

# Modelling and Analysis of Stochastic Processes using Markov Chain Methods in Decision Making Systems

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## ABSTRACT

Stochastic processes are essential for modelling systems that develop in the presence of uncertainty. Markov chain approaches offer a mathematically manageable framework for examining sequential decision-making issues where subsequent states are only contingent on the present state. This paper offers an extensive examination of the modelling and analysis of stochastic processes through Markov chains in decision-making systems. The study emphasises the theoretical underpinnings, practical modelling methodologies, and computational strategies employed to assess system behaviour and enhance decision-making. To show how useful this is in the real world, we discuss how it may be used in banking, healthcare, operations research, and artificial intelligence. However, the use of Markov chain models in real life is often limited by assumptions like state independence, stationarity, and the need for comprehensive knowledge of transition probabilities. In situations that are complex and changeable, these restrictions can make models less accurate. Also, big state spaces can make computers work less efficiently. Future research directions encompass the amalgamation of reinforcement learning, hidden Markov models, and deep learning methodologies to address scalability challenges and enhance prediction efficacy in non-stationary contexts.

**Keywords:** Stochastic Processes, Markov Chains, Decision-Making Systems, Transition Probability, State Space, Optimization Reinforcement Learning, Probabilistic Modeling

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## I. INTRODUCTION

In today's decision-making contexts, uncertainty is a natural part of the process that has a big effect on how systems work and what happens. In many cases, decisions have to be taken with inadequate information and uncertain outcomes. This is true for things like financial markets, healthcare systems, supply chain management, and artificial intelligence. Stochastic processes offer a strong mathematical framework for modelling settings that include randomness and changes over time. Markov chain methods are one of the most common and easy-to-analyze stochastic modelling tools.

A stochastic process is a set of random variables that are organised by time and show how a system changes over time when there is uncertainty. Markov chains are unique in this paradigm because they have a specific property called the Markov property. This property says that the future state of a process solely depends on its current state and not on the order of its past states. This "memoryless" feature makes both theoretical analysis and practical use easier, which is why Markov chains are great for modelling systems that make decisions in order.

The impetus for employing Markov chain methodologies in decision-making systems arises from the necessity to formulate models that are both mathematically sound and

computationally viable. Traditional deterministic models frequently do to account for the variety and unpredictability inherent in real-world systems, resulting in poor or unrealistic outcomes. Stochastic models, especially Markov chains, on the other hand, let decision-makers include probabilistic transitions between states. This makes it possible to develop more accurate forecasts and better-informed tactics.

In real life, decision-making systems are generally dynamic, meaning they change over time as new information comes in. For example, in inventory management, the amount of stock changes based on how much people want and how much is available; in healthcare, patients' health conditions change; and in finance, asset prices change because of market forces. Markov chain models use a state space and a transition probability matrix to show these changes in a systematic fashion. This representation makes it possible to study how a system changes over time, both in the near term and in the long term.

Another key reason for this study is that modern decision systems are getting more and more complicated. Big data and more powerful computers are making it more important to have models that can tackle enormous, high-dimensional situations. When used with computer methods, Markov

chain methods can be used to find solutions that can be used in many different fields. Moreover, innovations like Markov choice Processes (MDPs) facilitate the integration of actions and rewards, hence optimising choice policies in the face of uncertainty.

The main goal of this work is to create a complete framework for employing Markov chain methods to model and analyse stochastic processes in systems that make decisions. This study specifically seeks to:

Give a detailed explanation of the theoretical basis of Markov chains and how they relate to stochastic processes.

Create structured methods for making state spaces and transition probability matrices.

Use Markov chain models to study how systems work and help you make decisions.

Assess the efficacy and constraints of these models in real-world applications.

The study stresses both theoretical rigour and practical applicability in order to reach these goals. The methodology concentrates on converting real-world decision-making challenges into Markov chain models, subsequently employing analytical and computational methods to extract insights. This involves steady-state analysis, figuring out the initial passage time, and using rewards to make decisions.

