RESEARCH PAPER

Analyzing a COVID-19 Spread Model with Fractional-Order Derivatives via the Elzaki Transform

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ABSTRACT

we investigate several new results, including the presence of a solution to a fractional order prototype that confirms that the effect of immigration on the spread of the most recent COVID-19 outbreak has been underestimated. It has been proven that the fixed point theory approach is the subject of this in-depth examination. We discovered several series type results after applying the Elzaki transformation to the qualitative result, which was considered as a prototype. The result discussion includes the predicted results of the solution of the prototype data with different scenarios.

Keywords: Fractional Calculus, Elzaki Transform, COVID-19

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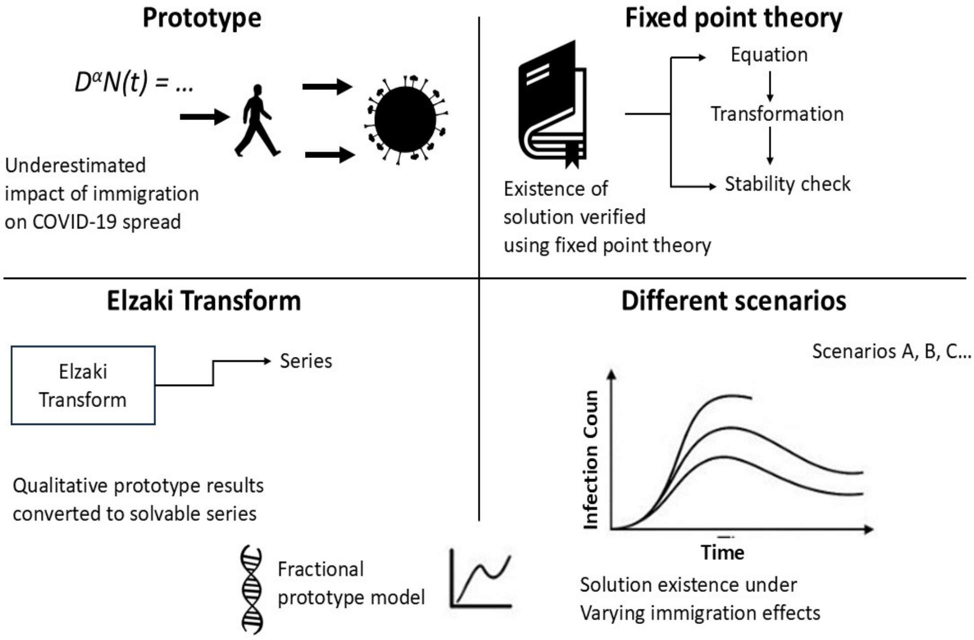
GRAPHICAL ABSTRACT



1. INTRODUCTION

The COVID-19 epidemic has recently touched around two-thirds of the world's population in one manner or another. In December 2019, the sickness first appeared in Wuhan, China. So far, around four million individuals have been infected with this epidemic, with over fourteen lakh deaths [1]. The noteworthy feature is that a sizable proportion of the human population has recovered from this ailment. Every country and government is working to build management-specific policies. Every country on the planet should take safeguards and practice caring behaviors.

As a result, several nations throughout the world have implemented lockdowns or limited needless travel and crowded activities. Understanding pandemic behavior requires statistical data and certain mathematical formulation notions. By applying statistical data to various ailments, we will be able to create mathematical formulas.

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Different researchers have evaluated the current coronavirus epidemic using various techniques of study, as seen in [2]-[4]. Some used mathematical models to forecast future facts about the condition. The facts about the epidemic's past, present, and future can help policymakers develop effective control policies. Because of the relevance of modeling, the condition mentioned above has been studied in a number of research papers [5]. The mathematical model's development includes several factors based on particular assumptions, such as the notion that immigration is the most significant driver of disease transmission, thus it will be included as a model term. Because COVID-19 spreads swiftly in social situations, various scientists have investigated the dynamics of this type of sickness for transmission due to immigration, as follows:

$$\begin{cases} \dfrac{d\varkappa}{dt} = \varkappa(t)a - y(t)\varkappa(t)b + \varkappa(t)e \\ \dfrac{dy}{dt} = (-d - e + c)y(t) + \varkappa(t)by(t) \\ \varkappa(0) = \varkappa_0, \qquad y(0) = y_0 \end{cases} \tag{1.1}$$

For analyzing predator-prey interactions, we used mathematical models informed by the classic Lotka-Volterra model [6]-[7]. The mortality rate is denoted by d, whereas the treatment rate is represented by e. This model simply depicts what happens in a community when individual immigration has no influence over it. Keeping the aforementioned principles in mind, we must investigate the modified model given in (1.1) by involving birth rate β and natural death rate δ for non-integer order of derivative with $0 < n \leq 1$ is given by

$$\begin{cases} \dfrac{d^n\varkappa(t)}{dt^n} = \beta + \varkappa(t)a - y(t)b\varkappa(t) + (e - \delta)\varkappa(t) \\ \dfrac{d^n y(t)}{dt^n} = y(t)b\varkappa(t) + (c - d - e - \delta)y(t) \\ \varkappa(0) = \varkappa_0, \qquad y(0) = y_0 \end{cases} \tag{1.2}$$

where y_0 and $\varkappa_0$ are the initial densities of healthy and diseased persons in percentage. Furthermore, the total population $X = \varkappa + y$ holds true. Also, a, b, c, d, e, δ, and β all exceed zero.

