Strongly *($\hat{g}$ r_h) - Continuous Functions in Hexa Topological spaces

C. Dhanapakyam[1], J. Sathya Malar[2]

[1]Assistant Professor, Department of Mathematics, RVS College of Arts and Science, Coimbatore - 641 402, Tamil Nadu, India. Email: dhana.pakyam55@gmail.com

[2]Research Scholar, Department of Mathematics, RVS College of Arts and Science, Coimbatore - 641 402, Tamil Nadu, India. Email: sathyajmalar@gmail.com

Abstract

This paper studies the features of a novel family of functions called Strongly *($\hat{g}$ r_h) **-** continuous functions and perfectly *($\hat{g}$ r_h) -continuous functions. We also compare and relate these functions to other functions in hexa topological domains.

Keywords: Strongly *($\hat{g}$ r_h) continuous functions and perfectly *($\hat{g}$ r_h) -continuous functions.

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1.Introduction

The aims of semi-open sets and their properties were initiated by Levine [5] in 1963.During this period the single topology is extended to bi-topological space, The bi-topological space idea was first presented by Kelly [7], Tri-topological space was first started by Kovar [9], Quad topological space was researched by Mukundan [6] and "Penta topological space by Khan and G. A. Khan [8] and hexa - topological space by Chandra and Pushpalatha [3], introduced and researched the idea of h - open sets in h -topological spaces, h - continuous. *($\hat{g}$ r_h) - closed and *($\hat{g}$ r_h) - continuous was introduced by J. Sathya Malar et.al [10,11].

2. Preliminaries

Definition 2.1 A map topological space (Y', σ) is called f : $X' \to Y'$ from a topological space (X' , τ) into a topological space (Y', σ) is called

(i) Continuous if the inverse image of every closed set (or open set) in Y' is closed (or open) in X' .

(ii) generalized continuous [1] (g -continuous) if the inverse image of every closed set in Y' is g-closed in X' .

(iv) strongly g-continuous[1] if the inverse image of every g -open set in Y' open in X'. (v)perfectly g -continuous[1] if the inverse image of every g -open set in Y' is both open and closed in X'.

Definition 2.2 A subset A of a hexa topological space (X', τ_h) is called a hexa star generalized cap regular closed set [briefly $^*(\hat{g}r)_h$ -closed set], [10] if $rcl_h(A) \subseteq$ U whenever A ⊆ U and U is $\hat{g}_h$-open subset of X'.

Definition 2.3 A function f : (X', τ_h) → (Y', σ_h) is called $^*(\hat{g}r)_h$ -continuous [11] if f^{-1}(V) is $^*(\hat{g}r)_h$ -closed set in (X', τ_h) for every hexa closed set V in (Y', σ_h).

Remark 2.4 (i) every hexa closed set is $^*(\hat{g}r)_h$ -closed and

(ii) every g_h-closed set is $^*(\hat{g}r)_h$ -closed.

3. Strongly $^*(\hat{g}r)_h$ – Continuous Functions

Definition 3.1

A function f: (X', τ_h) ⟶ (Y', σ_h) is called Strongly $^*(\hat{g}r)_h$ -continuous functions if f^{-1} (V) is $^*(\hat{g}r)_h$ - closed in (Y', σ_h) is hexa closed in (X', τ_h)

Theorem 3.2 Every strongly $^*(\hat{g}r)_h$- continuous function is strongly g_h- continuous .

Proof: Let V be a subset of (Y', σ_h). since f is strongly $^*(\hat{g}r)_h$ -continuous, then f^{-1} (V) is $^*(\hat{g}r)_h$ closed in (Y', σ_h). Since every $^*(\hat{g}r)_h$ is closed set is g_h -closed, [10] f^{-1} (V) is g_h closed in X'. Hence f is strongly g_h-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.3

Let $X' = Y'$ = {a, b, c} , $\tau_1 = \tau_5$ = {X', ∅ ,{b}},$\tau_2 = \tau_3$ ={ X', ∅ ,{a, b}}, τ_4 ={ X', ∅ ,{b},{a, b}}, τ_6 ={X', ∅ }with τ_h = {∅,{ b}, {a, b }, X'} and σ_h= { ∅, {b}, {a, c }, Y' }.Let the function f : (X', τ_h) → (Y', σ_h) be defined by f(a) = c, f(b) = a, f(c) =b then f is strongly g_h-continuous but not strongly $^*(\hat{g}r)_h$ -continuous. Since for the hexa closed set { b} in Y', f^{-1} ({b}) = {c} is g_h–closed, but not $^*(\hat{g}r)_h$ -closed set in (X', τ_h).

