RESEARCH PAPER

Stratified Flow over a Dipole and Conditions for the Non-Occurrence of Blocking

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ABSTRACT
An analysis is presented in the paper for the two-dimensional flow over a dipole and determination of the nature of the change of flow and the shape of the diving curve. The study investigates stratified flow dynamics and the critical conditions that prevent flow blocking, which has significant implications in geophysical and engineering fluid mechanics. Using theoretical framework based on inviscid stratified flow theory, we examine how the dipole-induced perturbation modifies the streamline patterns and establishes conditions for unblocked flow. The analysis reveals that the non-occurrence of blocking is directly linked to the Froude number and the dipole strength parameters. The diving curve characteristics are derived and classified according to the stratification intensity. These results provide deeper insight into flow past isolated obstacles in stratified environments and contribute to understanding of lee waves and wake dynamics.
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1. Introduction:

In a study on two-dimensional stratified flow in a channel, Yih (1960) proposed that the two-dimensional stratified flow over a barrier in a channel can be investigated by taking a suitable combination of sources, sink and doublets in place of barrier. Trustum (1964) considered it independently, the problem of two-dimensional channel flow over a barrier by applying an Oseen-type approximation to the general flow and discussed the validity of Long's hypothesis. In the study, Trustum simply remarked that in the case of the flow over a dipole (in place of the barrier), if the axis of the dipole be parallel to the direction of the uniform stream at infinity then practically there is no far- upstream influence (or the influence is negligible small). However, if the axis be perpendicular to the uniform stream at infinity, the far-upstream influence of the dipole is not negligible.

Drazin and Moore (1967) used the technique of diffraction theory to obtain the solution of Long's linearized equation for the steady flow of an incompressible inviscid fluid of variable density over an obstacle in an infinite channel. They extended the consideration to the flow over a dipole placed at the bottom of the channel with its axis parallel to the direction of the uniform stream by using some transformations and remarked in the same way as Yih did that the solution for the flow over a dipole can be used to describe the flow over any obstacle whose shape happened to coincide with a streamline. But it was pointed out a grave difficulty arises in making the shape of the obstacle to coincide exactly with the streamline.

In the study of Drazin and Moore (1967) on the flow past an obstacle it was indicated that the blocking may arise depending on the pressure condition at infinity, the value of the Froude number and the size of the obstacle. Blocking may occur also in the case of the flow over a dipole depending on the pressure at infinity and the strength of the dipole. So, it is my aim to examine and find a relation between the pressure at infinity, Froude number and strength the dipole to be placed at the bottom of the channel with its axis parallel to it and directed against the uniform flow.

If the pseudo-velocity U_0 at infinity on the negative side (i.e. at $x \to -\infty$) be not large enough,
then there is apparently a possibility that a layer of the stratified fluid in the lower region of the channel may not be able to cross the dipole. This will result in what may be called the blocking of the fluid by

the dipole. This leads rather to a contradiction to the remark by the Trustum (1964) that if the axis of the dipole be parallel to the channel wall then there is no possibility of blocking. So, it is proposing here to restudy the problem of the stratified flow over a dipole placed at the bottom of the infinite channel with its axis parallel to the uniform flow at infinity. An attempt is also made to find analytically the relationship between the pressure condition at infinity (on the negative side) and the strength of the dipole for the non-occurrence of blocking.

2. Governing Equation, Boundary Conditions and Solution:

Consider a two-dimensional stratified flow over a dipole placed at the origin at the bottom of an infinite horizontal channel formed by y □ 0 and y □ d . The axis of the dipole is horizontal and directed against the flow and the fluid of the purely dipole flow has constant density equal to that of the lowest stratum of the stratified fluid. The dipole flow affects the stratified fluid flow and at the same time it is affected by the stratified flow. Using the equation (1.6.3).

The physical flow is transformed into the pseudo-flow with uniform velocity at x □ □□; the stream function for the perturbed pseudo-flow in the channel in non-dimensional quantities is given by

(□² □ □²)□ □ □²y ,

(□ □ 1/F , F being the Froude number defined as F □ $\frac{U_0}{\sqrt{\alpha g d}}$)

The boundary conditions for □ are set as

□ □ y as x □ □,

□ □ □1 on |y| □ 1 for all x, y □ 0 for x □ 0.
□ { 0 on

Now writing □ □□ □ y , the above differential equation reduces to

(□² □ □²)□ □ 0 , (1)

with the boundary conditions

□ □ y as x □ □,

□0 on |y| □ 1 for all x,

□ □ □ 0 on y □ 0 for x □ 0.