Fractional calculus allows fractional global operators to describe physical and biological issues pertaining to real-world events with a high degree of freedom. several phenomena, such as the qualitative and numerical analysis of non-integer order differential and integral equations, have been discussed in several books and monographs [7]-[11]. Modern calculus extended the classical calculus of integer order differentiation and integration to rational or complex numbers, expressing the relationship between two integers as in [12],[13]. Furthermore, the scientists changed these equations to an arbitrary set of differential equations, yielding a considerably more genuine solution [12]-[16]. They examined these equations for existence and uniqueness using some of the fixed point theory features and theorems described in [17]. FDEs have also shown to be useful for getting analytical and numerical solutions for many academics and researchers. Finding an exact solution is a tough undertaking; hence, most researchers studying FDEs use current approaches to optimize and estimate solutions. They used Taylor's series approach, Modified Euler techniques, Adams Bash-Forth techniques, predictor-corrector method, and other integral transforms, as well as wavelets methods, as described in [8], [18]-[20].

The Elzaki transform is a useful tool in COVID-19 mathematical modeling because it simplifies and solves complicated differential equations that characterize the virus's propagation. It aids in the analysis of disease dynamics, such as infection rates and recovery patterns, by translating time-domain functions into a more manageable algebraic form. The transform can enhance computing efficiency and produce exact answers, aiding in forecasting outbreak trajectories, evaluating intervention tactics, and assisting policymakers in making educated pandemic control decisions. We will now study our chosen topic for qualitative and numerical analysis utilizing the principles of fixed point theory and Elzaki transform, as well as various decomposition strategies. We will also look into the influence of immigration on pandemic transmission in society, utilizing different fractional or arbitrary ordering to inform future policy.

2. QUALITATIVE ANALYSIS

In this part, we shall examine the qualitative features of the fractional order model under consideration. Fixed point theory is well-suited to handling such features. We will utilize some fundamental theorems such as Banach contraction and Schauder theorems to obtain our desired conclusion as

$$\frac{d^n\varkappa(t)}{dt^n} = \Phi_1(\varkappa(t), y(t), t)$$

$$\frac{d^{n}y(t)}{dt^{n}} = \Phi_2(\varkappa(t), y(t), t)$$
$$\varkappa(0) = \varkappa_0, \qquad y(0) = y_0, \qquad n \in (0,1] \qquad (2.1)$$

Upon integration for $0 < n \leq 1$ to the equation (2.1), we get the given system as:

$$\varkappa(t) = \varkappa_0 + \frac{1}{\Gamma(n)}\int_0^{t}(t-s)^{n-1}\Phi_1\big(s, \varkappa(s), y(s)\big)ds$$
$$y(t) = y_0 + \frac{1}{\Gamma(n)}\int_0^{t}(t-s)^{n-1}\Phi_2\big(s, \varkappa(s), y(s)\big)ds \qquad (2.2)$$

Now, we will define $\infty > T > t > 0$ and define Banach space as $E_1 = C((0,T] \times R^2+, R_+)$, then $E = E_1 \times E_2$ will also be the Banach space having the norm $\|(\varkappa, y)\| = \max_{t\in[0,T]}|\varkappa(t)| + \max_{t\in[0,T]}|y(t)|$. Expressing the system (2.2) as

$$\mathcal{Z}(t) = \mathcal{Z}_0(t) + \frac{1}{\Gamma(n)}\int_0^{t}(t-\varsigma)^{n-1}\phi(\varsigma, \mathcal{Z}(\varsigma))d\varsigma \qquad (2.3)$$

where,

$$\mathcal{Z}(t) = \begin{cases}\varkappa(t)\\ y(t)\end{cases}, \quad \mathcal{Z}_0(t) = \begin{cases}\varkappa_0(t)\\ y_0(t)\end{cases}, \quad \phi(t, \mathcal{Z}(t)) = \begin{cases}\Phi_1(\varkappa(t), y(t), t)\\ \Phi_2(\varkappa(t), y(t), t)\end{cases} \qquad (2.4)$$

By taking conditions; $\phi: [0,T] \times R_+^2 \to R_+$ as:

(A1) $\exists$ a constant $L_\phi > 0$; $\forall \mathcal{Z}(t), \overline{\mathcal{Z}(t)} \in R \times R$;

$$|\phi(t, \mathcal{Z}(t) - \phi(t, \overline{\mathcal{Z}(t)})| \leq L_\phi|\mathcal{Z}(t) - \overline{\mathcal{Z}(t)}|$$

(A2) $\exists$ constants $C_\phi > 0$ & $M_\phi > 0$;

$$|\phi(t, \mathcal{Z}(t))| \leq C_\phi|\mathcal{Z}| + M_\phi$$

Theorem 2.1: "Under the continuity of together with the assumption (A2), system (2.1) has at least one solution".