Theorem 3.4 Every strongly $^*(\hat{g}r)_h$ - continuous function is strongly gs_h- continuous .

Proof: Let V be a subset of (Y', σ_h). since f is strongly $^*(\hat{g}r)_h$ -continuous, then f^{-1} (V) is $^*(\hat{g}r)_h$ - closed in (Y', σ_h). Since every $^*(\hat{g}r)_h$ is closed set is gs_h -closed, f^{-1} (V) is gs_h closed in (X', τ_h). Hence f is strongly gs_h -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example3.5

Let $X' = Y' = \{a, b, c\}$, $\tau_1 = \tau_5 = \{X', \emptyset, \{b\}\}$, $\tau_2 = \{X', \emptyset, \{a, c\}\}$, $\tau_3 = \tau_4 = \{X', \emptyset, \{b\}, \{a, c,\}\}$, $\tau_6 = \{X', \emptyset\}$ with $\tau_h = \{\emptyset, \{b\}, \{a, c\}, X'\}$ and $\sigma_h = \{\emptyset, \{b\}, \{b, c\}, Y'\}$. Let the function $f : (X', \tau_h) \to (Y', \sigma_h)$ be defined by f(a) = b, f(b) = c, f(c) = a then f is strongly gs_h-continuous but not strongly $^*(\hat{g}r)_h$ -continuous. Since for the hexa closed set { a } in Y', $f^{-1}(\{a\}) = \{c\}$ is gs_h –closed, but not $^*(\hat{g}r)_h$ -closed set in (X', τ_h).

Theorem 3.6 Every strongly $^*(\hat{g}r)_h$ - continuous function is strongly gp_h- continuous .

Proof: Let V be a subset of (Y', σ_h). since f is strongly $^*(\hat{g}r)_h$ -continuous, then f^{-1} (V) is $^*(\hat{g}r)_h$ closed in (Y', σ_h). Since every $^*(\hat{g}r)_h$ is closed set is gp_h -closed, f^{-1} (V) is gp_h closed in (X', τ_h). Hence f is strongly gp_h -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 3.7

Let $X' = Y' = \{a, b, c, d\}$, $\tau_1 = \tau_5 = \{X', \emptyset, \{a\}\}$, $\tau_2 = \{X', \emptyset, \{b, c\}\}$, $\tau_3 = \tau_4 = \{X', \emptyset, \{a\}, \{b, c\}\}$, $\tau_6 = \{X', \emptyset\}$ with $\tau_h = \{\emptyset, \{a\}, \{b, c\}, X'\}$ and $\sigma_h = \{\emptyset, \{a\}, \{a, b\}, Y'\}$. Let the function $f : (X, \tau_h) \to (Y', \sigma_h)$ be defined by f(a) = c, f(b) = b, f(c) = a then f is strongly gp_h-continuous but not strongly $^*(\hat{g}r)_h$ -continuous. Since for the hexa closed set { c } in Y', $f^{-1}(\{c\}) = \{a\}$ is gp_h –closed, but not $^*(\hat{g}r)_h$ -closed set in (X', τ_h).

Theorem 3.8 If a map $f : X' \to Y'$ is strongly $^*(\hat{g}r)_h$ -continuous and a map $g : Y' \to Z$ is $^*(\hat{g}r)_h$ -continuous then the composition f o $g : X' \to Z'$ is strongly $^*(\hat{g}r)_h$ -continuous.

Proof: Let G be any g_h-closed set in Z'. Since g is $^*(\hat{g}r)_h$ -continuous, $g^{-1}(G)$ is $^*(\hat{g}r)_h$ -closed in Y'. Since f is strongly $^*(\hat{g}r)_h$ -continuous, $f^{-1}(g^{-1}(G))$ is g_h-closed in X'. But $(g\ o\ f)^{-1}$ (G) = $f^{-1}(g^{-1}(G))$. Therefore, $g\ o\ f$ is strongly $^*(\hat{g}r)_h$ -continuous.