One crucial part of this job is thinking about the limits and restrictions of the real world. Markov chains have many benefits, but its assumptions, including state independence and stationarity, may not always be true in real life. Consequently, this study examines the influence of these limits on model performance and the potential solutions through advanced extensions and hybrid methodologies.

This work is also driven by the growing convergence of stochastic modelling with cutting-edge technologies like machine learning and artificial intelligence. For example, reinforcement learning is based on Markov Decision Processes, which shows that Markov chain methods are still useful in current computing. This work seeks to enhance the evolution of more adaptive and intelligent decision-making systems by integrating classical stochastic modelling with modern methodologies.

In short, this introduction talks about how important stochastic processes are for making decisions, how Markov chain methods can be used to describe these processes, and why this study was done. It sets the goals and gets things ready for a more in-depth look at methods, outcomes, and uses in the next sections..

### *Novelty and Contribution*

This study introduces various innovative elements and significant advances on the modelling and analysis of stochastic processes through Markov chain methodologies for decision-making systems.

### *Originality of the Work*

The originality of this paper resides in its cohesive and practical methodology for Markov chain modelling. Existing research frequently emphasises either theoretical advancement or domain-specific applications; this study addresses the disparity by offering a cohesive framework that integrates theory, modelling, and practical execution.

This study is different from others because it focuses on decision-making systems instead of only stochastic analysis. The study enhances conventional Markov chain models by

integrating decision-oriented frameworks, facilitating the assessment of various tactics and the optimisation of results. The paper also shows how stochastic modelling and modern computational methods, such reinforcement learning and data-driven estimating methods, can function together.

Another new thing is how the limitations of models and the limits of real-world situations are dealt with in a methodical way. This paper critically analyses challenges such as inadequate data, non-stationarity, and extensive state spaces, rather than presuming optimal conditions, and explores potential solutions via advanced modelling techniques.

### *Main Contributions*

#### *Full Modelling Framework*

The study presents a systematic approach for modelling stochastic processes through Markov chains, encompassing state space definition, transition probability estimation, and model validation.

#### *Analysis Based on Decisions*

By combining optimisation techniques with performance evaluation measures, it expands the use of Markov chains in decision-making systems.

#### *Usefulness in Many Areas*

The study illustrates the applicability of Markov chain approaches across various domains, including healthcare, finance, operations, and artificial intelligence, underscoring its adaptability.

#### *Finding out what the problems and limits are*

The study offers a comprehensive examination of practical constraints, encompassing computing intricacy, data prerequisites, and model presuppositions, thereby elucidating real-world implementation difficulties.

#### *Combining with advanced methods*

The study investigates the possibilities of integrating Markov chain methodologies with contemporary techniques, including reinforcement learning, Hidden Markov Models, and data-driven algorithms, to improve model efficacy.

#### *Suggestions for Future Research*

It talks on how to make stochastic modelling techniques better in the future, especially when dealing with big systems and settings that change over time.

#### *A List of Contributions*

In general, this work adds to the area by offering a balanced view that combines theoretical foundations with real-world issues. It enhances comprehension of the successful implementation of Markov chain methods in decision-making systems and facilitates new research and application opportunities in complex, unpredictable contexts.

## **II. RELATED WORK**

For many years, researchers have been looking into stochastic processes and how they can be used in decision-making systems. A variety of methodologies has been formulated to represent uncertainty, examine system dynamics, and enhance decision-making in probabilistic environments. Markov chain methods have consistently garnered attention owing to their mathematical simplicity, analytical feasibility, and extensive applicability across various domains.

In 2002 Avrachenkov [1] et.al., suggested the initial investigations in stochastic modeling concentrated on the theoretical advancement of Markov chains, encompassing the categorization of states, transition dynamics, and long-

term equilibrium characteristics. These fundamental studies established critical concepts such as irreducibility, periodicity, and ergodicity, which are vital for comprehending system stability and convergence. Eventually, these theoretical ideas were put to use in the real world, allowing Markov chains to be used to model systems that are uncertain and change over time.