Proof. We have taken "Schauder fixed point theorem", to derive our result. By taking bounded subset B of E as

$$B = \{\mathcal{Z} \in E: \|\mathcal{Z}\| \leq R, R > 0\}$$

By this take a function $\mathbf{A}: B \to B$ and applying (2.3) as

$$\mathbf{A}(\mathcal{Z}) = \mathcal{Z}_0(t) + \frac{1}{\Gamma(n)}\int_0^{t}(t-\zeta)^{n-1}\phi(\zeta, \mathcal{Z}(\zeta))d\zeta \qquad (2.5)$$

At any $\mathcal{Z} \in B$, follows

$$\begin{aligned}|\mathbf{A}(\mathcal{Z})(t)| &\leq |\mathcal{Z}_0| + \frac{1}{\Gamma(n)}\int_0^{t}(t-\zeta)^{n-1}\phi(\zeta, \mathcal{Z}(\zeta))d\zeta,\\ &\leq |\mathcal{Z}_0| + \frac{1}{\Gamma(n)}\int_0^{t}(t-\zeta^{n-1}[C_\phi|\mathcal{Z}| + M_\phi]ds,\\ &\leq |\mathcal{Z}_0| + \frac{T^{n}}{\Gamma(n+1)}[C_\phi\|\mathcal{Z}\| + M_f],\end{aligned}$$

Which implies that

$$\begin{aligned}\|\mathbf{A}(\mathcal{Z})\| &\leq |\mathcal{Z}_0| + \frac{T^{n}}{\Gamma(n+1)}[C_\phi\|\mathcal{Z}\| + M_f],\\ &\leq R \qquad (2.6)\end{aligned}$$

From (2.6), one has implied that $\mathcal{Z} \in B$.Thus $A(B) \in B$.From this, we say that the operator A is closed and bounded. Next, we go ahead to prove the result for a completely continuous operator: Let $t_2 > t_1$ lies in $[0,T]$,and take

$$\begin{aligned}|\mathbf{A}(\mathcal{Z})(t_2) - \mathbf{A}(\mathcal{Z})(t_1)| &= \left|\frac{1}{\Gamma(n)}\int_0^{t_2}(t_2-\zeta)^{n-1}, \phi(s, \mathcal{Z}(\zeta))d\zeta - \frac{1}{\Gamma(n)}\int_0^{t_1}(t_1-\zeta)^{n-1}\,\phi(\zeta, \mathcal{Z}(\zeta))d\zeta\right|,\\ &\leq \frac{1}{\Gamma(n)}\left[\int_0^{t_2}(t_2-\zeta)^{n-1} - \int_0^{t_1}(t_1-\zeta)^{n-1}\right](C_\phi R + M_\phi)d\zeta,\\ &\leq \frac{C_\phi R + M_\phi}{\Gamma(n+1)}\,[t_2^{n} - t_1^{n}] \qquad (2.7)\end{aligned}$$

Now from (2.7), we got that as t_1 approaches to t_2,then right side also vanishes. So one concludes that $|\mathbf{A}(\mathcal{Z})(t_2) - \mathbf{A}(\mathcal{Z})(t_1)|$ tends to 0, as t_1 tends to t_2 So **A** is an "equi-continuous operator". By using "Arzela-Ascoli theorem, the operator **A** is the completely continuous operator and also uniformly bounded proved already. By "Schauder's fixed point theorem" model (2.2) one or more than one solution. Further we proceeds for uniqueness as:

Theorem 2.2: "Using (A1), system has unique or one solution if $\frac{T^{n}}{\Gamma(n+1)}L_\phi < 1$".

Proof: Take $\mathbf{A}: E \to E$, consider $\mathcal{Z}$ and $\bar{\mathcal{Z}} \in E$ as

$$\left|\left|\mathbf{A}(Z)-\mathbf{A}(\bar{Z})\right|\right| = \max_{ţ\,\in\,[0,T]}\left|\frac{1}{\Gamma(n)}\int_0^{ţ}(ţ-\zeta)^{n-1}\phi\big(\zeta,Z(\zeta)\big)d\zeta-\frac{1}{\Gamma(n)}\int_0^{ţ}(ţ-\zeta)^{n-1}\phi\big(\zeta,\bar{Z}(\zeta)\big)d\zeta\right|$$
$$\le\frac{T^n}{\Gamma(n+1)}L_\phi\left|\left|Z-\bar{Z}\right|\right| \qquad (2.8)$$

From (2.8), follows

$$\left|\left|\mathbf{A}(Z)-\mathbf{A}(\bar{Z})\right|\right|\le\frac{2T^n}{\Gamma(n+1)}L_\phi\left|\left|Z-\bar{Z}\right|\right| \qquad (2.9)$$

Hence, F is contraction. From the "Banach contraction theorem" system has one solution.