The function □ then defines the perturbation in the flow. The stream function for the purely dipole flow (without any external effect) satisfies the Laplace equation.

In the absence of the dipole, the equation (1) admits the simple solution □ □ 0 , giving □ □ y , which represents the unperturbed uniform parallel pseudo-flow. In the perturbed pseudo-flow, the solution is, as shown by Drazin and Moore (1967) , found in two types depending on □ . When □ □ □ , the solution does not contain wavy terms and when □ □ □ , the solution contains both wavy and non-wavy terms.

This can be seen from the following:

Assuming $\psi = \sum_{n=1}^{\infty} \phi_n(x) \sin n\pi y$, the equation (1) gives

$$\frac{d^2\phi_n}{dx^2} + (k^2 - n^2\pi^2)\phi_n = 0. \qquad (2)$$

The solution of this equation depends on the sign of $(k^2 - n^2\pi^2)$. If $0 < k < \pi$, then $(k^2 - n^2\pi^2)$ can never be positive and so the solution is of exponential type

$$\phi_n = A_n' \exp[(n^2\pi^2 - k^2)^{1/2} x] + A_n'' \exp[-(n^2\pi^2 - k^2)^{1/2} x]$$

where A_n' and A_n'' are arbitrary constants, and if $N\pi < k < (N+1)\pi$ for some positive N, then $(k^2 - n^2\pi^2)$ is positive for $n \le N$ and negative for $n \ge N+1$,
and so for $n \ge N+1$, the solution will be of the above exponential type while for $n \le N$, the solution will be sinusoidal type

$$\phi_n = A_n' \sin[(k^2 - n^2\pi^2)^{1/2} x] + A_n'' \cos[(k^2 - n^2\pi^2)^{1/2} x]$$

which is stationary wave.
Thus when $k > \pi$, the wave occur. To avoid the solution in which wave may occur, we shall restrict the consideration to the case when $k < \pi$.
The solution shown above cannot be accepted, for it cannot satisfy all the necessary boundary conditions for the flow over a dipole. Thus the above method of solution fails, and hence we are to look for other method of solution.
Since the disturbance of the flow is caused by the dipole at the origin, therefore the solution must be that one having a dipole singularity at the origin. Drazin and Moore (1967) used the Dirac delta-function for the dipole singularity at the origin and accordingly the boundary condition on $y = 0$ was set as $\psi = K\delta(x)$.
Following Drazin and Moore (1967), the solution of the equation (1) and having a dipole singularity at the origin is, for $k < \pi$, is written as

$$\psi = -K \sum_{n=1}^{\infty} \frac{n \sin n\pi y}{(n^2\pi^2 - k^2)^{1/2}} \exp[-|x| (n^2\pi^2 - k^2)^{1/2}] \qquad (3)$$

where K is some constant.
The perturbed pseudo-steam function

$$\Psi = y + \psi = y - K \sum_{n=1}^{\infty} \frac{n \sin n\pi y}{(n^2\pi^2 - k^2)^{1/2}} \exp[-|x| (n^2\pi^2 - k^2)^{1/2}]. \qquad (4)$$

It may be noted that it is through the constant K that the dipole strength is to be involved.
The relationship between K and the dipole strength can be established simply by arguing that the dipole flow in the neighbourhood of the origin is not affected by the stratified flow, i.e., the flow in the neighbourhood of the origin (dipole singularity) is purely dipole flow.