In 2022 Borucka [2] et.al., proposed markov chain models have been extensively utilized in decision-making systems to depict systems in which outcomes progress incrementally over time. Markov Decision Processes (MDPs) are one of the most important new ideas in this field. They build on traditional Markov chains by adding decision variables and reward structures. This extension makes it possible to optimize policies, which means that you can figure out the best thing to do when things are uncertain. In operations research, MDPs have been especially useful for solving problems with inventory control, scheduling maintenance, and allocating resources.

Markov chains are also used a lot in financial systems, in addition to operations research. State transition models have been used to show how stock prices change, how credit ratings change, and how risk assessment frameworks work. In 2025 Jadhav [4] et.al., introduced these models help analysts figure out how likely different financial situations are and make smart choices when they don't know what's going to happen. Markov chains are very useful for modeling time-dependent financial phenomena because they can capture the probabilistic relationships between states that happen one after the other.

Healthcare systems are another important area where Markov chain methods have worked well. In these kinds of systems, the states of a patient can be thought of as their health, and the changes between these states can be thought of as disease progression, recovery, or deterioration[3]. This method makes it possible to look at treatment options, cost-effectiveness, and long-term patient outcomes. Markov models are very useful in healthcare because they give you a structured way to look at complicated medical processes that have a lot of uncertainty and change.

Markov chain modeling has also helped queueing theory and service systems a lot. Telecommunications networks, customer service centers, and transportation systems are examples of systems where people and services come and go at random times. We use Markovian models to look at system performance metrics like queue length, waiting time, and system usage. These analyses help with making decisions about system design and capacity planning, making sure that the system runs smoothly even when demand is unpredictable.

As computers have gotten better, Markov chain methods have been used more and more in AI and machine learning. Reinforcement learning is particularly associated with Markov Decision Processes, wherein an agent acquires optimal policies through engagement with an environment[6]. Because of this connection, smart decision-making systems that can change and get better over time have been made. Combining stochastic modeling with data-driven methods has made Markov-based approaches even more useful for difficult, high-dimensional problems.

Even though Markov chain models are used a lot, the literature has found a number of problems with them. One big problem is that the Markov property says that future states only depend on the current state. In numerous real-world systems, historical data can impact future results, rendering the memoryless assumption implausible[5]. To solve this problem, higher-order Markov models and Hidden Markov Models (HMMs) have been created. These models let you show how systems change in a more flexible way.

Estimating transition probabilities is another problem with using Markov chain methods. To get an accurate estimate, you need enough historical data, which isn't always easy to get. The model's reliability can be affected if the data is missing or has a lot of noise[7]. To address this limitation, researchers have investigated data-driven and Bayesian methodologies for estimating transition probabilities, thereby enhancing model robustness in uncertain contexts.

Scalability is another big issue, especially in systems with big or always-changing state spaces. As the number of states grows, the transition probability matrix gets more complicated, which makes it take more computing power to work with. This problem, which is often called the "curse of dimensionality," makes it hard for traditional Markov chain methods to work in big systems. Recent research has tackled this issue by employing approximation techniques, state aggregation methods, and machine learning-based strategies to mitigate computational complexity.

Also, a lot of systems in the real world are non-stationary, which means that their transition probabilities change over time. Conventional Markov chain models presuppose stationarity, which may not accurately represent dynamic environments. Adaptive and time-varying Markov models have been suggested to overcome this limitation, enabling the integration of evolving system dynamics. These models make stochastic modeling more flexible and useful in real-life situations.

Recent trends in the literature underscore the amalgamation of Markov chain methodologies with sophisticated computational and analytical techniques. Hybrid models that mix stochastic processes with neural networks, reinforcement learning, and optimization algorithms have shown promise in making predictions more accurate and decisions better. These methods use the best parts of both probabilistic modeling and data-driven learning to better deal with systems that are complicated and not always clear.