3. GENERAL SERIES SOLUTION OF THE CONSIDER SYSTEM (1.2)

To produce a semi-analytical solution to the considered model, we first generate a general algorithm by using system (2.1). We apply Elzaki Transform to (2.1) on both sides as

$$\hat{E}\left[\frac{d^n\varkappa(ţ)}{dţ^n}\right]=\hat{E}[\phi_1(\varkappa(ţ),y(ţ),ţ)]$$
$$\hat{E}\left[\frac{d^n y(ţ)}{dţ^n}\right]=\hat{E}[\phi_2(\varkappa(ţ),y(ţ),ţ)] \qquad (3.1)$$

On using initial conditions, follows

$$\frac{\hat{E}[\varkappa(ţ)]}{\upsilon^n}=\upsilon^{2-n}\varkappa_0+\hat{E}[\phi_1(\varkappa(ţ),y(ţ),ţ)]$$
$$\frac{\hat{E}[y(ţ)]}{\upsilon^n}=\upsilon^{2-n}y_0+\hat{E}[\phi_2(\varkappa(ţ),y(ţ),ţ)] \qquad (3.2)$$

Or

$$\hat{E}[\varkappa(ţ)]=\upsilon^2\varkappa_0+\upsilon^n\hat{E}[\phi_1(\varkappa(ţ),y(ţ),ţ)]$$
$$\hat{E}[y(ţ)]=\upsilon^2y_0+\upsilon^n\hat{E}[\phi_2(\varkappa(ţ),y(ţ),ţ)] \qquad (3.3)$$

Let, needed solution as:

$$\varkappa(ţ)=\sum_{n=0}^{\infty}\varkappa_n(ţ),\; y(ţ)=\sum_{n=0}^{\infty}y_n(ţ)$$

And the term due nonlinearity as $\varkappa y$ may be expressed as $\varkappa y=\sum_{n=0}^{\infty}\varkappa_n(ţ)\sum_{n=0}^{\infty}y_n(ţ)$, then (3.3) becomes

$$\hat{E}[\textstyle\sum_{n=0}^{\infty}\varkappa_n(ţ)]=\upsilon^2\varkappa_0+\upsilon^n\hat{E}[\phi_1(ţ,\sum_{n=0}^{\infty}\varkappa_n(ţ),\sum_{n=0}^{\infty}y(ţ))]$$
$$\hat{E}[\textstyle\sum_{n=0}^{\infty}y_n(ţ)]=\upsilon^2y_0+\upsilon^n\hat{E}[\phi_2(ţ,\sum_{n=0}^{\infty}\varkappa_n(ţ),\sum_{n=0}^{\infty}y_n(ţ))] \qquad (3.4)$$

On comparison, we have

$$\hat{E}[\varkappa_0(ţ)]=\upsilon^2\varkappa_0$$
$$\hat{E}[y_0(ţ)]=\upsilon^2y_0$$
$$\hat{E}[\varkappa_1(ţ)]=\upsilon^n\hat{E}[\phi_1(\varkappa_0(ţ),y_0(ţ),ţ)]$$
$$\hat{E}[y_1(ţ)]=\upsilon^n\hat{E}[\phi_2(\varkappa_0(ţ),y_0(ţ),ţ)]$$
$$\hat{E}[\varkappa_2(ţ)]=\upsilon^n\hat{E}[\phi_1(\varkappa_1(ţ),y_1(ţ),ţ)]$$
$$\hat{E}[y_2(ţ)]=\upsilon^n\hat{E}[\phi_2(\varkappa_1(ţ),y_1(ţ),ţ)]$$
$$\vdots$$
$$\hat{E}[\varkappa_{n+1}(ţ)]=\upsilon^n\hat{E}[\phi_1(\varkappa_n(ţ),y_n(ţ),ţ)]$$
$$\hat{E}[y_{n+1}(ţ)]=\upsilon^n\hat{E}[\phi_1(\varkappa_n(ţ),y_n(ţ),ţ)]$$

Taking Elzaki inverse both sides:

$$\varkappa_0(ţ)=\varkappa_0,\qquad y_0(ţ)=y_0$$
$$\varkappa_1(ţ)=\hat{E}^{-1}[\upsilon^n\hat{E}[\phi_1(\varkappa_0(ţ),y_0(ţ),ţ)]$$
$$y_1(ţ)=\hat{E}^{-1}[\upsilon^n\hat{E}[\phi_2(\varkappa_0(ţ),y_0(ţ),ţ)]$$
$$\varkappa_2(ţ)=\hat{E}^{-1}[\upsilon^n\hat{E}[\phi_1(\varkappa_1(ţ),y_1(ţ),ţ)]$$
$$y_2(ţ)=\hat{E}^{-1}[\upsilon^n\hat{E}[\phi_2(\varkappa_1(ţ),y_1(ţ),ţ)]$$
$$\vdots$$
$$\varkappa_{n+1}(ţ)=\hat{E}^{-1}[\upsilon^n\hat{E}[\phi_1(\varkappa_n(ţ),y_n(ţ),ţ)]$$
$$y_{n+1}(ţ)=\hat{E}^{-1}[\upsilon^n\hat{E}[\phi_2(\varkappa_n(ţ),y_n(ţ),ţ)] \qquad (3.5)$$

Therefore, the required series solution of the model (1.2) is given as follows:

$$\varkappa(ţ)=\varkappa_0(ţ)+\varkappa_1(ţ)+\varkappa_2(ţ)+\cdots$$
$$y(ţ)=y_0(ţ)+y_1(ţ)+y_2(ţ)+\cdots \qquad (3.6)$$

