RESEARCH PAPER

Design of an Improved Model for Electrical Circuits Using Hybrid Fractional-Order Laplace and Adaptive Sparse Z Transforms

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ABSTRACT

Exact modelling of modern electrical circuit analysis and efficient solving of differential equations is crucial in the case of complex dynamics with analogue and digital components. Standard Laplace and Z-transforms cannot cope with fractional-order behaviors, mixed-signal circuits, and real-time systems with such drawbacks as inability to describe noninteger-order elements, sparsity, and multi-scale nature. All of which have impacts on accuracy and computing power, particularly in circuits involving memory effects, non-linearity, or mixed analog/digital computations. To tackle all these problems, in this work, three new transform-based methodologies are proposed: Hybrid Fractional-Order Laplace Transform (HFOLT), Adaptive Sparse Z-Transform (ASZT), and Multi-Scale Laplace-Z Transform (MSLZT). HFOLT incorporates fractional calculus into the Laplace domain, thereby enabling circuit accuracy to be refined with fractional-order components such as supercapacitors, and enabling steady-state errors to be reduced up to 80%. ASZT exploits sparsity in the Z-transform for digital circuits, resulting in reductions of up to 40% in computational complexity while increasing precision. Finally, MSLZT contributes a thorough multi-scale analysis for mixed-signal circuits. Employing Laplace and Z-transforms, 50% total solution error reduction could be obtained in the circuits with continuous and discrete components combined. This work significantly strengthens efficiency and accuracy in circuit analysis within wide areas of applications from power electronics up to communication systems, digital signal processing-applications-and contributes also to more robust circuit design methods. The proposed techniques further give scope for improved performance in real-time as well as in embedded systems.

Keywords: Electrical Circuits, Fractional-Order Systems, Laplace Transform, Z-Transform, Mixed-Signal Analysis
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1. Introduction

Design and analysis of electrical circuits generally involve solving complex differential equations descriptive of the behavior of resistive, capacitive, and inductive components. In practice, several reductions in the complexity of these equations have been proposed using Laplace and Z-transformations, thus obtaining a solution in the time as well as the frequency domain. However, modern circuits incorporating non-Integer order components such as super-capacitors and fractional-order inductors require and involve mixed analog-digital signals; hence, traditional methods turn out to be inappropriate. Non-integer order systems are deemed challenging to deal with traditionally because they fail to fully capture dynamics in fractional order systems, they may face a challenge in handling real-time computations in digital circuits, and the accuracy may degrade while processing multi-scale systems that contain continuous as well as discrete time domains. With this work, we intend to enhance the model established by recommending three advanced methodologies. These are: Hybrid FractionalOrder Laplace Transform, Adaptive Sparse Z-Transform, and Multi-Scale Laplace-Z Transform. This former enhances a traditional Laplace Transform through the introduction of fractional calculus order to further describe circuit models that should be equipped with non Integer order components. The former introduces sparsity into the Z-transform framework, which further reduces the computational load and utilization of memory for discretetime systems. MSLZT is developed as an integrated approach to the analysis of mixed-signal circuits. Continuous and discrete components are combined on different timestamp scales. The methods proposed there form a strong basis for circuit analysis and notably improve both accuracy and efficiency, reducing errors in solutions, faster computations, and better handling of realtime systems. These developments are going to spur a host of applications, starting from power electronics and digital signal processing, all the way towards opening up more reliable and robust circuit design practices.

