Characterisation of Somewhat Generalized Fuzzy (Swgfcm) and Somewhat Pairwise Fuzzy Continuous Maps (SWPWGFCM)

Mallikarjun N Hottinavar[1], Veeresh A Sajjanara[2]

[1,2]Department of Mathematics, School of Engineering, Presidency University, Bengaluru, Karnataka - 560064, India

Email: mallikarjunhottinavar@gmail.com / veeresha.sajjanara@presidencyuniversity.in

ABSTRACT: In this paper we present and describe the concept of somewhat generalized fuzzy continuous maps and somewhat pairwise generalized fuzzy continuous maps. Several significant results are developed, accompanied by appropriate examples that illustrate the underlying concepts and their applicability. Topologists have extensively studied various generalizations of continuous mappings in fuzzy topology, which have contributed to the advancement of modern technology. Topology, especially fuzzy topology, is fundamentally about concepts that are connected to continuity directly or indirectly.

KEYWORDS: Fuzzy topology, generalized fuzzy continuity, somewhat continuous maps, somewhat pairwise generalized fuzzy continuous maps fuzzy open sets, continuity concepts, topological structures

How to cite this article: Hottinavar MN, Sajjanara VA. Characterisation of Somewhat Generalized Fuzzy (Swgfcm) and Somewhat Pairwise Fuzzy Continuous Maps (Swpwgfcm). Int J Drug Deliv Technol. 2026;16(16s): 381-386. DOI: 10.25258/ijddt.16.16s.40

1. INTRODUCTION

In topological topology, continuous mappings are the key to retaining all the basic properties including convergence, compactness, and structural stability. They preserve the fundamental relationships in a space, which allows us to study invariants during transformation. Continuity thus constitutes a building block of classical topological theory, and forms the basis of various applications in mathematics and applied science fields. As fuzzy topological spaces developed, the classical idea of continuity was widely generalized in order to provide a treatment of uncertainty and imprecision. These fuzzy frameworks favor graded membership over crisp classification, leading to broader and more accurate mathematical models [9], [16]. Many extensions of continuity, including fuzzy semi-continuity, pre-continuity, weak continuity, and almost continuity in this generalized context, have been developed to better mimic even subtle transitions in the structure. Some examples are Azad [1], Bin Shahna [2], and Mukherjee and Sinha [3] who have proposed and investigated weaker forms of fuzzy continuous mappings. Further work included work on interrelations among different sorts of fuzzy continuity by Thakur and Saraf [4], [5] and Allam [6] that gave a better picture about their structural behavior. Such general mappings provide further power for the modeling of gradual transition and can be particularly helpful in the face of uncertainty, for example in the domain of decision-making and control. The analysis of fuzzy bi-topological spaces builds on this framework and considers two fuzzy topologies on a common object, thus allowing simultaneous consideration of different concepts of openness and continuity. Proposed by Kandil, this framework offers a more enriched environment for the analysis of interactions among the competing fuzzy topological entities. In this kind of space, all the types of continuity (both pairwise continuity and generalized continuity) are possible to be treated together [14], [17]. This two-stage approach allows for capturing systems where the approximation or convergence criteria can overlap, making this method an especially useful one in complex and multi-dimensional systems. In this broad approach, somewhat generalized fuzzy continuous maps appear as a natural extension of the current continuum notions. These mappings serve as an intermediate framework between a rigid continuity and a weaker generalized version, covering a more fine grained relationship among fuzzy topological structures. The theoretical basis for this kind of generalization is largely related to the theory of generalized topologies first introduced by Császár [11], [12] which codified general notions of openness and continuity more generally. Moreover, intermediate concepts of fuzzy weakly continuous mappings [15] and pairwise fuzzy continuity [17] have been added as well.

Hence, exploration of somewhat generalized fuzzy continuous maps is also needed for developing fuzzy topological theory. Not only does this provide a theoretical enrichment to the work of generalized continuity, but it extends the scope of fuzzy topological models on practical issues. This paper will thus continue in this direction by defining and describing these mappings within a well-defined fuzzy topological framework with evidence of rigorous results and examples.

2. PRELIMINARIES

Definition 2.1: Let X be a non-empty universal crisp set. A **fuzzy topology** on X is a non-empty collection τ of fuzzy sets on X satisfying the following conditions

i) Fuzzy sets 0 and 1 belong to τ
ii) Any arbitrary union of members of τ is in τ
iii) A finite intersection of members of τ is in τ

Here 0 and 1 represent the Zero Fuzzy Set and the Whole Fuzzy set on X, defined as, 0(x)=0,∀ x ∈ X 1(x)=1, ∀ x ∈ X and the pair (X,τ) is called Fuzzy Topological Space on X. For Convenience, we shall denote the fuzzy topological space simply as X.