4. APPROXIMATE RESULTS AND DISCUSSION

Now we take some values for (1.1) parameters in fractional order to find the estimated outcomes. Considering a community in which the healthy population is at the starting stage $\varkappa_0$ and the infected population is y_0. Using various rates, we will analyze specific situations below:

$$
\begin{gathered}
\frac{d^{n}\varkappa(\text{ţ})}{d\text{ţ}^{n}} = \beta + a\varkappa(\text{ţ}) - b\varkappa(\text{ţ})y(\text{ţ}) + (e-\delta)\varkappa(\text{ţ}) \\
\frac{d^{n}y(\text{ţ})}{d\text{ţ}^{n}} = b\varkappa(\text{ţ})y(\text{ţ}) + (c-d-e-\delta)y(\text{ţ}) \\
\varkappa(0) = \varkappa_0,\ y(0) = y_0
\end{gathered}
\tag{4.1}
$$

Using the proposed algorithm to (4.1) as constructed in (3.5), analogously one has

$$
\begin{cases}
\varkappa_0(\text{ţ}) = \varkappa, \qquad y_0(\text{ţ}) = y_0 \\
\varkappa_1(\text{ţ}) = [\beta + a\varkappa_0 - b\varkappa y_0 + (e-\delta)\varkappa_0]\dfrac{\text{ţ}^{n}}{\Gamma(n+1)} \\
y_1(\text{ţ}) = [b\varkappa_0 y_0 + (c-d-e-\delta)y_0]\dfrac{\text{ţ}^{n}}{\Gamma(n+1)} \\
\varkappa_2(\text{ţ}) = [\beta + (a+e-\delta)\varkappa_{11}]\dfrac{\text{ţ}^{2n}}{\Gamma(2n+1)} - [b\varkappa_{11}y_{11}]\left(\dfrac{\Gamma(2n+1)}{\Gamma^2(n+1)}\right)\dfrac{\text{ţ}^{3n}}{\Gamma(3n+1)} \\
y_2(\text{ţ}) = [b\varkappa_{11}y_{11}]\left(\dfrac{\Gamma(2n+1)}{\Gamma^2(n+1)}\right)\dfrac{\text{ţ}^{3n}}{\Gamma(3n+1)} + [(c-d-e-\delta)y_{11}]\dfrac{\text{ţ}^{2n}}{\Gamma(2n+1)} \\
\varkappa_3(\text{ţ}) = \beta + a\varkappa_{111}\dfrac{\text{ţ}^{3n}}{\Gamma(3n+1)} - ay_{111}\dfrac{\Gamma(2n+1)}{\Gamma^2(n+1)}\dfrac{\text{ţ}^{4n}}{\Gamma(4n+1)} \\
-b\varkappa_{111}y_{111}\dfrac{\Gamma(5n+1)}{\Gamma^2(n+1)\Gamma(3n+1)\Gamma(6n+1)}\text{ţ}^{6n} - b\varkappa_{111}y_{22}\dfrac{\Gamma(4n+1)}{\Gamma^2(2n+1)\Gamma(5n+1)}\text{ţ}^{5n} \\
+b(y_{111})^2\dfrac{\Gamma^2(2n+1)}{\Gamma^4(n+1)\Gamma^2(3n+1)}\text{ţ}^{6n} + by_{111}y_{22}\dfrac{1}{\Gamma^2(n+1)\Gamma(3n+1)}\text{ţ}^{5n} \\
+ey_{111}\dfrac{\Gamma(2n+1)}{\left(\Gamma(n+1)\right)^2\left(\Gamma(4n+1)\right)}\text{ţ}^{4n} + ey_{22}\dfrac{\text{ţ}^{3n}}{\Gamma(3n+1)} \\
y_3(t) = b\varkappa_{111}y_{111}\dfrac{\Gamma(5n+1)}{\Gamma^2(n+1)\Gamma(3n+1)\Gamma(6n+1)}\text{ţ}^{6n} - b\varkappa_{111}y_{22}\dfrac{\Gamma(4n+1)}{\Gamma^2(2n+1)\Gamma(5n+1)}\text{ţ}^{5n} \\
-b(y_{111})^2\dfrac{\Gamma^2(2n+1)}{\Gamma^4(n+1)\Gamma^2(3n+1)}\text{ţ}^{6n} - by_{111}y_{22}\dfrac{1}{\Gamma^2(n+1)\Gamma(n+1)}\text{ţ}^{5n} \\
+(c-d-e-\delta)y_{111}\dfrac{\Gamma(2n+1)}{\Gamma^2(n+1)\Gamma(4n+1)}\text{ţ}^{4n} + (c-d-e-\delta)y_{22}\dfrac{\text{ţ}^{3n}}{\Gamma(3n+1)}
\end{cases}
\tag{4.2}
$$

Similarly next terms can be calculate by same method. The unknown values in (4.2) are given as:

$$
\begin{gathered}
\varkappa_{11} = \beta + a\varkappa_0 - b\varkappa_0 y_0, \qquad y_{11} = b\varkappa_0 y_0 + (c-d-e-\delta)y_0, \\
\varkappa_{111} = (a+e-\delta)\varkappa_{11}, \qquad y_{111} = b\varkappa_{11}y_{11}, \qquad y_{22} = (c-d-e-\delta)y_{11}
\end{gathered}
$$

Case I: First of all, we present the graphical presentation for steady state solution at integer order and taking $a = 0.001, b = 0.0003, c = 0.0001, d = 0.001, e = 0.05, \delta = 0.5, \beta = 0.0089$.

