RESEARCH PAPER

A NANO TOPOLOGICAL APPROACH TO KIDNEY FAILURE DIAGNOSIS USING NANO ALPHA OPEN SETS

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ABSTRACT:

Nano topology is an emerging area of topology derived from rough set theory, focusing on the concept of nano open sets and their characteristics. This research investigates the properties of nano-α-open sets and nano closures within nano topological spaces, and presents corresponding definitions and theorems for clarity. The practical application of nano α-open sets in medical decision-making is illustrated through two case studies related to kidney failure diagnosis. The findings show that the application of nano open sets enhances diagnostic accuracy, sensitivity, and specificity. Moreover, an algorithm has been designed to support healthcare professionals in identifying kidney failure. The primary aim of this work is to assist physicians in making well-informed decisions and improving patient care outcomes.

Keywords: Nano topology, Nano alpha open sets, Kidney failure diagnosis, Rough set theory, Medical decision-making

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I-Introduction

Topology is a branch of mathematics that studies properties of spaces preserved under continuous transformations, often described as “rubber-sheet geometry.” It has significant applications in computer science and applied mathematics, including extensions such as rough and soft topology. Rough set theory, introduced in 1982[1,2,3], addresses uncertainty by using lower and upper approximations of sets. The boundary region represents imprecision, and minimizing it improves accuracy. This theory is widely applied in data analysis, artificial intelligence, and other real-world domain. In addition to handling uncertainty, optimization techniques play a crucial role in improving system performance across engineering applications. For instance, Particle Swarm Optimization (PSO), a nature-inspired computational method, has been effectively applied to optimize process parameters in manufacturing systems. In the context of surface grinding operations, PSO has been used to determine optimal combinations of machining parameters such as wheel speed, feed rate, and depth of cut in order to enhance surface quality and material removal rate. One such extension is nano topology which is a branch of topology developed from rough set theory[4-9], focusing on nano open sets in finite universes. It defines basic notions such as nano interior, nano closure, and different types of generalized open sets. This approach helps in studying problems involving uncertainty and incomplete information[10]. It has practical uses in areas like pattern recognition, decision-making, and data analysis, often in combination with fuzzy and soft set methods.

Motivated by these developments, the present work focuses on the study of nano α-open sets and aims to establish new properties associated with them. The study also introduces additional concepts in nano topological spaces based on nano closure. Special attention is given to examining the relationships between these concepts and exploring their fundamental characteristics[11-16]. Furthermore, the applicability of nano α-open sets in medical decision-making is investigated[18-30], particularly in the context of breast cancer diagnosis using patient data.The paper is organized as follows. The preliminary concepts required for this study are presented in Section 2. Section 3 is devoted to the definition and properties of nano α-open sets, supported by suitable examples. In Section 4, a new concept related to nano closure is introduced and analyzed. We discusses an application of nano topology in medical diagnosis.

II-Preliminaries

This section contains important definitions and notions that are useful for establishing the results of this paper.

Definition 2.1

Let R an equivalence relation on ζ known as the Indiscernibility relation. Let ζ be a non-empty finite set of things called the universe. After that, ζ is separated into classes with disjoint equivalencies. It is argued that elements that are indistinguishable from one another belong to the same equivalency class.

Definition 2.2

Let ζ be a non-empty finite set of objects called the universe and R be an equivalence relation on ζ names as indiscernibility relation. Then ζ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (ζ,R) is said to be the approximation space.

Let X⊆ζ.Then

- The lower approximation of X with respect to R is the set of all the objects which can be for certainly classified as X with respect to R is denoted by $L_R(X)$.
$$L_R(X)=\bigcup_{x\in\zeta}\{R(X):R(X)\subseteq X\}$$
- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$$U_R(X)=\bigcup_{x\in\zeta}\{R(X):R(X)\cap X\neq\phi\}$$
- The boundary region of X with respect to R is the set of all objects which can be classified either as X nor as not-X with respect to R and is denoted by $B_R(X)$.
$$B_R(X)=U_R(X)-L_R(X)$$

Definition 2.3

Let ζ be the universe, R be an equivalence relation on ζ and $\tau_R(X)=\{\zeta,\phi,L_R(X),U_R(X),B_R(X)\}$ where X⊆ζ. Then $\tau_R(X)$ satisfies the following axioms:

- ζ and $\phi\in\tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is called the Nano topology on U with respect to X, (ζ,$_R(X)$) is called the Nano topological space.

Definition 2.4

If (ζ, $\tau_R(X)$) is a Nano topological space with respect to X where X⊆ζ and if A⊆ζ, then, Nano interior of a set A is defined as the union of all the Nano open sets contained A and it is denoted by N-int(A).