Definition 2.2: Let X be a crisp set and let "$\mathcal{T}$" be a collection of fuzzy sets on X. Then "$\mathcal{T}$" is called $G_{\mathcal{F}}-$topologyon X if it satisfies following conditions

i) The fuzzy sets 0 and 1 are in $'\mathcal{T}'$ where $0,1: X \to I$ are defined as $0(x) = 0$ and $1(x) = 1$ for all $x \in X$
ii) If $\{\lambda_j\}$, $j \in J$ is any family of fuzzy sets on X where $\lambda_j \in \mathcal{T}$ then $\cup_{j\in J} \lambda_j \in \mathcal{T}$

The pair $(X,\mathcal{T})$ is called $G_{\mathcal{F}}-$ topological space

Definition 2.3: Let $(X,\mathcal{T})$ be $G_{\mathcal{F}}-$topological space. The members of the collection $'\mathcal{T}'$ are called $G_{\mathcal{F}}-$open sets in $G_{\mathcal{F}}-$topological space. The complement of $G_{\mathcal{F}}-$open set in X is called $G_{\mathcal{F}}-$ close set.

Definition 2.4: Let $(X,\mathcal{T})$ be $G_{\mathcal{F}}-$topological space. For a fuzzy set A in X the Closure of A is defined as $Cl_{\mathcal{T}}(A) = \inf\{K : A \subseteq K, K^C \in \mathcal{T}\}$. Thus $Cl_{\mathcal{T}}(A)$ is the smallest closed set in X containing the fuzzy $G_{\mathcal{F}}-$open set A. From the definition, if follows that $Cl_{\mathcal{T}}(A)$ is the intersection of all $G_{\mathcal{F}}-$closed sets in X containing A.

Definition 2.5: Let $(X,\mathcal{T})$ be $G_{\mathcal{F}}-$topological space. For a fuzzy set A in X, the interior of A, is defined as $I_{\mathcal{T}}(A) = \mathrm{Sup}\{Q : Q \subseteq A, Q \in \mathcal{T}\}$. Thus $I_{\mathcal{T}}(A)$ is the largest $G_{\mathcal{F}}-$open set in X contained in the fuzzy set A. From the definition, if follows that $I_{\mathcal{T}}(A)$ is the union of all $G_{\mathcal{F}}-$open set in X contained in A.

Remark 2.1: Let $(X,\mathcal{T})$ be$G_{\mathcal{F}}-$topological space. Then:

i) 0 and 1 are fuzzy $G_{\mathcal{F}}-$closed sets in X.
ii) Arbitrary intersection of $G_{\mathcal{F}}-$closed sets in X is $G_{\mathcal{F}}-$closed set in X.

Definition 2.6: Let $(X,\mathcal{T})$ be $G_{\mathcal{F}}-$topological space. Then a fuzzy set A in X is called $G_{\mathcal{F}}-\alpha-$open set if $A \subseteq I_{\mathcal{T}}(Cl_{\mu}(I_{\mathcal{T}}(A)))$. The fuzzy set λ in X is called $G_{\mathcal{F}}-\alpha-$closed set if λ^c is $G_{\mathcal{F}}-\alpha-$open set in X.

Definition 2.7: Let $(X,\mathcal{T})$ be $G_{\mathcal{F}}-$ topological space. Then a fuzzy set λ in X is called $G_{\mathcal{F}}-$semi open set if $\lambda \subseteq Cl_{\mathcal{T}}(I_{\mathcal{T}}(\lambda))$. The fuzzy set λ in X is called $G_{\mathcal{F}}-$semi closed set if λ^c is $G_{\mathcal{F}}-$semi open set in X.

Definition 2.8: Let $(X,\mathcal{T})$ be $G_{\mathcal{F}}-$ topological space. Then a fuzzy $\mathcal{S}$et λ in X is called $G^1_{\mathcal{F}}-$pre-open set if $\lambda \subseteq I_{\mathcal{T}}(Cl_{\mathcal{T}}(\lambda))$. The fuzzy set λ in X is called $G_{\mathcal{F}}-\beta-$open set if $\lambda \subseteq Cl_{\mathcal{T}}(I_{\mathcal{T}}(Cl_{\mathcal{T}}(\lambda)))$.

Definition 2.9: Let$(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two $G_{\mathcal{F}}-$topological spaces and let $\mathcal{F}: X \to Y$ be a mapping from set X to set Y. Then f is called $G_{\mathcal{F}}-$continuous mapping if $f^{-1}(\lambda)$ is $G_{\mathcal{F}}-$open in $(X,\mathcal{T}_1)$ for each $G_{\mathcal{F}}-$open set λ in $(Y,\mathcal{T}_2)$.

Remark 2.2: Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two $G_{\mathcal{F}}-$topological spaces and let $\mathcal{F}: X \to Y$ be a mapping from set X to set Y. Then following statements are all equivalent:

i) $\mathcal{F}$ is fuzzy continuous.
ii) For each fuzzy closed set λ in Y, $\mathcal{F}^{-1}(\lambda)$ is fuzzy closed set in X.

Remark 2.3: Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ be two $G_{\mathcal{F}}-$topological spaces and let $\mathcal{F}: X \to Y$ be a mapping from set X to set Y. Let B be a basis for $\mathcal{T}_2$ if for each $G_{\mathcal{F}}-$ basic open set μ in B, $\mathcal{F}^{-1}(\mu)$ is fuzzy open in X, then $\mathcal{F}: X \to Y$ is $G_{\mathcal{F}}-$continuous.