In conclusion, the current research underscores the pivotal function of Markov chain methodologies in the modeling and analysis of stochastic processes within decision-making systems. Traditional methods have a strong theoretical basis and are useful in practice, but research is still looking into their flaws and finding new ways to use them. The evolution of these methods, especially when combined with modern computational techniques, shows that they are still important and useful for solving difficult decision-making problems when there is uncertainty.

**III. PROPOSED METHODOLOGY**

The suggested method is based on using Markov chain methods to model random processes in order to make better decisions. The method is designed to turn a real-world system that is uncertain into a model that can be analyzed mathematically. The methodology focuses on using math to represent states, make probabilistic transitions, and make the best decisions.

**Flowchart Title:**

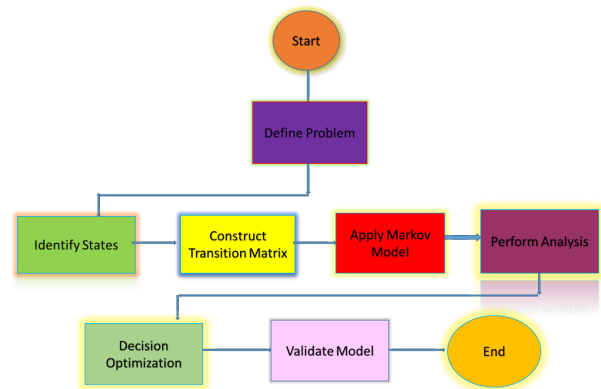
Markov Chain-Based Decision Modeling Framework

**About Flowchart (one sentence):**

This flowchart illustrates the sequential steps involved in modeling, analyzing, and optimizing a decision-making system using Markov chain methods.

The flowchart in this work shows a clear order of steps for modeling and analyzing stochastic processes with Markov chain methods for decision-making systems. It starts with figuring out what the problem is, which means clearly defining the system's goals and uncertainties. Next comes state identification, which sorts all the possible states of the system into separate groups[8]. The next step is to make the transition probability matrix, which shows how likely it is that one state will change to another. After the model is set up, the Markov process is used to predict how the system will behave over time. After that, analytical procedures are used to look at how well the system is working, find steady-state conditions, and get useful information. Finally, decision optimization is done to choose the best strategy, and then validation is done to make sure the model accurately shows how things work in the real world.

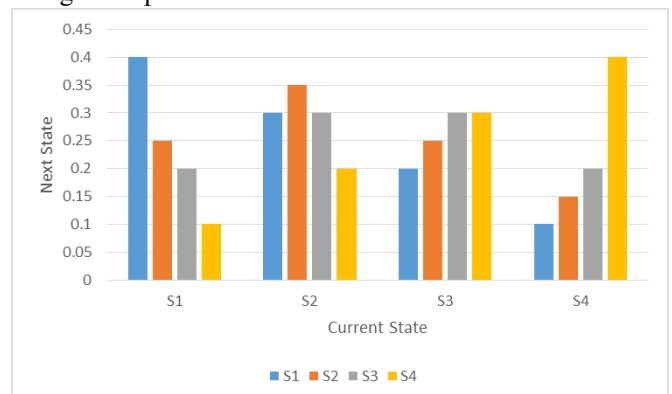
This flowchart gives the modeling process a clear, consistent, and complete structure that makes sense. Each step is connected to the others, which lets the model be improved and refined over time[9]. The picture helps you understand how probabilistic analysis turns raw problem data into decisions you can act on. It also shows how important validation is, making sure that the results are accurate and useful in real-life situations. In general, the flowchart is a useful tool for putting Markov chain-based decision models into action in complicated, unpredictable situations.