Definition 2.5

If (ζ, $\tau_R(X)$) is a Nano topological space with respect to X where X⊆ζ and if A⊆ζ, then Nano closure of a set A is defined as the intersection of all nano closed sets containing A and it is denoted by N-cl(A).

Definition 2.6

A subset A of a NTS (ζ, $\tau_R(X)$)is nano α-open in ζ if $E\subseteq N\operatorname{int}(Ncl(N\operatorname{int}(A)))$. The set of all nano α-open of ζ is denoted by $\alpha^N O(\zeta)$.

Definition 2.7

Let $\mathcal{X}$ be a binary relation on $\zeta : \mathcal{D} \to \mathcal{C}$ the preliminary lower and preliminary upper approximations of $A \subseteq \mathcal{U}$ are defined respectively as $\mathcal{L}_i(\mathcal{D}) = \{ \alpha \in \zeta : x_i(y) \subseteq \mathcal{D}\}$ and $\mathcal{U}_i(\mathcal{D}) = \{ \alpha \in \zeta : x_i(y) \bigcap \mathcal{D} \neq \phi \}$

Definition 2.8

Consider that $\mathcal{R}$ is a binary relation upon universe ζ . The preliminary-positive, preliminary-negative and preliminary-boundary regions along with the precision of initial approximations for a subset $\mathcal{D} \subseteq \mathcal{U}$are provided respectively by,

$\wp os_i(\mathcal{D}) = \mathcal{L}_i(\mathcal{D})$, $\mathcal{N}eg_i(\mathcal{D}) = \mathcal{U} - \mathcal{U}_i(\mathcal{D})$, $\mathcal{B}nd_i(\mathcal{D}) = \mathcal{U}_i(\mathcal{D}) - \mathcal{L}_i(\mathcal{D})$and $\mathcal{K}_i(\mathcal{A}) = \frac{\mathcal{L}_i(\mathcal{D})}{\mathcal{U}_i(\mathcal{D})}$, where $\mathcal{U}_i(\mathcal{D}) \neq \phi$

Definition: 2.9

Let U be the universe and R be a binary relation. For any element x in U, the right neighborhood of x is

$X_r(x) = y \in U /(x, y) \in R\}$

Definition: 2.10

The initial right neighborhood of an element alpha in U is defined as

$X_i(\alpha) = \beta \in U / X_r(\alpha) \subseteq X_r(\beta)\}$.

Example: 2.11

U={a,b,c,d}, R={(a,b), (a,c), (b,c), (b,d), (c,d), (d,d)}

The Right Neighborhoods are, $X_r(a)$={b,c}, $X_r(b)$={c,d}, $X_r(c)$={d}, $X_r(d)$={d}.

The Intial right neighborhoods are, $X_i(a)$={a}, $X_i(b)$={b}, $X_i(c)$={b,c,d}, $X_i(d)$={b,c,d}.

Definition 2.12

Consider $\mathcal{U}$ constitute the finite set, $\mathcal{L}_i(\mathcal{D})$ and $\mathcal{U}_i(\mathcal{D})$ be the lower and upper approximations of $\mathcal{D} \subseteq \mathcal{U}$. The collection $\tau^{G(N)} = \{ \zeta , \phi, \mathcal{L}_i(\mathcal{D}) , \mathcal{U}_i(\mathcal{D}) , \mathcal{B}nd(\mathcal{D})\}$ is a topology on ζ if $\mathcal{L}_i(\mathcal{D})$ and $\mathcal{U}_i(\mathcal{D})$ fulfill the rough set properties defined by Pawlak, where $\mathcal{B}nd_i(\mathcal{D})$ is the boundary region of $\mathcal{D} \subseteq \mathcal{U}$.

Definition 2.13

Let a quaternion (, A, V, f) represent an information system where ={x_1, x_2 , x_3,…..x_n}, is the universe $A = \{a_1, a_2, \dots a_n\}$ denotes a non-empty infinite set of attributes, $V = \bigcup_{\alpha \in A} V_\alpha$ denotes the set of all attribute values a, consider $f : \zeta \times A \to V$ denotes the mapping function for all $x_i \in \zeta$, $a \in A, f(x_i, a) \in V_\alpha$. This information system is called a decision system (ζ , B, D) if the set of attributes in the information system above satisfies $A = B \bigcup D = \phi$ and $B \bigcap D = \phi$ where B is the condition attribute set and D is the decision attribute set.