Remark 2.4: Let $(X,\mathcal{T}_1)$ and $(Y,\mathcal{T}_2)$ and $(Z,\mathcal{T}_3)$ be $G_{\mathcal{F}}-$topological spaces and $\mathcal{F}: X \to Y$, $\mathcal{G}: Y \to Z$, be maps. If '$\mathcal{F}$' and '$\mathcal{G}$' are $G_{\mathcal{F}}-$continuous maps, then $\mathcal{G}o\mathcal{F}: X \to Y$ is also $G_{\mathcal{F}}-$continuous map.

3. SOMEWHAT GENERALIZED FUZZY CONTINUOUS MAPS ($\mathcal{SWGFCM}$)

Definition 3.1: Suppose (Λ, t_Λ) and (Ω, t_Ω) be $\mathcal{GFTS}$ and suppose $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ be a map from (Λ, t_Λ) to (Ω, t_Ω). Then $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is called somewhat generalized fuzzy continuous map ($\mathcal{SWGFCM}$) if $\alpha \neq 0_\Omega$ is $\mathcal{GFOS}$ on "Ω" and $\mathcal{F}^{-1}(\alpha) \neq 0_\Lambda$, then $\exists$ "$\mathcal{GFOS}$" $\beta \neq 0_\Lambda$ on "Λ" such that $\beta \subseteq \mathcal{F}^{-1}(\alpha)$

Definition 3.2: Suppose (Λ, t_Λ) and (Ω, t_Ω) be $\mathcal{GFTS}$ and suppose $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ be a map from (Λ, t_Λ) to (Ω, t_Ω). Then $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is called somewhat generalized fuzzy pre-continuous map ($\mathcal{SWGFPCM}$) if $\alpha \neq 0_\Omega$ is $\mathcal{GFOS}$ on "Ω" and $\mathcal{F}^{-1}(\alpha) \neq 0_\Lambda$, then $\exists$ "$\mathcal{GFPOS}$" $\beta \neq 0_\Lambda$ on "Λ" such that $\beta \subseteq \mathcal{F}^{-1}(\alpha)$

Proposition 3.1: $\mathcal{GFCM} \Rightarrow\not\Leftarrow \mathcal{SWGFPCM}$

Example 3.1: Suppose $\Lambda = \{\Lambda_1, \Lambda_2\}$ and $\Omega = \{\Omega_1, \Omega_2\}$ such that $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.3}\right), \left(\frac{\Lambda_2}{0.6}\right)\right\}, \alpha_2 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.4}\right)\right\}, \alpha_3 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.6}\right)\right\}, \alpha_4 = \left\{\left(\frac{\Lambda_1}{0.2}\right), \left(\frac{\Lambda_2}{0.5}\right)\right\}, \alpha_5 = \left\{\left(\frac{\Lambda_1}{0.6}\right), \left(\frac{\Lambda_2}{0.3}\right)\right\}, \alpha_6 = \left\{\left(\frac{\Lambda_1}{0.6}\right), \left(\frac{\Lambda_2}{0.5}\right)\right\}$. Then $t_\Lambda = \{0,1,\alpha_1,\alpha_2,\alpha_3\}$ and $t_\Omega = \{0,1,\alpha_4,\alpha_5,\alpha_6\}$ are $\mathcal{GFTS}$ on Λ and Ω respectively. Now, consider the map $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ defined by $\mathcal{F}(\Lambda_1) = \Omega_1$ and $\mathcal{F}(\Lambda_2) = \Omega_2$. We observe that $\alpha = \left\{\left(\frac{\Lambda_1}{0.6}\right), \left(\frac{\Lambda_2}{0.6}\right)\right\} \neq 0_\Omega$ is $\mathcal{GFOS}$ on "Ω" therefore $\exists$ "$\mathcal{GFPOS}$" $\beta = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.4}\right)\right\} \neq 0_\Lambda$, such that $\beta \subseteq \mathcal{F}^{-1}(\alpha)$, Therefore, $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is $\mathcal{SWGFPCM}$ but not $\mathcal{GFCM}$, because $\alpha = \left\{\left(\frac{\Lambda_1}{0.6}\right), \left(\frac{\Lambda_2}{0.6}\right)\right\}$ is not $\mathcal{GFOS}$ on "Ω

Definition 3.3: Suppose (Λ, t_Λ) and (Ω, t_Ω) be $\mathcal{GFTS}$ and suppose $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ be a map from (Λ, t_Λ) to (Ω, t_Ω). Then $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is called somewhat generalized fuzzy semi-continuous map ($\mathcal{SWGFPCM}$) if $\alpha \neq 0_\Omega$ is $\mathcal{GFOS}$ on "Ω" and $\mathcal{F}^{-1}(\alpha) \neq 0_\Lambda$, then $\exists$ "$\mathcal{GFSOS}$" $\beta \neq 0_\Lambda$ on "Λ" such that $\beta \subseteq \mathcal{F}^{-1}(\alpha)$

Proposition 3.2: $\mathcal{GFCM} \Rightarrow\not\Leftarrow \mathcal{SWGFSCM}$

Example 3.2: We observe from 3.1, example that the $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is $\mathcal{SWGFPCM}$ but not $\mathcal{GFCM}$, reason is already mention there.