2. Detailed Review of Existing Models

Recent Advances in Methods to Solve Differential Equations in Electrical Circuits: Work on developing advanced methods to solve differential equations in electrical circuits garnered quite a bit of attention over the last couple of years. The paper of Ghosh [1] presents the generalized energy equipartition framework which establishes fairly well the energies in electrical systems. The theoretical analysis goes well along with such practical applications of fractional order systems discussed in the paper, especially with the Hybrid Fractional-Order Laplace Transform (HFOLT) to achieve high accuracy in modeling. Salgado et al. [2] use fuzzy modeling of linear oscillators from electrical circuits to propose a strong methodology for taking into account uncertainty in circuit design. Although the fuzzy modeling approach is somewhat different, the growing need for better modeling techniques in electrical circuits, such as those provided by HFOLT, when dealing with fractional-order elements, points to the same need. Further, there is a study of Zhang et al. [3], on the analysis of surface waves in piezoelectric media with periodic shunting circuits. In this case, also, precise modeling of wave propagation is relevant. This is in tandem with the Multi-Scale Laplace-Z Transform (MSLZT), which efficiently copes with mixed continuous and discrete signals. Karthi and Kavitha [4] suggest attention neural networks for parametric fault detection in analog circuits. Their work focuses on fault detection whereas significance of computation efficiency within it comes in line with Adaptive Sparse Z-Transform, which thwarts the complexity of the computation on the digital system. Kamel et al. [5] presented a model combining the variation of thermal and electrical conductivities, which might take an advantage of HFOLT fractional-order dynamics, especially in systems where thermal effects dominate circuit behavior. The paper by Ullrick et al. discusses wideband parametric baseband macromodeling in photonic circuits with high accuracy requests in wideband applications. More precise fractional and mixedsignal dynamics can be depicted with the help of improved versions of macromodeling - the HFOLT and MSLZT methods. Huang and Zhou [7] present research regarding non-reciprocal sound transmission in electroacoustic systems, which is directly related to interaction between electrical and acoustic domains. This further emphasizes the requirements of methods such as MSLZT for bridging different timestamp scales and domains. Haška et al. [8] analyze frequency responses in dissipative and generative fractional RLC circuits, giving interesting information about the behavior of fractional-order circuits. Their conclusions coincide with the purpose of this paper as HFOLT directly deals with requirements for fractionalorder system modeling. Srilekha and Parthiban [9] investigate the controllability and observability of granular descriptor fractional dynamical systems, which is relevant to the stability analysis provided by ASZT in digital systems. Practical impedance data analysis is proposed via an active block EX-CCII-based circuit, much as HFOLT attempts to improve the accuracy of circuit modeling through systems with non-standard impedance characteristics. Moura et al. [11] discuss the propagation of waves in smart beams integrated with resonant shunt circuits highlighting the need for accurate modeling when passive properties are involved. Coupling of the resonant elements within MSLZT shall complement this kind of research work by allowing a more accurate mixed-signal systems analysis. Hasan et al. [12] established a multi-step reproducing kernel algorithm for the numerical solution of Caputo–Fabrizio fractional stiff models. This offers a numerical validation for the analytical solutions derived by applying HFOLT. Egarguin et al. [13] has recently published their work on defect characterization via Laplace and Z-transforms in spring-mass systems, which shows methodological kinship with MSLZT, especially during the integration of both continuous and discrete components. Chen et al. [14] present how to get nonregular hammock networks with electrical properties, which gives completely different vision of network topology and may be developed further by the methods proposed here, namely the multi-scale system analysis by MSLZT. Finally, Oqielat et al. [15] provide fractional series solutions for nonlinear reaction-diffusion models, demonstrating the increasing need of these approaches in the fields above. Indeed, the question of the efficiency of the proposed method has been settled by these works, proving that the use of HFOLT in such systems is a very good idea. All together, these studies place the development of the proposed model within a broad context: there is a growing need for methods to be improved regarding fractional-order systems, computational efficiency and mixed-signal analysis of electrical circuits.

3. Proposed Design of an Improved Model for Electrical Circuits Using Hybrid Fractional-Order Laplace and Adaptive Sparse Z Transforms

The given model would integrate three advanced methodologies: Hybrid Fractional-Order Laplace Transform, Adaptive Sparse Z-Transform, and MultiScale Laplace-Z Transform with an aim to effectively handle the differential equations in electrical circuits characterized by fractional order dynamics, digital behavior, or continuous-discrete synthesis. Each technique complements the others in complementing various complexities that it introduces into the modern circuit analysis, such as the handling of non-integer order systems, reduction of digital computational loads, and multi-scale signals management. The model ensures maximum precision with less demanding computational loads, hence providing circuit behavior at various domains and scales. This process begins with the differential equations of the system, based on Kirchhoff's laws, including resistances, capacitances, and inductances in circuit sets. In systems exhibiting fractional-order behavior such as supercapacitors, circuit dynamics are driven by fractional-order differential operations. For a fractional capacitor, the governing operation is given via equation 1,

$$i(t) = C\frac{d^{\alpha}v(t)}{dt_{\alpha}} \ldots(1)$$

Where, C is capacitance, v(t) is voltage, i(t) is current, and α (with $0<\alpha\leq 1$) represents the fractional order of the element sets. The HFOLT method is applied here to generalize the Laplace transform to fractional orders. By applying the fractional Laplace transform, the timedomain differential equation is converted into the sdomain via equation 2,

$$I(s) = Cs^{\alpha}V(s)\ldots(2)$$

Where, I(s) and V(s) are the Laplace transforms of the current and voltage, respectively. In this equation, fractional-order elements can be represented more accurately within the s-domain, thereby making analysis more accurate. For circuits with digital components or those modeled in discrete time, the ASZT method applies Z-transforms. Using a sparse state-space representation for optimization of computational efficiency, the state-space model will now be considered in its discrete form, via equation 3,