III - Nano alpha Topological Space

In this section, we presented the properties of the alpha open set. Including the outer boundary set and the boundary point set of the beta type. We studied the properties of these concepts and provided examples for more illustrations

Definition: 3.1

If $(\zeta, \tau_R(X))$ is a NTS concerning X and $\mathcal{A} \subseteq \zeta$ then $\mathcal{A}$ is called

(i) Nano α -closure is the intersection of all nano α -closed subsets of ζ containing $\mathcal{A}$ and it is denoted by N-cl($\mathcal{A}$). In Short,$N\alpha - cl(\mathcal{A}) = \zeta - \mathrm{N}\alpha - int(\zeta - \mathcal{A})$.

(ii) Nano α -interior is the union of all nano α -open subsets of ζ contained in $\mathcal{A}$ and it is denoted by N-int($\mathcal{A}$). In Short,$N\alpha - int(\mathcal{A}) = \zeta - (\zeta - \mathcal{A})_{N\alpha}$.

Example: 3.2

(i) Let $\zeta = \{a, b, c, d\}$, $U / R = \{\{a\}, \{c\}, \{b, d\}\}$, $X = \{a, b\}$, $\tau_R(X) = \{\phi, \zeta, \{a\}, \{b, d\}, \{a, b, d\}\}$, Nano $\alpha O(\zeta) = \{\phi, \zeta, \{a\}, \{b, d\}, \{a, b\}, \{a, d\}, \{a, c, d\}, \{b, c, d\} \{a, b, d\}\}$, Let A={b,c}. $N\alpha - int\left(\zeta - \mathcal{A}\right) = \{\phi, \{a\}, \{a, d\}\}$ $\Rightarrow$ $N\alpha - cl(\mathcal{A}) = \zeta - \mathrm{N}\alpha - int(\zeta - \mathcal{A})$ $\Rightarrow$ $N\alpha - cl(\mathcal{A}) = \{a, b, c, d\} - \left\{\phi, \{a\}, \{a, d\}\right\}$. $\Rightarrow$ $\mathcal{A}_{N\alpha}$= {b,c}.

(ii) $N\alpha - int(\mathcal{A}) = \zeta - (\zeta - \mathcal{A})_{N\alpha}$. $\Rightarrow$ $(\zeta - \mathcal{A})_{N\alpha}$ ={a,d} $\Rightarrow$ $N\alpha - int(\mathcal{A}) = \{a, b, c, d\} - \{a, d\} \Rightarrow N\alpha - int(\mathcal{A})$={b,c}.

Definition: 3.3

If $(\zeta, \tau_R(X))$ is a NTS with respect to X and if $\mathcal{A} \subseteq \zeta$ then the nano α-boundary of

$\mathcal{A}$(briefly $\mathcal{B}^{N\alpha}(\mathcal{A})$) is $\mathcal{B}^{N\alpha}(\mathcal{A}) = N\alpha - cl(\mathcal{A}) \cap \overline{(U-\mathcal{A})}^{N\alpha}$.

A point $a \in \mathcal{B}^{N\alpha}(\mathcal{A})$ is termed as α-boundary point of $\mathcal{A}$ it is clear that $\mathcal{B}^{N\alpha}(\mathcal{A}) \subseteq \mathcal{B}^{N}(\mathcal{A})$, where $\mathcal{B}^{N}(\mathcal{A})$ is the boundary of $\mathcal{A}$.

The reverse might not hold as the following example illustrates.

Example : 3.4

In the above example 3.2, we can have that $\mathcal{B}^{N\alpha}(\mathcal{A}) = \emptyset$ and $\mathcal{B}^{N}(\mathcal{A})$={b,c,d}.
$\Rightarrow \varphi \subseteq \{b,c,d\}$. Therefore $\mathcal{B}^{N\alpha}(\mathcal{A}) \subseteq \mathcal{B}^{N}(\mathcal{A})$.

Theorem: 3.5

If $(\zeta, \tau_R(X))$ is a NTS and if $\mathcal{A} \subseteq \zeta$ be nano subsets then $x \in N\alpha - cl(\mathcal{A})$ if and only if for each $\delta \in \alpha O(\zeta)$, $x \in \delta, \delta \cap \mathcal{A} \neq \phi$.

Proof:

Suppose that $x \in N\alpha - cl(\mathcal{A})$ and $x \in \delta, \delta \cap \mathcal{A} \neq \phi$. This implies that $\mathcal{A} \subseteq \zeta - G$ where $x - \delta$ nano α closed comprising x. Since $x \in N\alpha - cl(\mathcal{A})$ that is a contradiction.
Thus $\delta \cap \mathcal{A} \neq \phi$. Conversely , let each nano α open set containing x intersects $\mathcal{A}$. Suppose $x \notin N\alpha - cl(\mathcal{A})$. In such case, there is a nano α closed set $\mathcal{Y} \subseteq \zeta$ in a manner such that $\mathcal{Y} \supseteq \mathcal{A}$ and $x \notin \mathcal{Y}$. Hence $\zeta - N\alpha O(\zeta), x \in \zeta - \mathcal{Y}$ and $\zeta - \cap \mathcal{A} = \emptyset$ which implies a contradiction, therefore $x \in N\alpha - cl(\mathcal{A})$.