Definition 3.4: Suppose (Λ, t_Λ) and (Ω, t_Ω) be $\mathcal{GFTS}$ and suppose $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ be a map from (Λ, t_Λ) to (Ω, t_Ω). Then $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is called somewhat generalized fuzzy beta-continuous map ($\mathcal{SWGF\beta CM}$) if $\alpha_1 \neq 0_\Omega$ is $\mathcal{GFOS}$ on "Ω" and $\mathcal{F}^{-1}(\alpha_1) \neq 0_\Lambda$, then $\exists$ "$\mathcal{GF\beta OS}$" $\alpha_2 \neq 0_\Lambda$ on "Λ" such that $\alpha_2 \subseteq \mathcal{F}^{-1}(\alpha_1)$

Proposition 3.3: $\mathcal{GFCM} \Rightarrow\not\Leftarrow \mathcal{SWGF\beta CM}$

Example 3.3: We observe from 3.1, example that, the $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is $\mathcal{SWGF\beta CM}$ but not $\mathcal{GFCM}$, reason is already mention there.

Definition 3.5: Suppose (Λ, t_Λ) and (Ω, t_Ω) be $\mathcal{GFTS}$ and suppose $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ be a map from (Λ, t_Λ) to (Ω, t_Ω). Then $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is called somewhat generalized fuzzy alpha-continuous map ($\mathcal{SWGF\alpha CM}$) if $\alpha_1 \neq 0_\Omega$ is $\mathcal{GFOS}$ on "Ω" and $\mathcal{F}^{-1}(\alpha_1) \neq 0_\Lambda$, then $\exists$ "$\mathcal{GF\beta OS}$" $\alpha_2 \neq 0_\Lambda$ on "Λ" such that $\alpha_2 \subseteq \mathcal{F}^{-1}(\alpha_1)$

Remark 3.4: $\mathcal{GFCM} \Rightarrow\not\Leftarrow \mathcal{SWGF\alpha CM}$

Example 3.4: We observe from 3.1, example that, the $\mathcal{F}: (\Lambda, t_\Lambda) \to (\Omega, t_\Omega)$ is $\mathcal{SWGF\alpha CM}$ but not $\mathcal{GFCM}$, reason is already mention there.

4. SOMEWHAT PAIRWISE GENERALIZED FUZZY CONTINUOUS MAPS ($\mathcal{SWPWGFCM}$)

Definition 4.1: A mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is called somewhat pairwise generalized fuzzy continuous map ($\mathcal{SWPWGFCM}$) if $\exists$ $t_{\Lambda_i} - \mathcal{GFOS}$ for (i = 1,2), $\alpha \neq 0_\Lambda$ on $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$, such that $\mathcal{F}^{-1}(\beta) \neq 0_\Lambda$ for any $t_{\Omega_i} - \mathcal{GFOS}$ for (i = 1,2), $\beta \neq 0_\Omega$

Example 4.1: Suppose $\Lambda = \{\Lambda_1, \Lambda_2\}$ and $\Omega = \{\Omega_1, \Omega_2\}$, such that $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.4}\right), \left(\frac{\Lambda_2}{0.5}\right)\right\}$, $\alpha_2 = \left\{\left(\frac{\Lambda_1}{0.3}\right), \left(\frac{\Lambda_2}{0.4}\right)\right\}, \alpha_3 = \left\{\left(\frac{\Omega_1}{0.4}\right), \left(\frac{\Omega_2}{0.4}\right)\right\}$, $\alpha_4 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.5}\right)\right\}$. Then $t_{\Lambda_1} = \{0,1,\alpha_1\}$, $t_{\Lambda_2} = \{0,1,\alpha_2\}, t_{\Omega_1} = \{0,1,\alpha_3\}, t_{\Omega_2} = \{0,1,\alpha_4\}$ are $\mathcal{GFTS}$ on Λ and Ω. Then the mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ defined as $\mathcal{F}(\Lambda_1) = \Omega_1$, $\mathcal{F}(\Lambda_2) = \Omega_2$ is $\mathcal{SWPWGFCM}$

Definition 4.2: The $\mathcal{FS}$ "α" of $\mathcal{GFBTS}$, $(\Lambda, t_\Lambda, t_\Omega)$ is called pairwise dense generalized fuzzy set ($\mathcal{PWDGFS}$) if $\exists$ $t_{\Lambda_i} - \mathcal{GFCS}$, (i = 1,2), "$\beta$" such that $\alpha \subseteq \beta \subseteq 1$

Definition 4.3: A mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is called somewhat pairwise generalized fuzzy pre-irresolute map ($\mathcal{SWPWGFIM}$) if $\exists$ $t_{\Lambda_i} - \mathcal{GFOS}$ for (i = 1,2), $\alpha \neq 0_\Lambda$ on $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$, such that $\alpha \subseteq \mathcal{F}^{-1}(\beta) \neq 0_\Lambda$ for any $t_{\Omega_i} - \mathcal{GFPOS}$ for (i = 1,2), $\beta \neq 0_\Omega$ on $(\Omega, t_{\Omega_1}, t_{\Omega_2})$

