathcal{A} = \{a_1, a_2, a_3\}$ and $\mathcal{B} = \{b_1, b_2\}$. Suppose $A = \left\{\left(\frac{a_1}{0.4}\right), \left(\frac{a_2}{0.6}\right), \left(\frac{a_3}{0.5}\right)\right\}$, $B = \left\{\left(\frac{b_1}{0.5}\right), \left(\frac{b_2}{0.7}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$, $\mathcal{T}_2 = \{0, 1\}$, $\mathcal{T}_1' = \{0, \beta, 1\}$ and $\mathcal{T}_2' = \{0, 1\}$ be FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$, Such that, $\mathcal{F}(a_1) = b_1$, $\mathcal{F}(a_2) = b_2$. Thus, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is $\mathcal{PwFIM}$

Definition 4.5: A mapping $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is said to be $\mathcal{PwF}\alpha\mathcal{CM}$ if inverse image of everb $\mathcal{T}_i' - \mathcal{FOS}$ in $\mathcal{B}$ is $\mathbb{F}_{(i,j)} - \alpha\mathbb{OS}$ in $\mathcal{A}$

Example 4.5: Let $\mathcal{A} = \{a_1, a_2, a_3\}$ and $\mathcal{B} = \{b_1, b_2, b_3\}$. Suppose, $= \left\{\left(\frac{a_1}{0.4}\right), \left(\frac{a_2}{0.4}\right), \left(\frac{a_3}{0.4}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.5}\right), \left(\frac{a_3}{0.5}\right)\right\}$, $\gamma = \left\{\left(\frac{b_1}{0.4}\right), \left(\frac{b_2}{0.0}\right), \left(\frac{b_3}{0.4}\right)\right\}$, $\theta = \left\{\left(\frac{b_1}{0.5}\right), \left(\frac{b_2}{0.0}\right), \left(\frac{b_3}{0.5}\right)\right\}$ and $\text{ETA} = \left\{\left(\frac{b_1}{0.5}\right), \left(\frac{b_2}{0.5}\right), \left(\frac{b_3}{0.5}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$, $\mathcal{T}_2 = \{0, \beta, 1\}$, $\mathcal{T}_1' = \{0, \gamma, \text{ETA}, 1\}$ and $\mathcal{T}_2' = \{0, \theta, 1\}$ be FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ Such that, $\mathcal{F}(a_1) = b_1$, $\mathcal{F}(a_2) = b_2$, $\mathcal{F}(a_3) = b_3$. Clearly $\mathcal{F}^{-1}(\gamma) = 0$, $\mathcal{F}^{-1}(\theta) = 0$, $\mathcal{F}^{-1}(\text{ETA}) = \beta$ are $\mathcal{T}_2 - \alpha\mathbb{OS}$ in $\mathcal{A}$. Thus, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is $\mathcal{PwF}\alpha\mathcal{CM}$

Definition 4.6: A mapping $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is said to be $\mathcal{PwF}\beta\mathcal{CM}$ if inverse image of everb $\mathcal{T}_i' - \mathcal{FOS}$ in $\mathcal{B}$ is $\mathbb{F}_{(i,j)} - \beta\mathbb{OS}$ in $\mathcal{A}$

Example 4.6: Let $\mathcal{A} = \{a_1, a_2\}$ and $\mathcal{B} = \{b_1, b_2\}$. Suppose, $\alpha = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.3}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.3}\right), \left(\frac{a_2}{0.2}\right)\right\}$, $\gamma = \left\{\left(\frac{b_1}{0.5}\right), \left(\frac{b_2}{0.7}\right)\right\}$ and $\theta = \left\{\left(\frac{b_1}{0.2}\right), \left(\frac{b_2}{0.6}\right)\right\}$. Let $\mathcal{T}_1 = \{0, \alpha, 1\}$, $\mathcal{T}_2 = \{0, \beta, 1\}$, $\mathcal{T}_1' = \{0, \gamma, 1\}$ and $\mathcal{T}_2' = \{0, \theta, 1\}$ be FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ Such that, $\mathcal{F}(a_1) = b_1$, $\mathcal{F}(a_2) = b_2$. Clearly $f^{-1}(\gamma) = 0$, $f^{-1}(0) = 0$, $f^{-1}(\gamma) = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.7}\right)\right\}$, $f^{-1}(\theta) = \left\{\left(\frac{a_1}{0.2}\right), \left(\frac{a_2}{0.6}\right)\right\}$ are $\mathcal{T}_i - \beta\mathbb{OS}$ in $\mathcal{A}$, (i = 1,2). Thus, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \to (\mathcal{B}, \mathcal{T}_1', \mathcal{T}_2')$ is $\mathcal{PwF}\beta\mathcal{CM}$

Remark 4.1: $\mathcal{Pw}0\mathcal{FCM} \Rightarrow\not\Leftarrow \mathcal{PwFSCM}$ $(\mathcal{PwF}\alpha\mathcal{CM})$, which is shown in Example 4.7

Example 4.7: Let $\mathcal{A} = \{a_1, a_2\}$ and $\mathcal{B} = \{b_1, b_2\}$.