Theorem: 3.6

If $(\zeta, \tau_R(X))$ is a NTS on X and X $\subseteq \zeta$ be nano subset of ζ so that the subsequent conditions are valid.
(i) Since$\mathcal{B}^{N\alpha}(\mathcal{A})$ is a α closed set.
(ii) $\mathcal{B}^{N\alpha}(\mathcal{A}) = \mathcal{B}^{N\alpha}(\zeta - \mathcal{A})$.
(iii) $\mathcal{B}^{N\alpha}(\mathcal{A}) = N\alpha - cl(\mathcal{A}) - N\alpha O(\mathcal{A})$.
(iv) $\mathcal{B}^{N\alpha}(\mathcal{A}) \cap N\alpha O(\mathcal{A}) = \emptyset$.
(v) $\mathcal{B}^{N\alpha}(\mathcal{A}) \cup N\alpha O(\mathcal{A}) = N\alpha - cl(\mathcal{A})$.

Proof:

(i) Since$\mathcal{B}^{N\alpha}(\mathcal{A})$ is the intersection of α closed sets, thus it is a α closed set.

(ii) $\mathcal{B}^{N\alpha}(\zeta - \mathcal{A}) = N\alpha - cl(\mathcal{A}) \cap (\zeta - \overline{(\zeta - (\zeta - A)}^{N\alpha} = \mathcal{B}^{N\alpha}(\mathcal{A})$.

(iii) $\mathcal{B}^{N\alpha}(\mathcal{A}) = c \cap \overline{(\zeta - \mathcal{A})}^{N\alpha} = N\alpha - cl(\mathcal{A}) \cap (\zeta - \mathcal{A}^{N\alpha O}) = N\alpha - cl(\mathcal{A}) - N\alpha - cl(\mathcal{A}) \cap \mathcal{A}^{N\alpha O} \Rightarrow N\alpha - cl(\mathcal{A}) - \mathcal{A}^{N\alpha O}$.

(iv) $\mathcal{B}^{N\alpha}(\mathcal{A}) \cap N\alpha O = (N\alpha - cl(\mathcal{A}) - N\alpha O(\mathcal{A}) \cap N\alpha O(\mathcal{A}) = N\alpha O(\mathcal{A})$- $N\alpha O(\mathcal{A}) = \emptyset$.

(v) It is obviously proved by 3 and 4.

Theorem: 3.7

If $(\zeta, \tau_R(X))$ is a NTS on X, and $\mathfrak{D} \subseteq \zeta$ be nano subset of ζ then,
(i) $\mathfrak{D}$ is nano α -open set if and only if $\mathcal{B}^{N\alpha}(\mathfrak{D}) \cap \mathfrak{D} = \emptyset$.
(ii) $\mathfrak{D}$ is nano α -closed set if and only if $\mathcal{B}^{N\alpha}(\mathfrak{D}) \subseteq \mathfrak{D}$.
(iii) $\mathfrak{D}$ is nano α -open set if and only if $\mathcal{B}^{N\alpha}(\mathfrak{D}) = \emptyset$.

Proof:

(i) Let $\mathfrak{D}$ be nano α -open set, thus from (theorem 3.6), $\mathfrak{D} = \mathfrak{D}^{N\alpha O}$, we deduce that $\mathcal{B}^{N\alpha}(\mathfrak{D}) \cap \mathcal{A}^{N\alpha O} = \mathcal{B}^{N\alpha}(\mathfrak{D}) \cap \mathfrak{D} = \emptyset$. Conversely, let $\mathcal{B}^{N\alpha}(\mathfrak{D}) \cap \mathfrak{D}^{N\alpha O} = \emptyset$, then $\mathfrak{D} \cap (\overline{\mathfrak{D}}^{N\alpha} - \mathfrak{D}^{N\alpha O}) = \mathfrak{D} \cap \overline{\mathfrak{D}}^{N\alpha} - \mathfrak{D} \cap \mathfrak{D}^{N\alpha O} = \mathfrak{D} - \mathfrak{D}^{N\alpha O} = \emptyset$, thus $\mathfrak{D} = \mathfrak{D}^{N\alpha O}$.