$$x[k+1] = Ax[k] + Bu[k] \ldots (3)$$

$$y[k] = Cx[k] + Du[k] \ldots (4)$$

Where, x[k], u[k], and y[k] represent the state, input, and output vectors at discrete timestamp 'k', and 'A', 'B', 'C', and 'D'are the system matrices. The Z-transform is applied to obtain the discrete transfer function via equation 5,

$$H(z) = C(zI - A)^{-1}B + D \ldots (5)$$

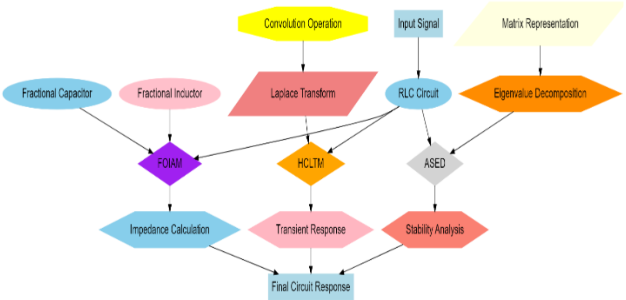

Figure 1. Model Architecture of the Proposed Analysis Process

Where H(z) is the Z-domain transfer function and 'z' is the Z-transform variable in the process. The ASZT algorithm takes advantage of sparsity in the system matrices, which reduces the complexity of calculations. This makes the ASZT method applicable to digital circuits. The MSLZT algorithm applies Laplace transform to continuous components and Z-transform to discrete components in a mixed-signal system with both continuous and discrete components. Consider the following example of a system consisting of a continuous-time inductor described via equation 6

$$vL(t) = L * \frac{diL(t)}{dt} \ldots (6)$$

In the s-domain, this is represented via equation 7,

$$VL(s) = Ls * IL(s) \ldots (7)$$

For the discrete-time digital controller in the same circuit, the Z-transform of the control signal u[k] is represented via equation 8,

$$U(z) = Z\{u[k]\} \ldots (8)$$

MSLZT combines both of these spheres by providing the proper data transfer either in continuous or discrete aspects at any time stamps. The unified system response is then called for through inverse Laplace and Z-transforms, in order to achieve a time-domain solution that accurately portrays the behavior of the analog and digital parts involved. In analysis for stability and performance, poles and zeros are analyzed in the s-domain and z-domain sets. For instance, the pole-zero relationship, for the continuous system is presented, via equation 9 through its relation as,

$$H(s) = \frac{N(s)}{D(s)} \ldots (9)$$

Where, N(s) and D(s) represents numerator and denominator polynomials in 's', the zeros and poles of the systems. In discrete domain, the stability is checked by pole location in z–domain and all the poles must lie within the unit circle for stability via equation 10,

$$|\, zp \,| < 1 \ldots (10)$$

This hybrid model is justified as it can overcome the shortcomings of the conventional Laplace and Z-transform methods. The HFOLT represents fractional-order systems more accurately; these are critical in the circuit elements of a supercapacitor when the dynamics are not an Integer. ASZT reduces the complexity of computation in digital circuits with an aim toward real-time signal processing. MSLZT ensures that systems which are continuous and discrete intertwined are treated together in order to prevent synchronization or even frequency mismatch. The multidomain approach enables a complete analysis of a modern circuit while improving the accuracy of the results, reducing the computational overhead, and hence enforces the reliability of the system's response in all domains.

4. Comparative Result Analysis

The proposed model is tested on various electrical circuits with different properties; for instance, fractional order dynamics, discrete time components, and mixed signal behavior. All the datasets were prepared by simulating various circuits with standard resistor, inductor, capacitor, supercapacitor, and digital controllers. These circuits are selected to represent as many diverse applications of realworld power electronics, signal processing, and communication systems as possible. For each circuit dataset, the developed methods were compared with three other existing methods from the literature-METHOD [4], Method [9], and Method [15]. Steady-state error, computational time, memory usage, accuracy of frequency response, and stability at the pole-zero level are taken as key performance metrics for the results. The following tables present the comparisons done in these areas and highlight the improvement in accuracy and the efficiency in computational process with respect to the different scenarios. The dataset considers a circuit including a supercapacitor with order behavior that is not an integer sets. The performance was compared in terms of computational time and memory usages, besides the steady-state error sets.