Example 4.2: Suppose $\Lambda = \{\Lambda_1, \Lambda_2, \Lambda_3\}$ and $\Omega = \{\Omega_1, \Omega_2, \Omega_3\}$ such that $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.1}\right), \left(\frac{\Lambda_2}{0.1}\right), \left(\frac{\Lambda_3}{0.1}\right)\right\}, \alpha_2 = \left\{\left(\frac{\Lambda_1}{0.3}\right), \left(\frac{\Lambda_2}{0.3}\right), \left(\frac{\Lambda_3}{0.3}\right)\right\}, \alpha_3 = \left\{\left(\frac{\Lambda_1}{0.8}\right), \left(\frac{\Lambda_2}{0.8}\right), \left(\frac{\Lambda_3}{0.8}\right)\right\}$, $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.9}\right), \left(\frac{\Lambda_2}{0.9}\right), \left(\frac{\Lambda_3}{0.9}\right)\right\}$. Then $t_{\Lambda_1} = \{0,1,\alpha_1\}$, $t_{\Lambda_2} = \{0,1,\alpha_2\}$, $t_{\Omega_1} = \{0,1,\alpha_3\}$, $t_{\Omega_2} = \{0,1,\alpha_4\}$ are $\mathcal{GFTS}$ on Λ and Ω. Then the mapping

$\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}, t_{\Lambda_3}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2}, t_{\Omega_3})$ defined as $\mathcal{F}(\Lambda_1) = \Omega_1$, $\mathcal{F}(\Lambda_2) = \Omega_2$, $\mathcal{F}(\Lambda_3) = \Omega_3$ is $\mathcal{SWPWGFIM}$

Definition 4.4: A mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is called pairwise generalized fuzzy almost continuous map ($\mathcal{PWGFACM}$) if $\mathcal{F}^{-1}(\alpha)$ is $t_{\Lambda_i} - \mathcal{GFOS}$ for (i = 1,2) in $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$ for every $(i,j) - \mathcal{GFROS}$ in $(\Omega, t_{\Omega_1}, t_{\Omega_2})$

Example 4.3: Suppose $\Lambda = \{\Lambda_1, \Lambda_2\}$ and $\Omega = \{\Omega_1, \Omega_2\}$, such that $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.6}\right), \left(\frac{\Lambda_2}{0.5}\right)\right\}$, $\alpha_2 = \left\{\left(\frac{\Lambda_1}{0.3}\right), \left(\frac{\Lambda_2}{0.5}\right)\right\}$, $\alpha_3 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.4}\right)\right\}$, $\alpha_4 = \left\{\left(\frac{\Omega_1}{0.6}\right), \left(\frac{\Omega_2}{0.5}\right)\right\}$, $\alpha_5 = \left\{\left(\frac{\Omega_1}{0.3}\right), \left(\frac{\Omega_2}{0.5}\right)\right\}$. Then $t_{\Lambda_1} = \{0,1,\alpha_1,\alpha_3\}$, $t_{\Lambda_2} = \{0,1,\alpha_2\}$, $t_{\Omega_1} = \{0,1,\alpha_3\}$, $t_{\Omega_2} = \{0,1,\alpha_4,\alpha_5\}$ are $\mathcal{GFTS}$ on Λ and Ω. Then the mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ defined as $\mathcal{F}(\Lambda_1) = \Omega_1, \mathcal{F}(\Lambda_2) = \Omega_2$ is $\mathcal{PWGFACM}$, because $0,1,\alpha_3$ are $(1,2) - \mathcal{GFROS}$ in $(\Omega, t_{\Omega_1}, t_{\Omega_2})$. Also $\mathcal{F}^{-1}(0)$, $\mathcal{F}^{-1}(1)$ and $\mathcal{F}^{-1}(\alpha_3) = \alpha_2$ are $t_{\Lambda_2} - \mathcal{GFOS}$ in $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$

Definition 4.5: A mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is called pairwise* generalized fuzzy continuous map ($\mathcal{PW^*GFCM}$) if $\mathcal{F}^{-1}(\alpha)$ is $t_{\Lambda_i} - \mathcal{GFOS}$ for (i = 1,2) in $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$ for every $(i,j) - \mathcal{GFOS}$ in $(\Omega, t_{\Omega_1}, t_{\Omega_2})$

Theorem 4.1: $\mathcal{PWGFCM} \Rightarrow\not\Leftarrow \mathcal{PW^*GFCM}$

Proof: Suppose $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PWGFCM}$. Then for every "α" in t_{Ω_1} , $\mathcal{F}^{-1}(\alpha)$ lies in t_{Λ_1} $\Rightarrow$ $\mathcal{F}^{-1}(\alpha)$ is a $t_{\Lambda_1} - \mathcal{GFOS}$ or $t_{\Lambda_2} - \mathcal{GFOS}$ on $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$ for any any $t_{\Omega_1} - \mathcal{GFOS}$ or $t_{\Omega_2} - \mathcal{GFOS}$ "α" on $(\Omega, t_{\Omega_1}, t_{\Omega_2})$. Thus $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PW^*GFCM}$