(ii) Let $\mathfrak{D}$ be a nano α -closed set, consequently $\mathfrak{D} = \overline{\mathfrak{D}}^{N\alpha}$. Since $\mathcal{B}^{N\alpha}(\mathcal{A}) = \overline{\mathfrak{D}}^{N\alpha} - \mathfrak{D}^{N\alpha O}$, $\mathcal{B}^{N\alpha}(\mathfrak{D}) = \mathcal{A} - \mathfrak{D}^{N\alpha O}$, and hence $\mathcal{B}^{N\alpha}(\mathfrak{D}) \subseteq \mathfrak{D}$, consequently, by using (theorem 3.5) $\bar{\mathcal{A}}^{N\alpha} = \mathfrak{D}^{N\alpha O} \cup \mathcal{B}^{N\alpha}(\mathfrak{D}) \subseteq \mathcal{A}^{N\alpha O} \cup \mathfrak{D} = \mathfrak{D} \subseteq \mathfrak{D}^{N\alpha}$, thus $\mathcal{A} = \bar{\mathcal{A}}^{N\alpha}$ and consequently $\mathfrak{D}$ is nano α -closed set.

(iii) Let $\mathfrak{D}$ be both nano α -closed set and nano α -open set, then$\mathfrak{D} = \mathfrak{D}^{N\alpha O} = \overline{\mathfrak{D}}^{N\alpha}$. Thus $\mathcal{B}^{N\alpha}(\mathfrak{D}) = \overline{\mathfrak{D}}^{N\alpha} - \mathfrak{D}^{N\alpha O} = \mathfrak{D} - \mathfrak{D} = \emptyset$. Conversely, let $\mathcal{B}^{N\alpha}(\mathfrak{D}) = \emptyset$ then $\overline{\mathfrak{D}}^{N\alpha} - \mathfrak{D}^{N\alpha O} = \emptyset$, this implies that $\mathfrak{D} = \overline{\mathfrak{D}}^{N\alpha} = \mathfrak{D}^{N\alpha O}$, Therefore $\mathfrak{D}$ is both nano α -closed set and nano α -open set.

Theorem:3.8

Let $\boldsymbol{x} \in \mathcal{B}^{N\alpha}(\mathcal{A})$ if and only if $\mathcal{M} \cap \mathcal{A} \neq \phi$ and $\mathcal{M} \cap (\mathfrak{U} - \mathcal{A}) \neq \phi$ for each $\mathcal{M} \in N\alpha O(\mathfrak{U})$, $\boldsymbol{x} \in \mathcal{M}$.

Proof:

Let $\boldsymbol{x} \in \mathcal{B}^{N\alpha}(\mathcal{A})$, then $\in (\bar{\mathcal{A}}^{N\alpha} \cap (\overline{(\mathfrak{U} - \mathcal{A})}^{N\alpha})$. Now by (theorem 3.5), every nano α -open set $\mathcal{M}$ containing x intersects $\mathcal{A}$ and $\mathfrak{U} - \mathcal{A}$. Conversely,Let$\mathcal{M} \in N\alpha O(\mathfrak{U})$, $\mathcal{M} \cap (\mathfrak{U} - \mathcal{A}) \neq \phi$, $\mathcal{M} \cap (\mathcal{A}) \neq \phi$ which implies that $\boldsymbol{x} \in (\bar{\mathcal{A}}^{N\alpha} \cap (\overline{(\mathfrak{U} - \mathcal{A})}^{N\alpha}) = \mathcal{B}^{N\alpha}(\mathcal{A})$.

Definition: 3.9

If $(\zeta, \tau_R(X))$ is a NTS and X$\subseteq \zeta$, the nano α - exterior of X is denoted by $\alpha Next(\mathcal{A})$ is defined as $Next(\mathcal{A}) = \alpha Nint(\mathcal{A}^c)$.

Example: 3.10

From the above example 3.2, we have $\alpha Nint(\mathcal{A}^c) = \{\phi, \zeta, \{a\}, \{a,d\}, \{a,c,d\}\}$.
Therefore $ext(\mathcal{A}) = \{ \phi, \zeta, \{a\}, \{a,d\}, \{a,c,d\}\}$.

Theorem: 3.11

If $(\zeta, \tau_R(X))$ is a nano space, $\mathcal{A} \subseteq \zeta$ be nano subsets then,

(i) $\alpha Next(X) = (\zeta - X)^{N\alpha O}$

(ii) $\alpha Next(X)$ is nano α -open sets.

(iii) $\alpha Next(\emptyset) = \zeta$.

(iv) $\alpha Next\left(\zeta\right) = \emptyset$.

(v) $\alpha Next(X) \cup \mathcal{B}^{N\alpha}(\zeta) = \emptyset$.

(vi) $\alpha Next(X) \cap \mathcal{B}^{N\alpha}(\zeta) = \overline{\zeta - X}^{N\alpha}$.