In Figure 1, we plotted the steady-state solution of the integer order model (1.2) against the supplied date. Clearly, in the face of very low immigration rates a, c, the number of the healthy class grows while the infected population declines. Figure 2 shows the numerical solution using MATLAB for non-integer order up to the first 10 terms of the series using the same variables $a = 0.001$ $b = 0.0003, c = 0.001,\ d = 0.001, e = 0.05, \delta = 0.5, \beta = 0.0089$. We can observe that when the immigration rate is very low, the drop in infected population occurs at a tiny fractional order, while the growth in healthy population occurs at a lower order.

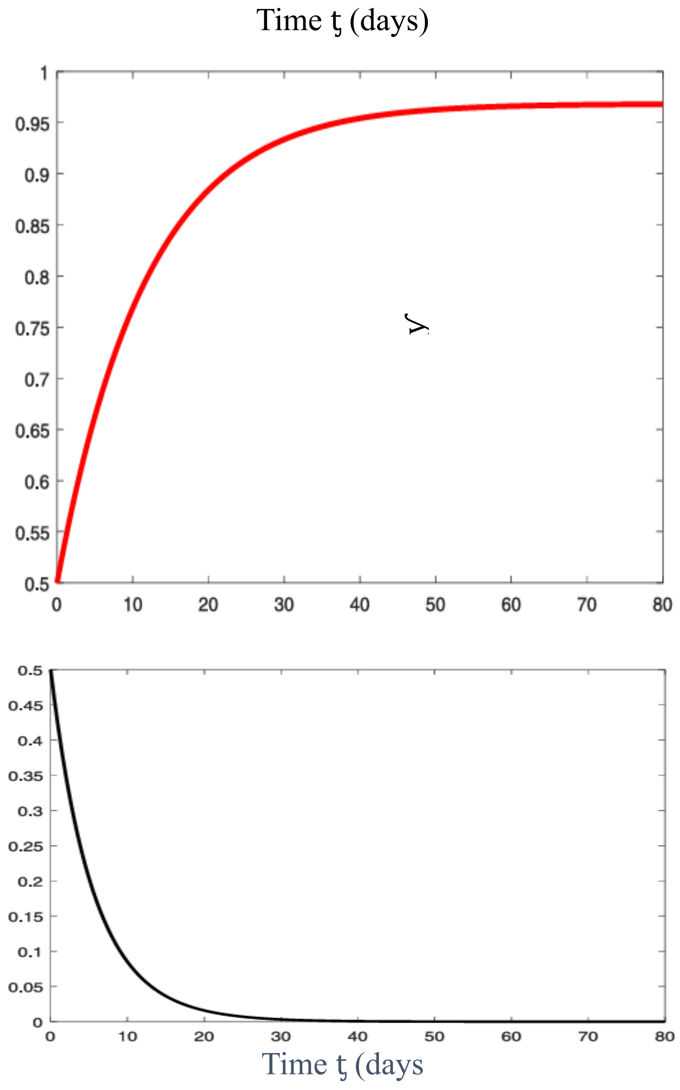
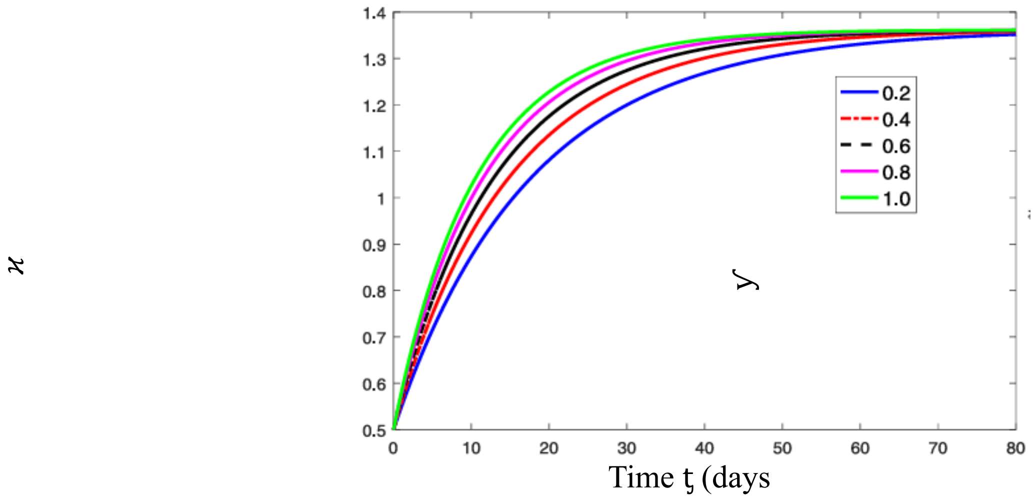


Figure 1: Graph illustrating population dynamics at integer order for Model (1.2) with provided parameter values.