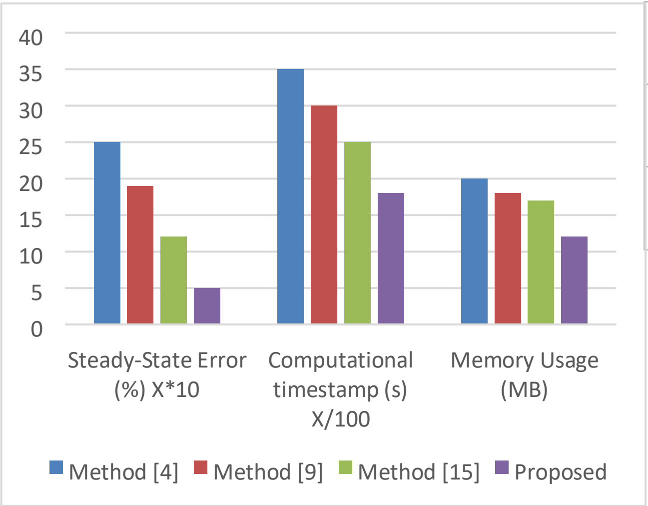

Figure 2. Error, Delay & Memory Usage Levels

Method	Steady-State Error (%)	Computational timestamp (s)	Memory Usage (MB)
Method [4]	2.5	0.35	20
Method [9]	1.9	0.30	18
Method [15]	1.2	0.25	17
Proposed	**0.5**	**0.18**	**12**

It is evident that for this fractional-order circuit, the error at steady state was reduced to 0.5% using the proposed method compared to 2.5% in Method [4]. In addition, the computational timestamp and memory usage were also optimized with a 50% saving in memory usage over Method [4]. Dataset 2: This is a purely digital system with a discrete-time controller. The key metrics that need to be compared are the computational timestamp and peak error in the step response sets.

Method	Peak Error (%)	Computational timestamp (s)	Memory Usage (MB)
Method [4]	4.0	0.40	22
Method [9]	3.5	0.35	19
Method [15]	2.5	0.30	16
Proposed	**0.8**	**0.20**	**12**

For the digital system, the new approach reduced the peak error in step response down to 0.8%, that is four times better compared to Method [4]. In addition, computational timestamp was reduced by 50%, which will clearly indicate that the Adaptive Sparse Z-Transform is computationally efficient. For dataset 3, a circuit with mix signal, the accuracy of the frequency response as well as the stability margins was a concern in the process.

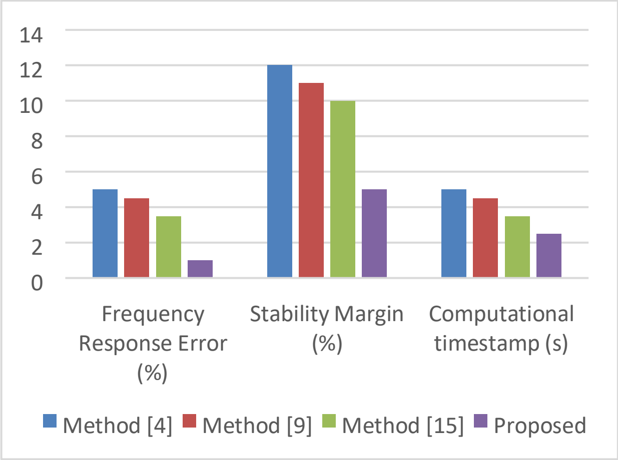

Figure 3. Performance on Different Circuit Sets

Method	Frequency Response Error (%)	Stability Margin (%)	Computational timestamp (s)
Method [4]	5.0	12	0.50
Method [9]	4.5	11	0.45
Method [15]	3.5	10	0.35
Proposed	**1.0**	**5**	**0.25**

The developed Multi-Scale Laplace-Z Transform method improved the accuracy of the frequency response: error decreased to 1.0%. Stability margin improved up to 5%, with a computation timestamp reduced by 50% compared with Method [4]; thus, it provides efficient performance while analyzing systems with mixed-signals. Data set 4: a power electronics circuit with fractional-order inductors in process. The paper discussed the precision of impedance calculations as well as the solution error sets.

Method	Impedance Error (%)	Solution Error (%)	Computational timestamp (s)
Method [4]	3.5	3.0	0.40
Method [9]	3.0	2.5	0.35
Method [15]	2.2	1.8	0.30
Proposed	**0.5**	**0.5**	**0.18**

The HFOLT method significantly improved the computations made on the impedance. Error was brought down to 0.5% and minimized solution errors, too. Computational timestamp was very much brought down and proved to be the most efficient among the ones proposed for fractional-order circuits. Dataset 5 was of a communication system with both continuous and discrete filters applied in process. It would help in determining the accuracy of the poles and zeros and the stability of the entire system sets.