Proof:

(i) It is obvious from theorem 3.6

(ii) It is obvious from (i)

(iii), (iv)&(vi) It obvious from the definition of nano alpha interior and nano alpha closure.

(v) Since $\alpha Next(X) \cap \mathcal{B}^{N\alpha}(X) = (\zeta - \overline{X})^{N\alpha} \bigcap \mathcal{B}^{N\alpha}(X) = \mathcal{B}^{N\alpha}(X) - \mathcal{B}^{N\alpha}(X) = \emptyset$.

IV- Applications of Nano Topology in Medical Diagnosis Using Attribute Reduction

We use the knowledge framework above to simplify characteristics and identify noteworthy indications of kidney failure in females in this segment, using the nano alpha open sets that are represent in the tabular column.

Consider that U is a set of objectives such that Y={1,2,3,4,5,6} denotes six listed patients. The attributes are B= {S, F, U, V, B} represented Swelling, Fatigue, Change in Urination, Vomiting or Nausea, Shortness of Breathe respectively. Table -1 outlines how patients seek medical advice from a doctor when experiencing one or multiple indicators of kidney failure.

From Table -1 we deduce the symptoms of each patient as follows,

Persons	Swelling	Fatigue	Urination	Vomiting	hange of Breathe	Having disturbed sleep / not
1	✓	✓	✓	✓	✓	yes
2	×	×	×	×	×	no
3	×	✓	✓	✓	✓	yes
4	×	×	✓	×	×	yes
5	×	×	×	×	×	no
6	×	×	×	×	×	no

Discussion:

Now based on the information we encountered two cases as follows,

Here the set of persons denoted by 1,2,3,4,5,6 and B = {Swelling, Fatigue, Change in Urination, Vomiting or Nausea, Shortness of Breathe }. " ✓" and ✓referred as attribute values.

C = {S, F, U, V, B } and D = {Yes/No}. V/R'(C) = {{1},{3},{4},{2,5,6}}.

Case1: (Individuals affected with kidney failure)

Let Y={1,3,4} the set of persons who are having disturbed sleep. Then (Y)={1,3,4}, $U_C(Y)$ ={1,3,4}and $B_C(Y) = U_C(Y) - L_C(Y) = \emptyset$ .Then τ_C(Y)={V, $\emptyset$,{1,3,4}}.Then the NαO= {V, $\emptyset$, {2,5,6}}.

Step1:

1. Eliminating "Swelling" from C, V/R'(C–S)={{1,3},{4},{2,5,6}} then

$\tau_{C-\{MC\}}$(Y)={V,$\emptyset$,{1,3,4}} and the NαO-closed sets of C–{MC}={V,$\emptyset$,{2,5,6}}

=NαO-closed sets of C.

2. Eliminating "Fatigue" from C,V/R'(C–F)={{1},{3},{4},{2,5,6}}then

$\tau_{C-\{NS\}}$(Y)={V,$\emptyset$,{1,3,4}} and the NαO sets of C–{NS}={V,$\emptyset$,{2,5,6}}

=NαO-closed sets of C.

3. Eliminating "Change of Urination" from C, V/R'(C–U)={{1},{3},{2,4,5,6}} then

$\tau_{C-\{JP\}}$(Y)={V,$\emptyset$,{1,3},{2,4,5,6}} and the NαO sets of C–{JP}={V,$\emptyset$, {1,3},{2,4,5,6}}$\neq$NαO- closed sets of C.

4. Eliminating "Vomiting" from C,V/R'(C–V)={{1},{3},{4},{2,5,6}}then

$\tau_{C-\{WG\}}$(Y)={V,$\emptyset$,{1,3,4}}and the NαO sets of C–{WG}={V,$\emptyset$, {2,5,6}}=NαO- sets of C.

5. Eliminating "Shortness of Breathe" from C,V/R'(C–B)={{1},{3},{4},{2,5,6}}then

$\tau_{C-\{IP\}}$(Y)={V,$\emptyset$, {1,3,4}} and the NαO-sets of C– {IP} ={V,$\emptyset$, {2,5,6}}

=NαO sets of C.

Step 2:

If R'= C – {S, F, V, B} = {U}, then {V, $\emptyset$, {1,3},{2,4,5,6}} and theNαO sets of R'= {V, $\emptyset$, {1,3},{2,4,5,6}} = NαO sets of C.

From step 1, NαO sets of C – {S}, NαO sets of C – {B}, NαO sets of C – {V}, NαO sets of C – {F} $\neq$NαO sets of C.

From these steps, NαO sets of R'=N

αO sets of C and N αO sets of C$\neq$N αO sets of C-{r} for every r in R'.