Consider FS′s $= \left\{\left(\frac{a_1}{0.4}\right), \left(\frac{a_2}{0.5}\right)\right\}$, $\beta = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.5}\right)\right\}$, $\gamma = \left\{\left(\frac{b_1}{0.5}\right), \left(\frac{b_2}{0.6}\right)\right\}$ and $\theta = \left\{\left(\frac{a_1}{0.5}\right), \left(\frac{a_2}{0.5}\right)\right\}$ on $\mathcal{A}$. Again $\varphi = \left\{\left(\frac{b_1}{0.7}\right), \left(\frac{b_2}{0.8}\right)\right\}$, $\epsilon = \left\{\left(\frac{b_1}{0.7}\right), \left(\frac{b_2}{0.9}\right)\right\}$, $\rho = \left\{\left(\frac{b_1}{0.6}\right), \left(\frac{b_2}{0.7}\right)\right\}$ and $\pi = \left\{\left(\frac{b_1}{0.7}\right), \left(\frac{b_2}{0.7}\right)\right\}$ on $\mathcal{B}$. Let $\mathcal{T}_1 = \{0, \alpha, \text{BETA}, 1\}$, $\mathcal{T}_2 = \{0, \gamma, \theta, 1\}$, $\mathcal{T}_1{}' = \{0, \pi, \epsilon, 1\}$ and $\mathcal{T}_2{}' = \{0, \rho, \pi, 1\}$ be the FT's given on $\mathcal{A}$ and $\mathcal{B}$. Then, consider $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (\mathcal{B}, \mathcal{T}_1{}', \mathcal{T}_2{}')$ Such that, $\mathcal{F}(a_1) = b_1$ and $\mathcal{F}(a_2) = b_2$. Therefore $\mathcal{F}^{-1}(0) = 1'$, $\mathcal{F}^{-1}(1) = 0'$, $\mathcal{F}^{-1}(\pi) = \left\{\left(\frac{a_1}{0.7}\right), \left(\frac{a_2}{0.8}\right)\right\}$, $\mathcal{F}^{-1}(\epsilon) = \left\{\left(\frac{a_1}{0.7}\right), \left(\frac{a_2}{0.9}\right)\right\}$, $\mathcal{F}^{-1}(\rho) = \left\{\left(\frac{a_1}{0.6}\right), \left(\frac{a_2}{0.7}\right)\right\}$ and $\mathcal{F}^{-1}(\pi) = \left\{\left(\frac{a_1}{0.7}\right), \left(\frac{a_2}{0.7}\right)\right\}$. Hence, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (\mathcal{B}, \mathcal{T}_1{}', \mathcal{T}_2{}')$ is $\mathcal{PwFSCM}$ ($\mathcal{PwF}\alpha\mathcal{CM}$) but not $\mathcal{PwFCM}$, because π, ϵ, ρ and φ are $\mathbb{F}_{(i,j)} - \mathbb{SOS}$ ($\mathbb{F}_{(i,j)} - \alpha\mathbb{OS}$), but not $\mathbb{F}_{(i,j)} - \mathbb{OS}$ in $\mathcal{A}$.

Remark 4.2: $\mathcal{PwFCM} \Rightarrow\not\Leftarrow \mathcal{PwFPCM}$ ($\mathcal{PwF}\beta\mathcal{CM}$), which is shown in Example 4.8

Example 4.8: In Example 4.7, the mapping, $\mathcal{F}: (\mathcal{A}, \mathcal{T}_1, \mathcal{T}_2) \rightarrow (\mathcal{B}, \mathcal{T}_1{}', \mathcal{T}_2{}')$ is $\mathcal{PwFPCM}$ ($\mathcal{PwF}\beta\mathcal{CM}$) but not $\mathcal{PwFCM}$, because π, ϵ, ρ and φ are $\mathbb{F}_{(i,j)} - \mathbb{POS}$ ($\mathbb{F}_{(i,j)} - \beta\mathbb{OS}$), but not $\mathbb{F}_{(i,j)} - \mathbb{OS}$ in $\mathcal{A}$.

5. CONCLUSION

The proposed fuzzy bi-topological spaces offer a flexible and powerful model for dealing with the problem of ambiguity and unpredictability. In the fusion of fuzzy logic with dual topological structures, FbTS enhances classical topology and allows us to more easily analyze complex systems in more than one dimension. In this paper, the understanding of fuzzy open sets and mappings has been investigated, and their fundamental features and the relationships between open sets and mappings are developed on the basis of the FbTS model.

In addition, connections between various classes of fuzzy open sets with fuzzy continuous maps were formed (with appropriate examples to explain the connections). In addition, their results give the theoretical footing for fuzzy topology and to understand structural behavior in these spaces

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