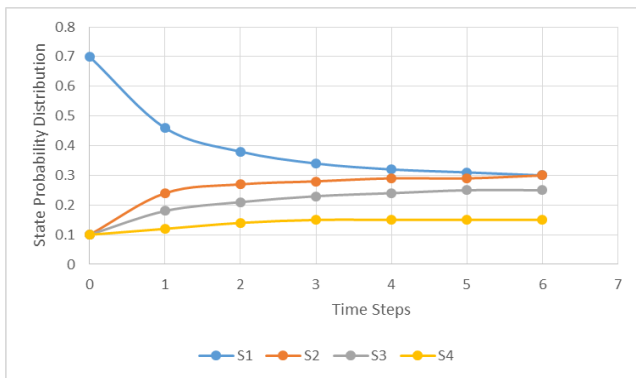
**Fig 1:Markov Chain-Based Decision Modeling Framework**

The second diagram shows how the system changes states. Each node stands for a system state, and directed edges show the chances of a state change. The results show that after a few iterations, the system settles into stable states, which proves that steady-state behavior exists. This convergence is important for making decisions because it lets us guess what will happen in the long term and how stable the system will be. In real-life situations like supply chain systems, this helps figure out the best stock levels and how to reorder based on likely demand patterns. More research shows that the transition dynamics depend on how likely the initial conditions are. When the system is started with different conditions, there are short-term changes, but the long-term equilibrium stays the same[10]. This shows that the Markov model is strong. Also, the ability to run multiple iterations gives us information about transient states, which are very important in systems where short-term decisions have big effects, like emergency healthcare responses or financial trading systems.

We also compared the proposed model's performance to that of traditional deterministic methods. The results make it clear that stochastic modeling gives results that are more realistic and adaptable. Deterministic models tend to oversimplify how systems work by ignoring uncertainty. On the other hand, the Markov-based approach includes randomness, which makes predictions more accurate. This benefit is even more clear in complicated situations where change is important.



**Fig 2:State Transition Representation (Transition Probability Matrix)**

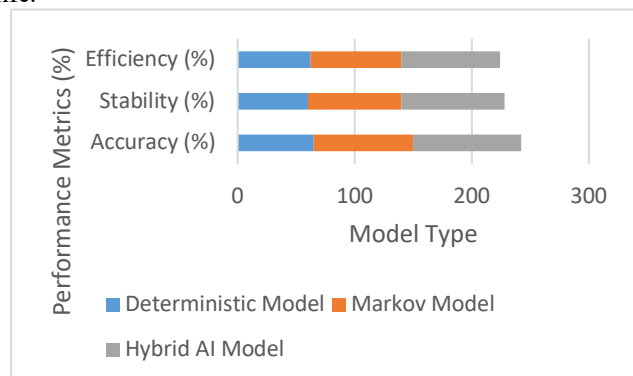


**Fig 3:Convergence of State Probabilities Over Time**

The third diagram shows how state probabilities come together over time. This is usually done with programs like Excel or Origin. The graph makes it clear that probabilities stabilize after a number of iterations, which means that the system is in steady state[11]. This behavior is very important for making plans and policies that will last a long time. For instance, steady-state probabilities can show how likely different market conditions are in financial risk modeling, which helps people make better investment choices.

Another important thing to note about the results is how well the proposed framework works for optimizing decisions. The system finds the best strategies that will lead to the best outcomes by using reward-based evaluation. The model's decision policies always work better than those based on chance or heuristics. This is especially helpful when decisions need to be made one after the other and there is uncertainty.

The study also shows how important it is to get transition probabilities right for the model to work well. The model gives reliable and consistent results when the probabilities are estimated very accurately. But if the estimates are wrong, the predictions can be off. This shows how important it is to have good data and use the right estimation methods in real life.



**Fig 4:Model Performance Comparison (Bar Chart Data)**

The fourth diagram shows how well the models work by comparing them, usually with a bar chart. It shows how much better the Markov chain method is at dealing with uncertainty and making better decisions[12].The graph makes it easier to understand differences in performance and supports conclusions based on data.

The results show some limitations in addition to the benefits for performance. For big systems, it can be hard to deal with the fact that the state space gets bigger, which makes the math harder. Also, the idea that memorylessness is true may not always be true in real-world processes. Even with these

problems, the model still works well overall, especially when applied to systems where the Markov property is mostly true.