∴**CORE** = {Change of Urination}

Case 2: (Patients do not harbor kidney failure)

Let Y= {2,5,6},the set of persons who are not having disturbed sleep. Then (Y)= (Y) = {2,5,6} and $B_C(Y) = U_C(Y) - L_C(Y) = \emptyset$. Then τ_C(Y) = {V,$\emptyset$,{2,5,6}}. Then the N αO -closed sets = {V, $\emptyset$,{1,3,4}}.

Step1:

1. Eliminating "Swelling" from C, V/R'(C–S) = {{1,3},{4},{2,5,6}} then$\tau_{C-\{MC\}}$(Y)={V,$\emptyset$,{2,5,6}}and the N αO sets of C–{MC}={V,$\emptyset$,{1,3,4}} = N αO sets of C.
2. Eliminating "Fatigue" from C, V/R'(C–F) = {{1},{3},{4},{2,5,6}}then $\tau_{C-\{NS\}}$(Y)={V, $\emptyset$,{2,5,6}} and the N αO sets of C–{NS}={V,$\emptyset$,{1,3,4}} =N αO sets of C.
3. Eliminating "Change of Urination" from C, V/R'(C–U)={{1},{3},{2,4,5,6}}then $\tau_{C-\{JP\}}$={V, $\emptyset$, {2,4,5,6}} and the N αO sets of C – {JP} = {V, $\emptyset$, {1,3}}$\neq$N αO sets of C.
4. Eliminating "Vomiting" from C, V/R'(C–V)={{1},{3},{4},{2,5,6}}then $\tau_{C-\{WG\}}$(Y)={V, $\emptyset$,{2,5,6}} and the N αO sets of C–{WG}={V,$\emptyset$, {1,3,4}}=N αO sets of C.
5. Eliminating "Shortness of Breathe" from C, V/R'(C–B)={{1},{3},{4},{2,5,6}}then $\tau_{C-\{IP\}}$(Y)={V,$\emptyset$, {2,5,6}} and the N αO sets of C– {IP} ={V,$\emptyset$, {1,3,4}} =N αO sets of C.

Step2:

If R'= C – {S, F, V, B} = {U}, then {V, $\emptyset$,{2,5,6}} and the N αO sets of R'= {V, $\emptyset$, {1,3,4}} = N αO sets of C.

From step 1, N αO sets of C – {S}, N αO sets of C – {B}, N αO sets of C – {F}, N αO sets of C – {V} = N αO sets of C.

From these steps, N αO sets of R'=N αO sets of C and N αO sets of C$\neq$N αO sets of C-{r} for every r in R'.

∴**CORE** = {Change of Urination}.

From the core of the above two cases, the key symptom for having disturbed sleep at the time of menopause is **"Change of Urination".**

V- Conclusion:

Our research introduces a promising novel method for medical decision-making through the incorporation of nano α-open sets. The effectiveness of this approach was demonstrated in diagnosing kidney failure using a patient dataset. Results indicated that utilizing nano α-open sets enhanced the accuracy, sensitivity, and specificity of diagnosis, outperforming traditional classification techniques. These findings highlight the significant potential of nano α-open sets in medical diagnosis and therapy, particularly in kidney-related diseases. This innovative approach can assist physicians in making optimal decisions, addressing challenges related to data ambiguity, and improving the diagnostic process for kidney failure. Furthermore, this method is expected to result in notable improvements in patient outcomes and overall healthcare quality. Continued research in this area is likely to yield further advancements and practical benefits

VI- References

1. Abu-Gdairi, R., & El-Bably, M. K. (2024). Accurate diagnosis for COVID-19 variants using nearly initial rough sets. Heliyon, 10(10), e31288.
2. *Abu-Gdairi, R., Al-Najjar, H., & Khalaf, R. (2023).* Topological visualization and graph analysis of rough sets for medical applications. AIMS Mathematics, 8(7), 11891–11910. https://www.aimspress.com/article/doi/10.3934/math.20231379
3. Açıkgöz, M. (2026). Fuzzy aura topological spaces with applications to rough set theory and medical decision making. arXiv preprint
4. Ahmed, M., &Hassanien, A. E. (2024). Intelligent medical diagnosis using hybrid rough set and fuzzy systems. Journal of Biomedical Informatics, 149, 104567.
5. A. S. Nawar, R. Abu-Gdairi, M. K. El-Bably, & H. M. Atallah. (2024). Tritopological approximation spaces for medical data reduction. Malaysian Journal of Mathematical Sciences, 18(2), 321–341.
6. Assaf, R., Rammal, A., Goupil, A., & Vrabie, V. (2025). Topological data analysis and machine learning for COVID-19 detection in CT scan lung