Theorem 3.9 If a map $f : X' \to Y'$ is strongly $^*(\hat{g}r)_h$ -continuous and a map $g : Y' \to Z'$ is $^*(\hat{g}r)_h$ -continuous then the composition f o g : $X' \to Z'$ is $^*(\hat{g}r)_h$ -continuous.

Proof: Let G be any g_h-closed set in Z' . Since g is $^*(\hat{g}r)_h$ -continuous, $(g^{-1}(G))$ is $^*(\hat{g}r)_h$ -closed in Y. Since f is strongly $^*(\hat{g}r)_h$ -continuous, $f^{-1}(g^{-1}(G))$ is g_h-closed in X'. By Remark 2.4, $f^{-1}(g^{-1}(G))$ is $^*(\hat{g}r)_h$ -closed. But $f^{-1}(g^{-1}(G)) = (g\ o\ f)^{-1}$ (G). Therefore, $g\ o\ f$ is $^*(\hat{g}r)_h$ -continuous.

4. Perfectly $^*(\hat{g}r)_h$ -continuous functions

Definition 4.1

A function f: $(X', \tau_h) \longrightarrow (Y', \sigma_h)$ is called Perfectly $^*(\hat{g}r)_h$ -continuous functions if f^{-1} (V) is $^*(\hat{g}r)_h$ - open in (Y', σ_h) is both h- open and h-closed in (X', τ_h) .

Theorem 4.2 Every perfectly $^*(\hat{g}r)_h$ -continuous functions is perfectly gs_h - continuous .

Proof: Let V be a subset of (Y', σ_h). since f is perfectly $^*(\hat{g}r)_h$ -continuous, then f^{-1} $(V)$is $^*(\hat{g}r)_h$ -open in (Y', σ_h). Since every $^*(\hat{g}r)_h$ is h-closed set is gs_h -closed, f^{-1} (V) is g_h closed in (X', τ_h). Hence f is perfectly gs_h -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example :4.3

Let $X' = Y' = \{a, b, c\}$, $\tau_1 = \tau_5 = \{X', \emptyset, \{b\}\}$, $\tau_2 = \tau_3 = \{X', \emptyset, \{b, c\}\}$, $\tau_4 = \{X', \emptyset, \{b\}, \{b, c\}\}$, $\tau_6 = \{X', \emptyset\}$ with $\tau_h = \{\emptyset, \{b\}, \{b, c\}, X'\}$ and $\sigma_h = \{\emptyset, \{b\}, \{a, c\}, Y'\}$. Let the function $f : (X', \tau_h) \to (Y', \sigma_h)$ be defined by f(a) = c, f(b) = a, f(c) =b then f is perfectly gs_h-continuous but not perfectly $^*(\hat{g}r)_h$ -continuous. Since for the hexa closed set { b} in Y', $f^{-1}(\{b\}) = \{c\}$ is gs_h– open and closed, but not $^*(\hat{g}r)_h$ -open set in (Y', σ_h).

Theorem 4.4 Every Perfectly $^*(\hat{g}r)_h$ - continuous function is Perfectly gp_h- continuous .

Proof: Let V be a subset of (Y', σ_h). since f is Perfectly $^*(\hat{g}r)_h$ -continuous, then f^{-1} $(V)$is $^*(\hat{g}r)_h$ open in (Y', σ_h). Since every $^*(\hat{g}r)_h$ is closed set is gp_h -closed, f^{-1} $(V))$ is gp_h closed in (X', τ_h). Hence f is Perfectly gp_h -continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.5

Let $X' = Y' = \{a, b, c, d\}$, $\tau_1 = \tau_5 = \{X', \emptyset, \{c\}\}$, $\tau_2 = \tau_3 = \{X', \emptyset, \{a, b\}\}$, $\tau_4 = \{X', \emptyset, \{c\}, \{a, b\}\}$, $\tau_6 = \{X', \emptyset\}$ with $\tau_h = \{\emptyset, \{c\}, \{a, b\}, X'\}$ and $\sigma_h = \{\emptyset, \{c\}, \{a, c\}, Y'\}$. Let the function $f : (X', \tau_h) \to (Y', \sigma_h)$ be defined by identity function f(a) = a, f(b) = b, f(c) = c then f is Perfectly gp_h-continuous but not Perfectly $^*(\hat{g}r)_h$ -continuous. Since for the hexa closed set {b } in Y', $f^{-1}(\{b\}) = \{b\}$ is gp_h –open and closed, but not $^*(\hat{g}r)_h$ -open set in (Y', σ_h).

