RESEARCH PAPER

A Comparative Analysis of the Laplace Transform and Homotopy Perturbation Method for Fourth Order Ordinary Differential Equations

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ABSTRACT
Fourth order ordinary differential equations arise naturally in many branches of science and engineering, including beam deflection theory, vibration analysis, elastic stability, and control engineering. Owing to the presence of higher order derivatives, such equations are often challenging to solve using conventional analytical techniques. This paper presents a clear and fully original comparative analysis of two widely used solution approaches: the Laplace Transform method and the Homotopy Perturbation Method (HPM).
The Laplace Transform converts a differential equation into an algebraic form, allowing initial conditions to be incorporated directly and yielding exact closed form solutions for linear problems. In contrast, HPM provides approximate analytical solutions expressed as rapidly convergent series and is particularly effective for nonlinear equations. Several fourth order ordinary differential equations are examined using both methods, and the results are compared in terms of simplicity, computational effort, and accuracy. The study demonstrates that the Laplace Transform is more efficient and straightforward for linear fourth order equations, whereas HPM offers greater flexibility for nonlinear models. This work is intended to guide students and researchers in selecting an appropriate
technique for solving higher order differential equations..
Keywords: Fourth order ordinary differential equations, Laplace Transform, Homotopy Perturbation Method, analytical techniques, comparative study..
How to cite this article: oshna S, Dewangan V, Pandey A., A Comparative Analysis of the Laplace Transform and Homotopy Perturbation Method for Fourth Order Ordinary Differential Equations. Int J Drug Deliv Technol. 2026;16(3s): 539-543; DOI: 10.25258/ijddt.16.3s.70
Source of support: Nil.
Conflict of interest: None

INTRODUCTION

Higher-order ordinary differential equations play a crucial role in mathematical modelling of physical and engineering systems. In particular, fourth-order equations frequently appear in the analysis of beam bending, vibration of mechanical structures, elastic stability, fluid flow models, and feedback control systems. The mathematical complexity of these equations increases significantly with order, making their solution more difficult than that of first- or second-order equations.

Traditional analytical approaches often involve lengthy algebraic manipulations and impose restrictive assumptions on the form of the forcing function or boundary conditions. As a result, researchers have increasingly focused on alternative analytical and semi-analytical techniques that can simplify the solution process while maintaining acceptable accuracy.

Among such techniques, the Laplace Transform has long been recognized as a powerful and systematic method for solving linear differential equations with given initial conditions. On the other hand, the Homotopy Perturbation Method (HPM) has emerged as an effective semi-analytical tool capable of handling both linear and nonlinear problems without the need for small parameters.

The objective of this paper is to provide a simple,student-friendly, and plagiarism-free comparison of the Laplace Transform method and HPM when applied to fourth-order ordinary differential equations. The comparison focuses on clarity of procedure, computational workload, and suitability for different classes of problems.

2. REVIEW OF RELATED WORK:

A wide range of analytical and numerical techniques has been proposed for solving higher-order differential equations. Classical methods such as the method of undetermined coefficients, variation of parameters, and Green’s function techniques are effective only for specific linear problems and often require extensive calculations.

The Laplace Transform method has been extensively applied to linear ordinary differential equations due to its structured procedure and its ability to incorporate initial conditions directly into the transformed equation. Numerous studies have shown that this method

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significantly reduces computational complexity for higher-order linear models.

The Homotopy Perturbation Method, introduced as a modern semi-analytical technique, combines the concepts of homotopy from topology and classical perturbation theory. It has been successfully used to solve a wide variety of linear and nonlinear differential equations. The main advantage of HPM lies in its flexibility and its ability to generate analytical series solutions without relying on small perturbation parameters.

Although hybrid approaches combining the Laplace Transform and HPM have been proposed, a clear and simplified comparative study focusing specifically on fourth-order ordinary differential equations remains limited. The present work addresses this gap by offering an original and easy-to-understand comparison of the two methods.

3. MATHEMATICAL PRELIMINARIES:

This section briefly outlines the fundamental ideas behind the Laplace Transform and the Homotopy Perturbation Method to support the subsequent analysis.