Thus in the neighbourhood of the origin, the equation (4) will reduce (by dropping the first term and also putting □ □ 0 , i.e. , there is no stratification effect and also no term involving y alone can occur, the stream function simply satisfying Laplace equation) to

$$\square \;\square\; \square K \square \; \square \sin n\square\, y.\exp(\square n \square\, x) \square \square \frac{K}{2} \frac{\sin \square\, y}{\cosh \square\, x \square \cos \square\, y}.$$
$n \square 1$

The above result simplifies to (on expending in powers of x and y and neglecting terms of order $O(x^3)$, $O(y^3)$ and higher order terms)

$$\square \;\square\; \square \frac{K}{\square} \frac{y}{x^2 \square y^2}. \qquad (5)$$

Again, considering the purely dipole flow in the neighbourhood of the origin, it is seen that the physical stream function $\square_1$ for the dipole flow may be taken as ($\square_1$ also satisfies the Laplace equation)

$$\square_1 \;\square\; \square_1 \square \frac{y}{x^2 \square y^2}, \; (x,y \text{ small}) \qquad (6)$$

where $\square_1$ is the physical dipole strength.

Since the density of the fluid in the dipole flow is constant, the pseudo-stream function for the dipole flow is een to be given by

$$\square_2 \;\square\; \square\square \frac{y}{x^2 \square y^2} \qquad (7)$$

where $\square_2 \;\square\; \frac{\square_1}{Ud}$ and $\square \;\square\; \frac{\square_1}{U}$ are the non-dimensional dipole strength. (Here x, y are non-dimensional co-ordinates as used in the pseudo-flow of the stratified fluid).

Now, in the neighbourhood of the origin, the equations (5) and (7) represent the same flow and so, on comparison, we find

$$K \;\square\; \square\square \,. \qquad (8)$$

This is the relation between the constant K and the non-dimensional dipole strength □ . So , writing K in terms of □ , the equation (4) now becomes

$$\square \;\square\; y \;\square\; \square\square \sum_{n \square 1}^{\infty} \frac{n \sin n\square\, y}{(n^2 \square^2 \;\square\; \square^2)^{1/2}} \exp[\square \,|x|\, (n^2\square^2 \;\square\; \square^2)^{1/2}]. \qquad (9)$$

This gives the perturbed pseudo-stream function of the flow in the channel.

3. Discussion:

The nature of the change of the flow with □ and □ is noticed. For the flow not contain waves, □ should be less than □ and also the non-occurrence of blocking, $\square \;\square\; \frac{1}{\square}$ and $p \geq \square \frac{\square^2}{8} \square \frac{\square}{2} F_0^{-2}$.

The case □ □ 0 implies □ □ 0 and so, in the case □ □ 0 , the stratified fluid flow reduces to the homogeneous irrotational flow of constant density □ □ 1 everywhere. In that case, there is no distinction between the pseudo-stream function and the natural stream function. The flow is simply described by the stream function $\square$, where

$$\square \ \square\, y \,\square\, \square\square \sum_{n\square 1}^{\square} \sin n\square y.\exp(\square n\square x) \square y \square \frac{\square\square}{2} \frac{\sin\square y}{\cosh\square x \square \cos\square y} \qquad (10)$$

where $\square$ is not equal to zero , but small enough, the flow pattern of the stratified fluid will not differ much from that of the homogeneous fluid flow given by the equation (10).

Again $\square \ \square\, 0$, we have

$$u \,\square\, 1 \square \square\square^{3} \sum_{n=1}^{\square} \frac{n^2 \cos n\square y}{(n^2\square^2 \square \square^2)^{1/2}} \exp[\square x (n^2\square^2 \square \square^2)^{1/2}],$$

and $v \,\square\, \square\square^{2} \sum_{n\square 1}^{\infty} n \sin n\square y.\exp[\square x (n^2\square^2 \square \square^2)^{1/2}]$.

From these, it can be seen that the effect of $\square$ is to make a fluid move faster than that in the corresponding homogeneous fluid.

The streamline $\square \ \square\, 0$ demarcates the stratified fluid flow from the dipole flow region and so $\square \ \square\, 0$ defines the stratified flow while $\square \ \square\, 0$ defines the dipole flow.