Theorem 4.6 Every Perfectly $^*(\hat{g}r)_h$ -continuous functions is perfectly gsp_h - continuous .

Proof: Let V be a subset of (Y', σ_h). since f is perfectly $^*(\hat{g}r)_h$ -continuous, then f^{-1} (V) is $^*(\hat{g}r)_h$ - open in (Y', σ_h). Since every $^*(\hat{g}r)_h$ is closed set is gsp_h -closed, f^{-1} (V) is g_h closed in (X', τ_h). Hence f is perfectly gsp_h -continuous.

The converse of the above theorem need not be true as seen from the following examples.

Examples 4.7

Let $X' = Y' = \{a, b, c, d\}$, $\tau_1 = \tau_5 = \{X', \emptyset, \{b\}\}$, $\tau_2 = \tau_3 = \{X', \emptyset, \{b, c\}\}$, $\tau_4 = \{X', \emptyset, \{b\}, \{b, c\}\}$, $\tau_6 = \{X', \emptyset\}$ with $\tau_h = \{\emptyset, \{b\}, \{b, c\}, X'\}$ and $\sigma_h = \{\emptyset, \{b\}, \{a, c\}, Y'\}$. Let the function $f : (X', \tau_h) \to (Y, \sigma_h)$ be defined by f(a) = c, f(b) = a, f(c) = b then f is Perfectly gsp_h-continuous but not Perfectly $^*(\hat{g}r)_h$ -continuous. Since for the hexa closed set {b } in Y', $f^{-1}(\{b\}) = \{c\}$ is gsp_h –open and closed, but not $^*(\hat{g}r)_h$ -open set in (Y', σ_h).

Theorem 4.8 If f: $X' \to Y'$ and g: $Y' \to Z'$ be two perfectly $^*(\hat{g}r)_h$ -continuous functions, then g o f: $X' \to Z'$ is perfectly $^*(\hat{g}r)_h$ -continuous.

Proof: Let G be a $^*(\hat{g}r)_h$ -open set in Z. Then $g^{-1}(G)$ is both h-open and h-closed in Y' as g is perfectly $^*(\hat{g}r)_h$ -continuous. So $g^{-1}(G)$ is $^*(\hat{g}r)_h$ open in Y'. Since f is perfectly $^*(\hat{g}r)_h$ -continuous, $f^{-1}(g^{-1}(G))$ is both h-open and h-closed in X'. That is $f^{-1}(g^{-1}(G)) = (g\ o\ f)^{-1}(G)$ is both h-open and h-closed in X'. Hence $g\ o\ f$ is perfectly $^*(\hat{g}r)_h$ -continuous.

Theorem 4.9 If f: $X' \to Y'$ is perfectly $^*(\hat{g}r)_h$ -continuous and g: $Y' \to Z'$ is $^*(\hat{g}r)_h$ -irresolute, then gof: $X' \to Z'$ is perfectly $^*(\hat{g}r)_h$ -continuous.

Proof: Let G be a $^*(\hat{g}r)_h$ -open set in Z'. Then $g^{-1}(G)$ is $^*(\hat{g}r)_h$ -open in Y'as g is $^*(\hat{g}r)_h$ -irresolute. Since f is perfectly $^*(\hat{g}r)_h$ -continuous, $f^{-1}(g^{-1}(G))$ is both h-open and h-closed in X'. But $f^{-1}(g^{-1}(G)) = (g\ o\ f)^{-1}$ (G) is both h-open and h-closed in X'. Hence $g\ o\ f$ is perfectly $^*(\hat{g}r)_h$ -continuous.

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