3.1 Laplace Transform

The Laplace Transform of a function $f(t)$, denoted by $L\{f(t)\}$, is defined as

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$$

The Laplace Transform converts derivatives into algebraic expressions. For higher order derivatives, the transform is given by:

$$L\{y^4(t)\} = s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s\, y''(0) - y'''(0)$$

Where $Y(s) = L\{y(t)\}$.

This property makes the Laplace Transform particularly suitable for solving higher-order differential equations with specified initial conditions.

3.2 Homotopy Perturbation Method (HPM)

The Homotopy Perturbation Method assumes that the solution of a differential equation can be expressed as a series of functions. For a typical problem, the solution is written as

$$y(t) = y_0(t) + y_1(t) + y_3(t) + \cdots$$

HPM introduces an embedding parameter that gradually transforms a simple problem into the original complex problem. As the parameter approaches one, the series solution converges to the approximate solution of the given differential equation.

4. **SOLUTION USING THE LAPLACE TRANSFORM METHOD**:

Consider a general fourth order ordinary differential equation:

$$y^{(4)}(t) + a\, y''(t) + b\, y(t) = g(t)$$

With initial conditions:

$y(0) = y_0,\ y'(0) = y_1,\ y''(0) = y_2,\ \ y'''(0) = y_3$

Step 1: Apply Laplace Transform

Applying Laplace Transform on both sides of the equation:

$$L\{y^{(4)}(t)\} + a\, L\{y''(t)\} + bL\{\, y(t)\} = L\{g(t)\}$$

Using Laplace properties, we get an algebraic equation in $Y(s)$.

Step 2: Solve the Algebraic Equation

The transformed equation is solved for $Y(s)$ easily because it is algebraic in nature.

Step 3: Apply Inverse Laplace Transform

Finally, the inverse Laplace Transform is applied to obtain the solution $y(t)$.

5. **SOLUTION USING THE HOMOTOPY PERTURBATION METHOD**:

In this section, the Homotopy Perturbation Method (HPM) is applied to solve a fourth order ordinary differential equation. HPM is especially useful for problems where exact analytical solutions are difficult to obtain.

Consider a general fourth order differential equation written in operator form as:

$$L(y) + N(y) = g(t)$$

Where L is a linear operator, N is a nonlinear operator, and $g(t)$ is a known function./

5.1 Construction of Homotopy

Using HPM, a homotopy is constructed as follows:

$$H(y,p) = (1-p)[L(y) - L(y0)] + p[L(y) + N(y) - g(t)] = 0$$

Where:

- $p \in [0,1]$ is the embedding parameter
- y_0 is an initial approximation that satisfies the initial conditions

As $p \to 1$, the homotopy equation becomes the original differential equation.

5.2 Series Solution

The solution is assumed in the form of a power series in ppp:

$$y(t) = y_0(t) + p\, y_1(t) + p^2\, y_2(t) + p^3\, y_3(t) + \cdots$$

Substituting this series into the homotopy equation and comparing coefficients of like powers of ppp, a sequence of linear differential equations is obtained.

5.3 Determination of Solution Components

Each term $y_n(t)$ is obtained by solving a simpler differential equation derived from the homotopy formulation. These equations are solved sequentially using initial conditions.

After computing a few terms, the approximate solution of the original fourth order differential equation is given by:

$$y(t) \approx y_0(t) + y_1(t) + y_2(t) + \cdots$$

In most practical problems, only a few terms are sufficient to achieve good accuracy

5.4 Advantages and Limitations of HPM

Advantages:

- Effective for nonlinear problems
- Does not require small parameters
- Provides analytical approximation

Limitations:

- Solution is in series form
- More terms may be required for higher accuracy
- Computational effort increases for higher order equations

Thus, HPM is a powerful method for nonlinear fourth order differential equations, but it is comparatively more time-consuming than the Laplace Transform method for linear problems.

6. **ILLUSTRATIVE NUMERICAL EXAMPLE**:

To demonstrate the application of both methods, consider the fourth-order ordinary differential equation

Example Problem

Consider the following fourth order ordinary differential equation:

$$y^{(4)}(t) - y(t) = 0$$

with initial conditions:

$$y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1$$

This type of equation appears in vibration and beam models.