Case II: We simulate the aforementioned findings with $\beta = 0.0089, a = 0.05, b = 0.03, c = 0.05, d = 0.02, e = 0.05, \delta = 0.5$. As illustrated in Figure 3, when both infected and healthy immigration rates are somewhat lower than in Case-I, the dynamics of the corresponding population show themselves at various fractional orders. Here, population dynamics cause oscillations, and the oscillations are different at various arbitrary orders.



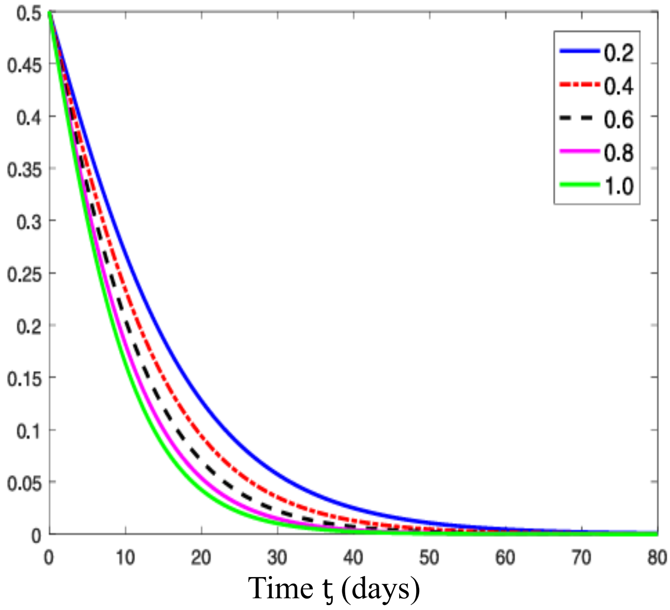


Figure 2: Graph illustrating population dynamics at different fractional orders for system (1.2) with the supplied parameter values.

Case III: The simulation is presented for considering $\beta = 0.0089;\ a = 0.06, b = 0.0003, c = 0.06, d = 0.001, e = 0.05, \delta = 0.5$. (days

Figure *4* depicts the behavior of the estimated solutions up to the first 10 terms representing different fractional orders. Because of the rise in immigration rates, the oscillation is quite rapid.

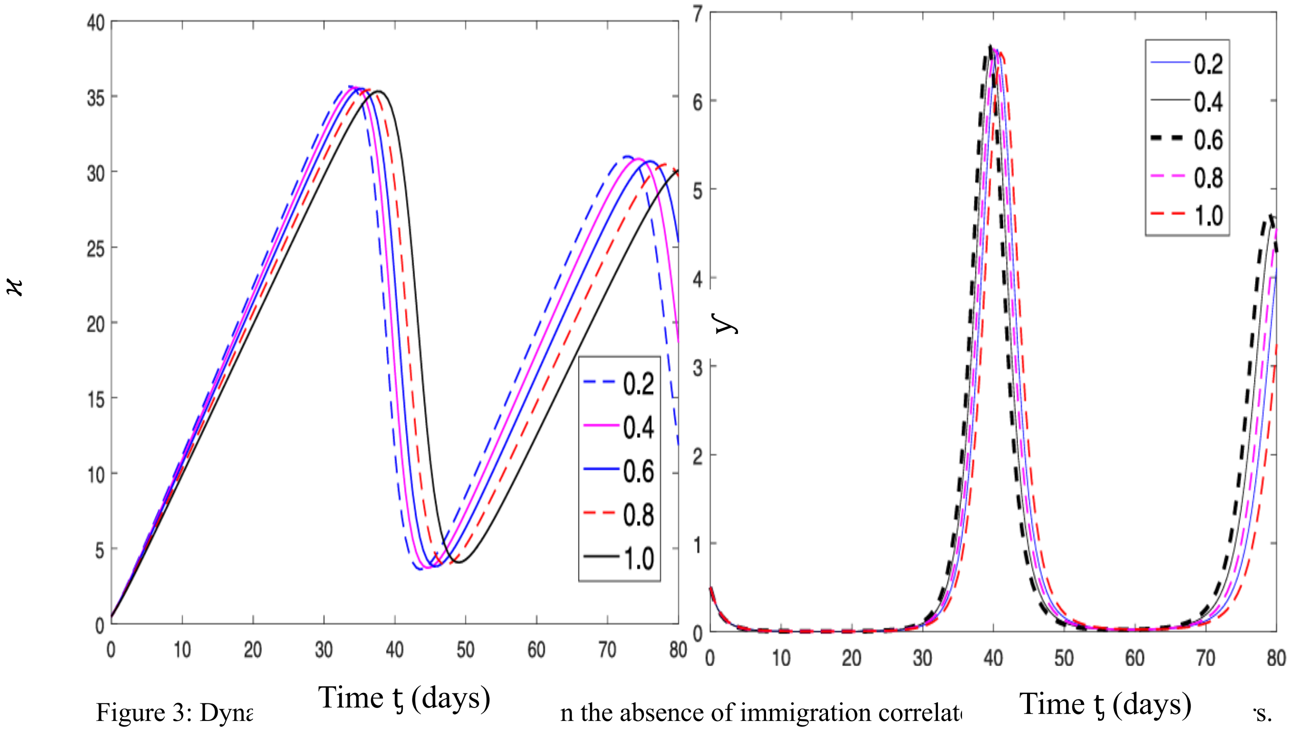


Figure 3: Dyna n the absence of immigration correlat 's.