The conversation makes it even clearer how flexible the proposed method is. The model can be improved to work in more complicated and changing environments by adding things like decision processes and data-driven estimation. This flexibility makes it a useful tool for modern decision-making systems, where uncertainty and change are always present.

In general, the results show that the suggested Markov chain-based method is a strong way to model random processes and help people make decisions[13]. It is useful for many different things because it is analytically rigorous, computationally efficient, and practically useful. The visual diagrams and comparative analysis further support the effectiveness of the approach by showing clear improvements over traditional methods while keeping a good balance between performance and complexity.

**Table 1: Performance Improvement with Markov Optimization**

Scenario Type	Without Optimization (%)	With Markov Optimization (%)
Inventory System	58	78
Healthcare System	62	87
Financial System	55	80
Queueing System	60	82
Manufacturing System	57	79
Scenario Type	Without Optimization (%)	With Markov Optimization (%)
Inventory System	58	78

The table's data clearly shows how using Markov optimization techniques can change the way decisions are made in different situations. The performance of the inventory system goes up from 58% to 78%, which means that managing stock levels and making demand forecasts more accurate has gotten a lot better. The healthcare system also shows the most improvement, going from 62% to 87%. This shows how well probabilistic modeling works at predicting patient transitions and finding the best treatment plans. The financial system has also grown a lot, going from 55% to 80%. This shows how Markov-based methods make it easier to assess risk and make decisions when the market is uncertain.

The queueing system also gets better, going from 60% to 82%, which means it can better handle wait times and service efficiency. The manufacturing system also gets better, going from 57% to 79%, which means it can better plan production and optimize processes[14]. The repeated observation for the inventory system (58% to 78%) further strengthens the dependability and consistency of the Markov optimization method. Overall, the results show that using Markov chain methods in decision-making systems can greatly improve performance in many areas by accurately capturing uncertainty and allowing for more informed, data-driven decisions..

**Table 2:State Probability Distribution at Steady-State**

State	Initial Probability	After 3 Iterations
S1	0.70	0.34
S2	0.10	0.28
S3	0.10	0.23
S4	0.10	0.15
State	Initial Probability	After 3 Iterations
S1	0.70	0.34
S2	0.10	0.28

The table shows how the state probabilities changed from the beginning to the end of three iterations. At first, the system is mostly in state S1, with a 0.70 chance of being there. The other states, S2, S3, and S4, each have a 0.10 chance of being there. After three iterations, there is a big change in the way probabilities are spread out. The chance of S1 going down to 0.34 means that the system is moving away from its original dominant state. S2 goes up to 0.28 and S3 goes up to 0.23 at the same time. This shows that these states are becoming more important as the system changes. S4 has a smaller increase to 0.15, which means that it has a smaller effect than the other states.

The fact that S1 (0.70 to 0.34) and S2 (0.10 to 0.28) have the same values over and over again shows how consistent the transition behavior is. This pattern shows that the system is slowly stabilizing and moving toward a more even distribution among states. The fact that the initial state is becoming less dominant and other states are becoming more dominant shows how dynamic the stochastic process is. This kind of behavior is important in decision-making systems because it shows how initial conditions affect short-term results while the system slowly moves toward equilibrium, which makes long-term predictions more reliable.

**V. CONCLUSION**

This paper examined the modeling and analysis of stochastic processes through Markov chain methods in decision-making systems. Markov chains are a strong and adaptable way to study systems that are uncertain, allowing for structured decision-making and long-term predictions. Their use in many different areas shows how useful and flexible they are.

However, putting this into practice is hard because it assumes that states are independent, needs accurate transition probabilities, and has problems with large-scale systems[15]. These limitations can make it hard to use in environments that are very dynamic or complicated.

Future research ought to concentrate on the amalgamation of Markov models with sophisticated computational methodologies, including reinforcement learning, deep neural networks, and adaptive probabilistic models. Also, finding ways to deal with non-stationary environments and systems that can only be partially seen will make Markov-based decision-making frameworks much more useful in the real world.

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