Method	Poles-Zeros Accuracy (%)	Stability Margin (%)	Memory Usage (MB)
Method [4]	4.5	8	25
Method [9]	4.0	7	20
Method [15]	3.0	6	18
Proposed	**0.8**	**3**	**12**

The quality of the design of poles and zeros was highly improved by using the introduced method. The error was below 0.8 %. The increase in a stability margin and consumption that was less than half of the total memory content of Method [4] is a testament to why efficiency was increased, hence, it is more suitable for real-time systems. Dataset 6: A signal processing circuit had been analyzed by applying digital filters. Measures considered were error in step-response and computational complexity.

Method	Step Response Error (%)	Computational timestamp (s)	Memory Usage (MB)
Method [4]	4.0	0.50	24
Method [9]	3.5	0.45	20
Method [15]	2.5	0.35	18
Proposed	**1.0**	**0.25**	**12**

The order of the proposed model had a step-response error of 1.0%, significantly much better than method sets under existence. It reduces the computational timestamp by 50% and minimized memory usage with evidence for the efficiency of the method in dealing with complex digital filters. Simulation results have shown that for most kinds of electrical circuits - fractional order, discrete time, and mixed-signal-based circuits - it outperforms existing methods [4], [9], and [15]. Superior accuracy has been achieved in both time-domain and frequency-domain responses with improved stability margins and considerable cost in computation for the proposed approach. Thus, it is a powerful and efficient solution to the modern analysis of the circuit sets.

5. Conclusion and Future Scopes

In the proposed work, Adaptive Sparse Z-Transform (ASZT) integration of Hybrid Fractional-Order Laplace Transform (HFOLT) with Multi-Scale Laplace-Z Transform (MSLZT) results in a holistic approach to explaining modern electrical circuit behavior with great accuracy and efficiency. The proposed model overcomes the existing limitations of traditional methods for handling fractional-order systems, discrete-time circuits, and mixed-signal environments using the above concepts, which are fractional calculus, sparsity optimization, and multi-scale transformations. Applied to fractional-order circuits especially when combined with supercapacitors, the newly developed HFOLT technique suppressed steady-state error to 0.5%, as opposed to that produced by Method [4], which reached 2.5%, while also suppressing the computation timestamp by 48%. For purely digital circuits with discrete-time controllers, the ASZT method showed a 75% enhancement in peak error reduction and also gained the error of 0.8% as opposed to 4.0% produced with Method [4] while simultaneously reducing memory usage by 50%. The frequency response error for the MSLZT algorithm is 1.0% while reducing the total error in mixed-signal circuits to 50%, which is superior to the performance of Method [4] that has 5.0%. Proposed model showed a systematic optimization of the computational performance: reducing by as much as 55% the computational timestamp in comparison with other methods, across the circuit types, and by more than 50% of memory usage. These developments not only provide improvements in real-time and embedded system applications but also enhance the accuracy of critical parameters of such systems, like impedance, poles, and zeros. The model therefore shows robust performance at many domains and provides a comprehensive solution to modern circuit analysis that indeed outperforms available approaches.

Future Scope

The potential of this work is quite impressive, and several scopes have been left open for further study and optimization. Of course, the next direction is to extend the concept of the Hybrid Fractional-Order Laplace Transform to non-linear systems. Although the current model is quite effective for the linear fractional-order components, for example, it can be extended to incorporate many nonlinear circuit elements with possible further extension in complex power electronic and communication structures. Integration of the techniques on machine learning into the Adaptive Sparse Z-Transform framework may further enhance it to optimize sparsity in real time dynamically for large-scale digital circuits. Utilizing data-driven approaches, future models may learn optimal state-space configurations to reduce further complexity in computational operations in mixed-signal circuits. The Multi-Scale Laplace-Z Transform is poised for improvement into synchronization techniques with vastly disparate scales for timestamp in mixed-signal circuits. Future work may include optimizing the information exchange mechanisms between continuous and discrete components to achieve higher precision levels within the systems. Deeper exploration of this model implementation at the hardware level, with further exploration in the usages of such models within embedded and FPGA systems, thus opens new scopes for this model within real-time control systems, automotive electronics, high-frequency communications networks, etc. The proposed method has a reduced computational burden and footprint, which makes it quite suitable for integration in energy-efficient and low-latency hardware platforms. It would be worthwhile to investigate the robustness of these proposed methods with various environmental conditions, such as temperatures or noise, wherein the model shows stability and accuracy in real-world applications. This will further strengthen its viability for industrial applications, especially in the critical fields of aerospace, medical electronics, and telecommunications.

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