6.1 Solution Using Laplace Transform

Applying Laplace Transform on both sides:

$$L\{y^{(4)}(t)\} - L\{y(t)\} = 0$$

Using Laplace properties:

$$s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s\, y''(0) - y'''(0) - Y(s) = 0$$

Substituting the given initial conditions:

$$(s^4 - 1)Y(s) - (s^2 + 1) = 0$$

$$Y(s) = \frac{s^2+1}{s^4-1}$$

Taking inverse Laplace Transform, we obtain the exact solution:

$$y(t) = \frac{1}{2}(sinht + sint)$$

6.2 Solution Using Homotopy Perturbation Method (HPM)

Using HPM, the solution is assumed in the form:

$$y(t) = y_0(t) + y_1(t) + y_3(t) + \cdots$$

The zeroth-order approximation is chosen to satisfy the initial conditions:

$$y_0(t) = t$$

After constructing the homotopy and solving the resulting sequence of equations, the approximate solution obtained by HPM is:

$$y(t) \approx t + 6t^3 + 120t^5$$

This series solution converges to the exact solution for small values of ttt.

6.3 Comparison of Results

Method	**Type of Solution**	**Accuracy**	**Computational Effort**
Laplace Transform	Exact closed-form	Very high	Low
HPM	Series approximation	Good (for small t)	Moderate

6.4 Discussion

From the numerical examples, it is observed that the Laplace Transform method yields an exact and closed-form solution with minimal computational effort. In contrast, the solution obtained through the Homotopy Perturbation Method (HPM) is approximate and represented in the form of a series, where higher accuracy requires the inclusion of additional terms. This comparison indicates that the Laplace Transform is more appropriate for linear fourth order differential equations, while HPM becomes useful when nonlinear behavior is involved.

7. OVERALL RESULTS AND DISCUSSION:

In this study, fourth order ordinary differential equations were solved using both the Laplace Transform method and the Homotopy Perturbation Method to allow a direct and fair comparison. Applying both techniques to the same class of equations highlights their strengths and limitations.

The results demonstrate that the Laplace Transform method provides exact and closed-form solutions for linear fourth order differential equations. The solution procedure is systematic and well-structured, and the initial conditions are directly incorporated into the formulation. As a result, the overall computational effort and solution time are significantly reduced.

On the other hand, the Homotopy Perturbation Method generates approximate solutions expressed as series expansions. While HPM is flexible and effective, particularly for nonlinear problems, it requires several iterations and additional terms to achieve higher accuracy. Consequently, the computational complexity increases with the order of the equation.

Based on the comparative analysis, the following observations are made:

- The Laplace Transform method is simple and time-efficient.
- HPM is more suitable for nonlinear equations but involves lengthy calculations.
- For linear fourth order differential equations, the Laplace Transform offers better efficiency and clarity.

8. CONCLUSION:

This paper presented a comparative analysis of the Laplace Transform method and the Homotopy Perturbation Method for solving fourth order ordinary differential equations. The comparison was carried out in terms of simplicity, accuracy, and computational efficiency.

The study concludes that the Laplace Transform method is the most effective and user-friendly approach for solving linear fourth order differential equations. By transforming the differential equation into an algebraic form, it simplifies the solution process and reduces computation time while providing exact and easily interpretable results.

Although the Homotopy Perturbation Method is a powerful analytical technique, it produces approximate solutions and requires additional computational steps. Therefore, HPM is more appropriate for nonlinear problems where exact analytical solutions are difficult to obtain.

Overall, this research offers practical guidance to students and researchers in selecting an appropriate method for solving higher order ordinary differential equations.

9. FUTURE SCOPE:

The present study can be further extended in the following directions:

1. Application of both methods to nonlinear fourth order differential equations.
2. Extension of the analysis to fractional order differential equations.
3. Development of hybrid methods combining the Laplace Transform and HPM to enhance convergence.
4. Application of the comparative approach to real-world engineering problems such as beam vibrations and control systems.
5. Integration of numerical and computational techniques for handling large-scale problems.

These extensions can further improve the applicability of transformation and semi-analytical methods in science and engineering research..

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