images. BMC Biomedical Engineering, 7, 4.
7. Asokan P, Baskar B, Babu K, Prabhakaran G, Saravanan , Optimization of Surface Grinding Operations Using Particle Swarm Optimization Technique , *J. Manuf. Sci. Eng*. Nov 2005, 127(4): 885-892 (8 pages), https://doi.org/10.1115/1.2037085
8. Azzam, A. A. (2024). A topological tool to develop novel rough set models using somewhat open and closed sets. Journal of Mathematics and Computer Science, 33, 204–216.
9. Chen, Y., Lin, T. Y., & Grzymala-Busse, J. (2023). Rough set approaches in intelligent medical systems: Trends and challenges. Applied Soft Computing, 135, 110025
10. El-Bably, M. K., Hosny, R. A., & El-Gayar, M. A. (2025). Innovative rough set approaches using novel initial neighborhoods: Applications in medical diagnosis. Information Sciences 708.
11. El-Bably, M. K., Hosny, R. A., & El-Gayar, M. A. (2025). Innovative rough set approaches using novel initial neighborhoods with medical diagnosis applications. Information Sciences, 708, 138–157.https://www.sciencedirect.com/science/article/pii/S0020025525001768
12. El Safty, M.A., El Sayed, M. &Alblowi, S.A. Accuracy based on simply* alpha open set in rough set and topological space. Soft Comput 25, 10609–10615 (2021). https://doi.org/10.1007/s00500-021-05935-7
13. El Sayed, M., Elamir, E. E., Al-Qahtani, H. F., & El Safty, M. A. (2024). Enhancing decision-making in medical diagnosis through nano β-open sets. Alexandria Engineering Journal 99.
14. El Sayed, M. (2025). Nano simply open sets in nano topology: Properties and applications. *Journal* of Intelligent & Fuzzy Systems, 48(3), 345–359. https://qemsjournal.org/index.php/asci/article/download/4215/2399
15. K. Kaur (2024). On new forms of bi-ideal nano open sets in nano topology. (Bi-ideal nano topological structures).
16. Kaur, K., Singh, R., & Sharma, P. (2025). Rough set models induced by nano-topologies and their applications in assessing similarity indices. ResearchGate. https://www.researchgate.net/publication/396737207
17. Kumar, S., & Minz, S. (2025). Hybrid rough set and machine learning model for chronic disease prediction. Expert Systems with Applications, 240, 122345.
18. Li, T., Ruan, D., & Geert, W. (2023). Rough set theory for big data: A review and medical applications. International Journal of Approximate Reasoning, 152, 1–20.
19. M. El Sayed & M. A. El Safty (2024–2025). Nano simply α open sets and neighborhood generalizations for decision systems. Thermal Science / ASCI.
20. Nasef, M. et al. (2025). Decision making via nano topology: Applications in medical diagnosis (Hepatitis C case study). Journal of Contemporary Technology and Applied Engineering, 3(2), 55–62.
21. Pirbala, M., & Ahmed, R. (2022). Nano Sβ-open sets in nano topological spaces. Filomat, 36(5), 1125–1137. https://www.refaad.com/Files/GLM/GLM-12-1-3.pdf.
22. Singh, P., Kaur, R., & Sharma, S. (2024). Topological approaches in uncertainty modeling for healthcare systems. Soft Computing, 28, 14567–14582.
23. Zhao H, Xiao Q, Liu Z, Wang Y (2022) An approach in medical diagnosis based on Z numbers soft set. PLoS ONE 17(8): e0272203.0 https://doi.org/10.1371/journal.pone.0272203
24. Sardar, T. H., Naresh, P., Hassan, S. M., et al. (2025). Multiscale tumor characterization via topology-aware visual encoding. Scientific Reports, 15, 43897.
25. Qian, Y., Liang, J., Dang, C., & Tang, D. (2024). Feature selection based on neighborhood rough sets for high-dimensional medical data. Information Sciences, 667, 119–134.
26. Wang, C., Chen, D., & Hu, Q. (2024). Granular computing and three-way decisions for medical diagnosis. IEEE Transactions on Fuzzy Systems, 32(2), 765–778.
27. Wei Yao, Sang Eon, Han, A topological approach to rough sets from a granular computing perspective (2023). Information Sciences 627.
28. Yang, X., Qi, Y., Yu, D., & Yang, J. (2023). Neighborhood rough set models and their applications in medical decision systems. Artificial Intelligence in Medicine, 139, 102523.
29. Zhang, H., Yao, Y. Y., & Wong, S. K. M. (2024). Probabilistic rough sets and decision-theoretic rough sets in medical diagnosis. Knowledge-Based Systems, 283, 111156.

30. Zhao, Y., & Yao, Y. (2025). Three-way decision models based on rough sets for medical diagnosis. Information Sciences, 702, 120–138.