Example 4.4`: Suppose $\Lambda = \{\Lambda_1, \Lambda_2, \Lambda_3\}$ and $\Omega = \{\Omega_1, \Omega_2, \Omega_3\}$ such that, $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.4}\right), \left(\frac{\Lambda_2}{0.5}\right), \left(\frac{\Lambda_3}{0.5}\right)\right\}$, $\alpha_2 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.5}\right), \left(\frac{\Lambda_3}{0.5}\right)\right\}$, $\alpha_3 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.6}\right), \left(\frac{\Lambda_3}{0.6}\right)\right\}$, $\alpha_4 = \left\{\left(\frac{\Omega_1}{0.4}\right), \left(\frac{\Omega_2}{0.5}\right), \left(\frac{\Omega_3}{0.4}\right)\right\}$, $\alpha_5 = \left\{\left(\frac{\Omega_1}{0.5}\right), \left(\frac{\Omega_2}{0.6}\right), \left(\frac{\Omega_3}{0.6}\right)\right\}$. Then $t_{\Lambda_1} = \{0,1,\alpha_1,\alpha_2\}$, $t_{\Lambda_2} = \{0,1,\alpha_3\}$, $t_{\Omega_1} = \{0,1,\alpha_4\}$, $t_{\Omega_2} = \{0,1,\alpha_5\}$ are $\mathcal{GFTS}$ on Λ and Ω. Then consider, the mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ defined as, $\mathcal{F}(\Lambda_1) = \Omega_1$, $\mathcal{F}(\Lambda_2) = \Omega_2$, $\mathcal{F}(\Lambda_3) = \Omega_3$. Implies $\mathcal{F}^{-1}(\alpha_4) \in t_{\Lambda_1}$ and $\mathcal{F}^{-1}(\alpha_5) \in t_{\Lambda_1}$. Implies $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PW^*GFCM}$ but not $\mathcal{PWGFCM}$, because $\mathcal{F}^{-1}(\alpha_5) \notin t_{\Lambda_2}$

Definition 4.6: A mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is called pairwise* generalized fuzzy completely continuous map ($\mathcal{PW^*GFCCM}$) if $\mathcal{F}^{-1}(\alpha)$ is $t_{\Lambda_i} - \mathcal{GFROS}$ for (i = 1,2) in $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$ for every $(i,j) - \mathcal{GFOS}$ in $(\Omega, t_{\Omega_1}, t_{\Omega_2})$

Theorem 4.2: $\mathcal{PW^*GFCCM} \Rightarrow\not\Leftarrow \mathcal{PW^*GFCM}$

Proof: Suppose $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PW^*GFCCM}$. Then for every "α" in t_{Ω_1} , $\mathcal{F}^{-1}(\alpha)$ lies in t_{Λ_1}, $\Rightarrow$ $\mathcal{F}^{-1}(\alpha)$ is a $t_{\Lambda_1} - \mathcal{GFROS}$ or $t_{\Lambda_2} - \mathcal{GFROS}$ on $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$ for any any $t_{\Omega_1} - \mathcal{GFOS}$ or $t_{\Omega_2} - \mathcal{GFOS}$ "α" on $(\Omega, t_{\Omega_1}, t_{\Omega_2})$. Thus $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PW^*GFCM}$

Example 4.5: Suppose $\Lambda = \{\Lambda_1, \Lambda_2, \Lambda_3\}$ and $\Omega = \{\Omega_1, \Omega_2, \Omega_3\}$ such that $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.5}\right), \left(\frac{\Lambda_3}{0.5}\right)\right\}$, $\alpha_2 = \left\{\left(\frac{\Lambda_1}{0.4}\right), \left(\frac{\Lambda_2}{0.4}\right), \left(\frac{\Lambda_3}{0.5}\right)\right\}$, $\alpha_3 = \left\{\left(\frac{\Lambda_1}{0.3}\right), \left(\frac{\Lambda_2}{0.3}\right), \left(\frac{\Lambda_3}{0.3}\right)\right\}$, $\alpha_4 = \left\{\left(\frac{\Lambda_1}{0.4}\right), \left(\frac{\Lambda_2}{0.4}\right), \left(\frac{\Lambda_3}{0.4}\right)\right\}$, $\alpha_5 = \left\{\left(\frac{\Omega_1}{0.4}\right), \left(\frac{\Omega_2}{0.4}\right), \left(\frac{\Omega_3}{0.4}\right)\right\}$, $\alpha_6 = \left\{\left(\frac{\Omega_1}{0.4}\right), \left(\frac{\Omega_2}{0.4}\right), \left(\frac{\Omega_3}{0.5}\right)\right\}$, $\alpha_7 = \left\{\left(\frac{\Omega_1}{0.3}\right), \left(\frac{\Omega_2}{0.3}\right), \left(\frac{\Omega_3}{0.3}\right)\right\}$. Then $t_{\Lambda_1} = \{0,1,\alpha_1,\alpha_2\}$, $t_{\Lambda_2} = \{0,1,\alpha_3,\alpha_4\}$, $t_{\Omega_1} = \{0,1,\alpha_5,\alpha_6\}$, $t_{\Omega_2} = \{0,1,\alpha_7\}$ are $\mathcal{GFTS}$ on Λ and Ω. Then consider, the mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ defined as $\mathcal{F}(\Lambda_1) = \Omega_1$, $\mathcal{F}(\Lambda_2) = \Omega_2$, $\mathcal{F}(\Lambda_3) = \Omega_3$. Implies $\mathcal{F}^{-1}(\alpha_5), \mathcal{F}^{-1}(\alpha_6) \in t_{\Lambda_1}$ and $\mathcal{F}^{-1}(\alpha_7) \in t_{\Lambda_2}$. Implies $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PW^*GFCM}$ but not $\mathcal{PW^*GFCCM}$, because $\mathcal{F}^{-1}(\alpha_6)$ is not $t_{\Lambda_i} - \mathcal{GFROS}$ for (i = 1,2)