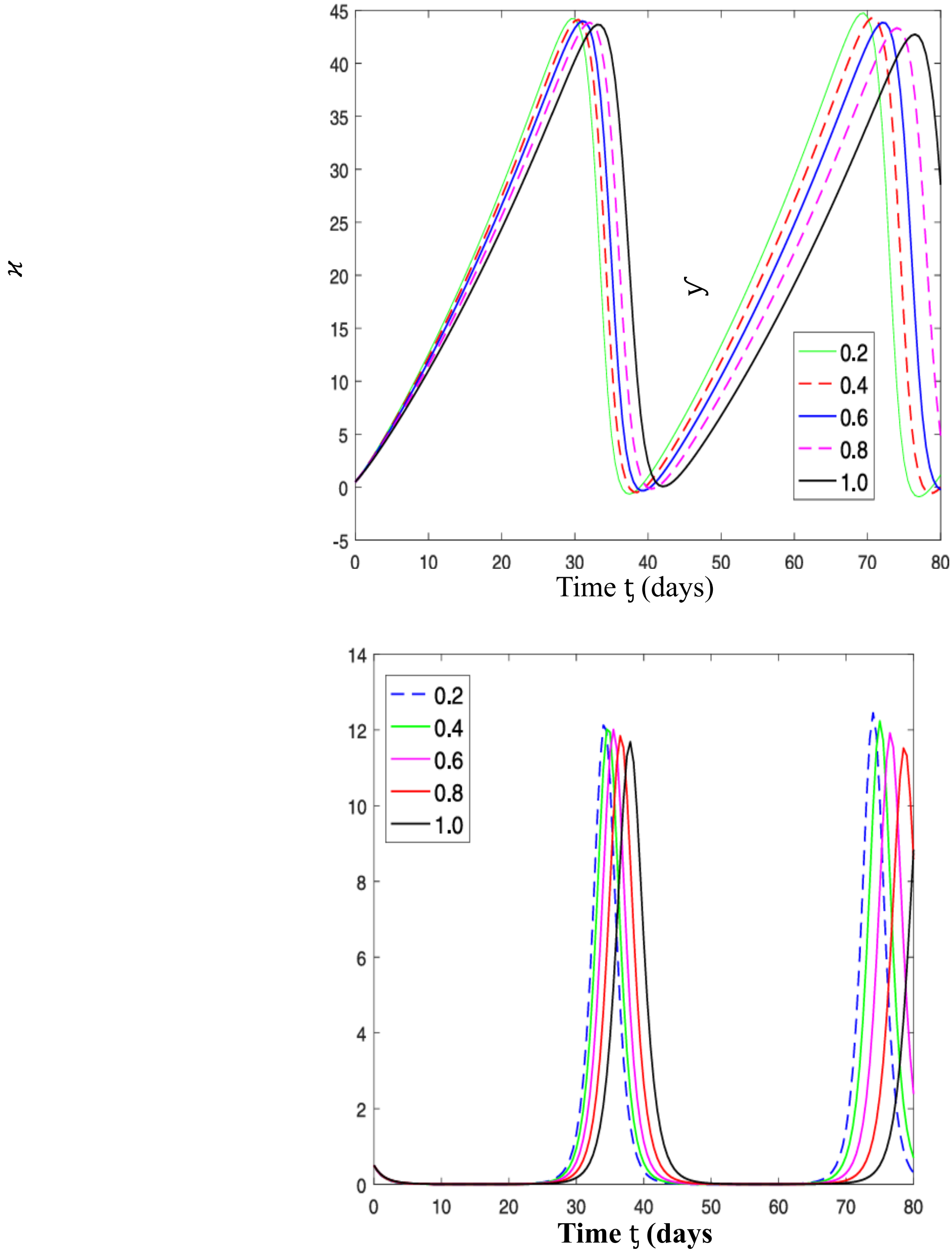


Figure 4: Population dynamics in the context of immigration exhibit distinct fractional orders of behavior.

5. CONCLUSION

Using various experimental values from [20] for a, b, c, d, and e, we may achieve the necessary comparable result for the integer order system of (1.1). We can see that by raising the rate of protection and cure while reducing the pace of immigration, we may gradually reduce the number of diseased people to a minimum or toward stability. By analyzing such a dynamic model, a person may readily learn how to prevent healthy people from infection and discourage diseased individuals from immigrating. This concept may be used to any population that has social events, whether locally or worldwide. Furthermore, fixed point theory demonstrates that such models exist in the actual world or represent real-world situations. However the rate of decay and increase has been demonstrated using fractional differentiation in global terms. Thus, fractional calculus may be utilized to fully describe the prototype. If the recover class is included, we can expand this model to examine the influence of immigration on the cured population as well. The simulation depicts the dynamics of healthy and sick people, as well as some near-term predictions in fractional form. This prototype is only an illustration of how the mentioned characteristics influence the transmission dynamics of the current new coronavirus-19 illness.

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