When $\square$ is small, the boundary of the dipole flow region can be taken as p approximately given by (assuming $\square$ small)

$$x^2 \square y^2 \ \square \ \square \qquad (11)$$

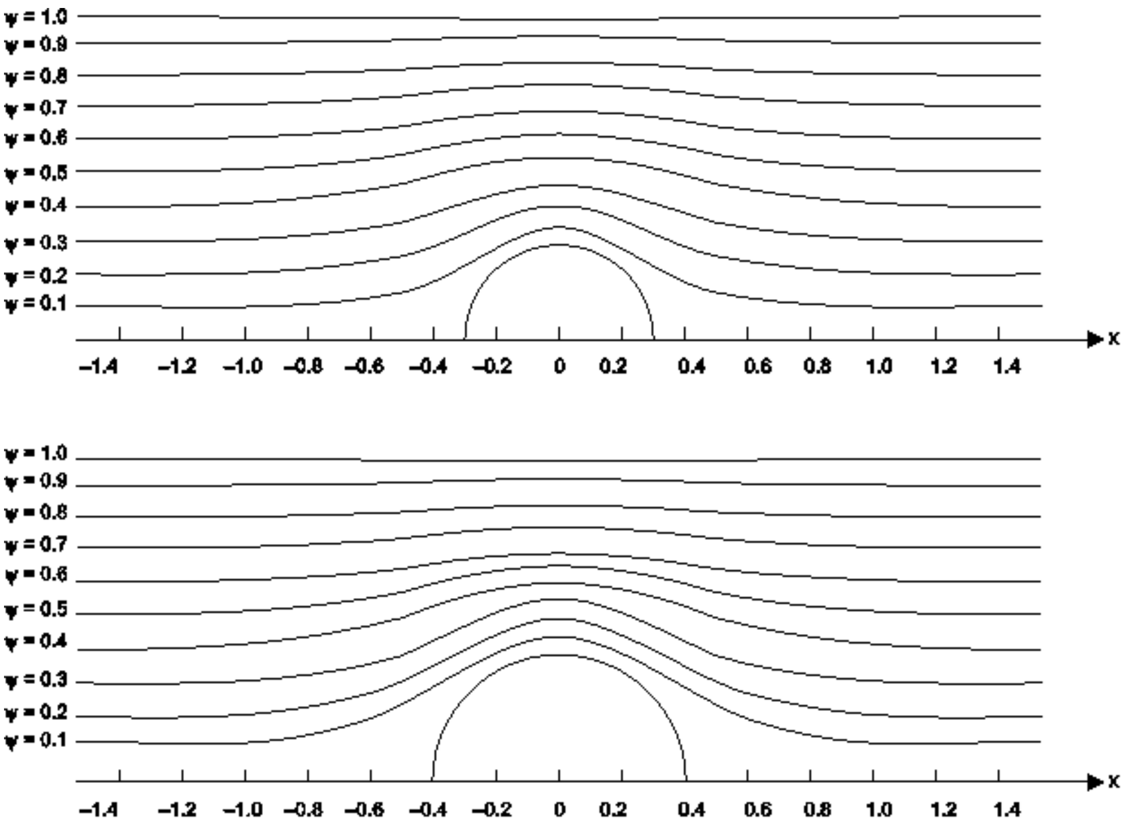
In the case of $\square \ \square\, 0$, two stagnation points symmetrically placed on either side of the origin appear on the lower boundary. As $\square$ increases from 0 to $\frac{1}{\square}$, the stagnation points shift away from the origin by equal amounts and accumulation of fluid occur near those points. This can also be verified from the stream pattern in Figure-1 . As result of shifting away of the stagnation points, the shape of the demarcating curve (i.e. the part of streamline $\square \ \square\, 0$) is found flattened extending more on both sides of the x-axis. This can be verified mathematically from the equation

$$\square \ \square\square \frac{K}{\square} \frac{y}{x^2 \square y^2} \qquad (12)$$

and

$$y \,\square\, \frac{2}{\square} y \tan \square \square y_0 \quad \square\, 0 \ \square \frac{\square}{2} \qquad (13)$$

and is confirmed from the graph of the streamline pattern(Figure-1(a)). This is actually what is expected and is partly due to the influence of stratified flow. When there is no possibility of blocking, the maximum height of the dipole flow region is $y \,\square\, \frac{1}{2}$ and this corresponds to $\square \ \square\, \frac{1}{\square}$.



Streamlines for the flow over a dipole of strength μ placed at the origin for μ = 0.4 and (a) μ = 0.024 (b) μ = 0.2

Figures-1 (a), 1 (b)

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