Definition 4.7: A mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is called somewhat pairwise* generalized fuzzy pre-irresolute map ($\mathcal{SWPW^*GFIM}$) if $\exists$ $t_{\Lambda_i} - \mathcal{GFROS}$ for (i = 1,2), $\alpha \neq 0_\Lambda$ on $(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$, such that $\alpha \subseteq \mathcal{F}^{-1}(\beta) \neq 0_\Lambda$ for any $t_{\Omega_i} - \mathcal{GFPOS}$ for (i = 1,2), $\beta \neq 0_\Omega$ on $(\Omega, t_{\Omega_1}, t_{\Omega_2})$

Theorem 4.3: $\mathcal{PW^*GFCCM} \Rightarrow\not\Leftarrow \mathcal{SWPWGFCM}$

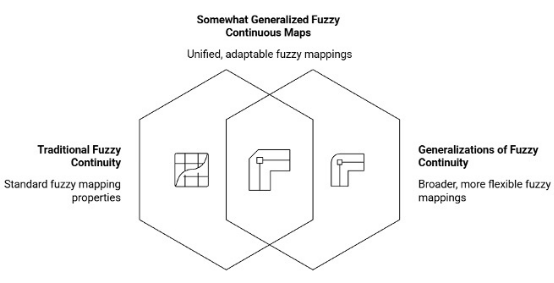
Proof: Suppose $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{PW^*GFCCM}$. Then for every "$\alpha \neq 0_\Omega$" is $t_{\Omega_1} - \mathcal{GFOS}$ or $t_{\Omega_2} - \mathcal{GFOS}$ in $(\Omega, t_{\Omega_1}, t_{\Omega_2})$ and $\mathcal{F}^{-1}(\alpha) \neq 0_\Lambda \Rightarrow$ $\mathcal{F}^{-1}(\alpha)$ is a $t_{\Lambda_1} - \mathcal{GFROS}$ or $t_{\Lambda_2} - \mathcal{GFROS}$ on

$(\Lambda, t_{\Lambda_1}, t_{\Lambda_2})$. Taking $\beta = \mathcal{F}^{-1}(\alpha)$ it follows that $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{SWPWGFCM}$

Example 4.6: Suppose $\Lambda = \{\Lambda_1, \Lambda_2, \Lambda_3\}$ and $\Omega = \{\Omega_1, \Omega_2, \Omega_3\}$ such that, $\alpha_1 = \left\{\left(\frac{\Lambda_1}{0.4}\right), \left(\frac{\Lambda_2}{0.4}\right), \left(\frac{\Lambda_3}{0.4}\right)\right\}$, $\alpha_2 = \left\{\left(\frac{\Lambda_1}{0.1}\right), \left(\frac{\Lambda_2}{0.1}\right), \left(\frac{\Lambda_3}{0.1}\right)\right\}$, $\alpha_3 = \left\{\left(\frac{\Lambda_1}{0.5}\right), \left(\frac{\Lambda_2}{0.5}\right), \left(\frac{\Lambda_3}{0.5}\right)\right\}$, $\alpha_4 = \left\{\left(\frac{\Omega_1}{0.5}\right), \left(\frac{\Omega_2}{0.5}\right), \left(\frac{\Omega_3}{0.5}\right)\right\}$, $\alpha_5 = \left\{\left(\frac{\Omega_1}{0.3}\right), \left(\frac{\Omega_2}{0.1}\right), \left(\frac{\Omega_3}{0.3}\right)\right\}$, $\alpha_6 = \left\{\left(\frac{\Omega_1}{0.5}\right), \left(\frac{\Omega_2}{0.4}\right), \left(\frac{\Omega_3}{0.5}\right)\right\}$. Then $t_{\Lambda_1} = \{0,1,\alpha_1\}$, $t_{\Lambda_2} = \{0,1,\alpha_2\}$, $t_{\Omega_1} = \{0,1,\alpha_4,\alpha_5\}$, $t_{\Omega_2} = \{0,1,\alpha_6\}$ are $\mathcal{GFTS}$ on Λ and Ω. Then consider, the mapping $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ defined as, $\mathcal{F}(\Lambda_1) = \Omega_1$, $\mathcal{F}(\Lambda_2) = \Omega_2$, $\mathcal{F}(\Lambda_3) = \Omega_3$. Thus we have $\alpha_2 \subseteq \mathcal{F}^{-1}(\alpha_4) = \alpha_3$, $\alpha_2 \subseteq \mathcal{F}^{-1}(\alpha_5) = \alpha_2$, $\alpha_2 \subseteq \mathcal{F}^{-1}(\alpha_6) = \alpha_1$. Implies $\mathcal{F}: (\Lambda, t_{\Lambda_1}, t_{\Lambda_2}) \to (\Omega, t_{\Omega_1}, t_{\Omega_2})$ is $\mathcal{SWPWGFCM}$ but not $\mathcal{PW^*GFCCM}$, because $\mathcal{F}^{-1}(\alpha_4)$ and $\mathcal{F}^{-1}(\alpha_6)$ is not $t_{\Lambda_i} - \mathcal{GFROS}$ for $(i = 1,2)$

5. CONCLUSION

We have investigated pairwise generalized fuzzy completely continuous maps and their structural properties in fuzzy bi-topological spaces. First, a somewhat pairwise fuzzy completely continuous map is introduced. Here are main findings and examples, with their examples and counter-examples provided through a discussion. The connection among the differences between several threads of continuity is reported and there is a need for exploring them (continuity, compactness, and discontinuities in complex mappings). These were significant progress which lay the groundwork for future work that may be employed in functional analysis and applied mathematics.



The results discussed in this paper are concluded as follows:

$$\mathcal{PWGFCM} \Rightarrow\nLeftarrow \mathcal{PWGFSCM}$$

$$\mathcal{PWGFCM} \Rightarrow\nLeftarrow \mathcal{PWGFPCM}$$

$$\mathcal{PWGFCM} \Rightarrow\nLeftarrow \mathcal{PW^*GFCM}$$

$$\mathcal{PW^*GFCCM} \Rightarrow\nLeftarrow \mathcal{SWPWGFCM}$$

REFERENCES

1. K. K. Azad, “On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity,” J. Math. Anal. Appl., 82 (1981), 14–32.
2. A. S. Bin Shahna, “On fuzzy strongly semi-continuity and fuzzy pre-continuity,” Fuzzy Sets and Systems, 44 (1991), 303–308.
3. M. N. Mukherjee and S. P. Sinha, “On some weaker forms of fuzzy continuous and fuzzy open mappings in fuzzy topological spaces,” Fuzzy Sets and Systems, 32 (1989), 103–114.
4. S. S. Thakur and R. K. Saraf, “Fuzzy almost semi-continuous mappings,” Delta J. Sci., 19 (1995), 507–509.
5. S. S. Thakur and R. K. Saraf, “Fuzzy pre-open mappings,” Acta Cien. Indica, 21M(4) (1996), 507–509.
6. A. A. Allam, “The interrelation among some types of fuzzy continuity,” Arabian J. Sci. Eng., 18(1) (1993), 57–61.
7. M. E. Abd El-Monsef, S. N. El-Deeb, and R. A. Mahmoud, “β-open sets and β-continuous mapping,” Bull. Fac. Sci. Assiut Univ. A, 12 (1983), 77–90.
8. A. S. Mashhour, M. E. Abd El-Monsef, and S. N. El-Deeb, “On pre-continuous and weak pre-continuous mappings,” Proc. Math. Phys. Soc. Egypt, 53 (1982), 47–53.
9. R. H. Warren, “Neighborhood bases and continuity in fuzzy topological spaces,” Rocky Mountain J. Math., 8 (1978), 459–470.
10. R. H. Warren, “Continuity of mappings of fuzzy topological spaces,” Notices Amer. Math. Soc., 21 (1974), 447.
11. A. Császár, “Generalized topology, generalized continuity,” Acta Math. Hungar., 96 (2002), 351–357.

12. A. Császár, "Generalized open sets," Acta Math. Hungar., 75 (1997), 65–87.
13. A. S. Bin Shahna, "Mappings in fuzzy topological spaces," Fuzzy Sets and Systems, 61 (1994), 209–213.
14. S. S. Thakur and R. Malviya, "Semi-open sets and semi continuity in fuzzy bitopological spaces," Fuzzy Sets and Systems, 79 (1996), 251–256.
15. M. E. Abd El-Monsef, I. M. Hanafy, and S. N. El-Deeb, "Fuzzy weakly continuous functions," Bull. Inst. Math. Acad. Sinica, 19 (1991), 75–85.
16. Z. G. Quan, "Fuzzy continuous functions and its properties," Fuzzy Sets and Systems, 43 (1991), 159–171.
17. S. Sampath Kumar, "On fuzzy pairwise α-continuity and fuzzy pairwise pre-continuity," Fuzzy Sets and Systems, 62 (1994), 231–238.