

A systematic review of effectiveness of Concrete Representational Abstract (CRA) intervention for individuals with mathematical disability

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Abstract

Learning is essential for human evolution, yet some individuals face challenges in the most fundamental aspect of learning due to learning disabilities like dyslexia, dysgraphia & dyscalculia. Dyscalculia affects approximately 3-7% of the population (Haberstroh & Korne, 2019). Despite its prevalence, dyscalculia remains understudied (Salisa & Meiliasari, 2023). One of the most appreciated multisensory approaches of intervention to teach mathematics is Concrete Representational Abstract intervention (CRA). The aim of the present review paper was to review existing research on the Concrete Representational Abstract (CRA) approach, clarify concepts, highlight methodologies of studies conducted and identify gaps in existing research. Using the PRISMA model, the paper analysed existing literature across various databases and synthesized the findings. As a result of systematic literature review, the results indicate that the CRA approach is effective for students with dyscalculia and those struggling with mathematics, enhancing their conceptual understanding compared to conventional approach. It helps in improving number recognition, number sense, counting, computational fluency, grasp geometrical patterns, understanding fractions etc and apply mathematical concepts to real life setting. It also considers the needs of diverse range of students. Given the limited research in India using the CRA approach, the paper also highlights its implications for parents, teaching professionals and policy makers. It proves as an effective strategy to help children with special needs, learn basic math skills. Finally, an attempt is made to discuss limitations and directions for future research.

Keywords: Dyscalculia, Concrete Representational Abstract, Mathematical Disability, Learning Disability, Mathematics Intervention

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Introduction

One language that is universal and has the same meaning across the globe is ‘mathematics’. Mathematics is not just confined to a classroom; it extends far beyond to applicational areas of life. It is a special skill that is required in various fundamental areas of our daily life. It also lays a foundation to many other higher-level subjects. Learning mathematics is important because it fosters the ability to think rationally and critically, develop on problem solving and conceptual thinking, advance in the fields of science, technology, economy, design, architecture and creative endeavours.

Along with the increasing class levels and the diversity of class population the complexity of mathematics skills also increases. Research has proven that mathematics is perceived to be one of the difficult subjects for most students in higher education (Langoban, 2020). There are

different cognitive skills required to learn mathematics including working memory, problem solving, spatial reasoning, number sense, conceptual thinking, metacognition etc. (Schoenfeld, 2022) [28]. Considering these factors, teaching mathematics is a special task and becomes demanding. Moreover, teaching mathematics to students with special needs presents unique challenges. Students with special needs includes students with intellectual disability, autism, attention deficit hyperactivity disorder, emotional and behavioural disorders, physical disability, speech and language problems and dyscalculia.

Mathematics disability

Mathematics learning disability is clinically termed as Dyscalculia. ‘Dyscalculia is a type of learning disability characterised by difficulties in acquiring adequate arithmetic skills in the context of otherwise normal

intelligence and age-appropriate school education'. The individual has difficulty in understanding math facts, number sense, arithmetic facts, calculation etc. It affects the individuals' ability to acquire arithmetic skills (APA, 2013) It also affects an individual's ability to understand concepts of time, money, spatial ability, reading maps and musical concepts (Schelke *et al*, 2017) [27]. Inclusive classroom setting creates a special learning environment where students with unique needs are valued and feel respected and are provided with equal opportunities with or without their disabilities. All the types of students are helped without making them feel any exclusion (Ekeh & Oladayo, 2012) [10]. Providing differentiated instruction is practiced providing students with knowledge, encourage learning and adapt with their needs. When students with special needs are included in regular classroom, it helps in building their confidence, experience greater levels of social acceptance, increase academic achievement, motivation and social interaction. However, it can be relatively challenging for teachers to practice inclusion with diverse learners. Therefore, teachers need to learn different strategies and techniques and be equipped with resources (Webster, 2014) [3]. In this regard various evidence-based intervention strategies have been effective in closing learning gaps. Some of the strategies include, explicit instruction, visual representations, multisensory approach, using assistive technology, concrete representational abstract approach etc.

Concrete Representational Abstract (CRA) Approach of Intervention

The CRA strategy is a three-stage intervention for mathematics where students learn through visually manipulating objects, followed by pictorial representations and using abstract symbols to create meaningful connections (Khan & Khan, 2021). The CRA instructional sequence has three stages including: Concrete stage, Representational stage and Abstract Stage

Concrete stage: In the concrete stage, physical materials also known as concrete manipulatives are given to understand and solve a mathematics problem. It can include cubes, blocks, bars, sticks etc. The teachers model learning by helping to visualise the concept. Hands on materials provide guided learning.

Representational stage: In this stage, the concrete model is transformed into representational pictures. It can include, dots, lines, tallies circle etc drawn on paper to imprint the concept. Students draw pictures and understand the problems previously discussed which again helps in forming meaningful connections.

Abstract Stage: In the abstract stage, the concept is taught using symbolic level. It can include, numbers, symbols, notations used to depict mathematical problems discussed earlier. Using this in conjunction with concrete and representational aspects creates an interconnected sequence and helps to learn the targeted skill (Khan & Khan, 2021). The steps involved in teaching CRA strategy to students include:

Concrete Level: Teach the mathematical concept using concrete objects, encourage practice using the

manipulatives and provide reinforcement. Eg: Show 5 blocks and 2 blocks and then join them together. Help the child count. Provide different opportunities, assess the understanding at concrete level before shifting to representational level

Representational level: Introduce previous mathematical concept in pictorial form Eg: Draw 5 squares and 2 squares on a paper, ally the similar strategy of counting using modelling. Appreciate practice using various pictorial alternatives, which develops flexibility. After assessing the mastery, move to abstract level

Abstract level: Teach the previously learnt concept using only numbers, symbols and notations. Eg: $(5 + 2 =)$ Provide ample opportunities to learn and absorb the concept. Appraise the knowledge and revisit previous stages if required while providing feedback and reinforcement (Khan & Khan, 2021).

The CRA has its roots in the work of Jerome Bruner. It involves both behaviouralist and constructivist theories for cognitive development (Khan & Khan, 2021). It also involves discovery learning strategies which is important to facilitate learning in individuals with learning disabilities as they seem to have deficits in procedural learning (Fawcett & Nicolson, 2007) [11].

Theoretical Framework

Jerome Bruner hypothesized the theory of cognitive development (Bruner, 1966) [8]. He believed that children learn information through active experience of the world around them. He believed that intellectual development happens in three stages and the fundamental information can be processed at any age levels. The three modes of representation include:

- **Inactive stage:** In the enactive stage, knowledge is stored in the form of motor activities. Individuals learn by 'DOing' and encode information in the form of muscle memory.
- **Iconic stage:** In the iconic stage, information is stored through sensory images or visual representations. Individuals learn by 'seeing' and storing in form of mental images.
- **Symbolic stage:** In the symbolic stage knowledge is stored primarily as language or symbols. Here individuals are not restricted to actions or images and can store arbitrary information. (Bruner, 1966) [8].

According to him the aim of education is to create independent learners. Education needs to facilitate symbolic thinking in children and facilitate problem solving rather than just imparting knowledge. His main premise was 'Students are active learners who construct their own knowledge' (Bruner, 1960).

Therefore, purpose of the current paper is to highlight methodologies of studies conducted and identify gaps in existing research and discuss scope for further research.

Aims and Objectives:

The aim of this paper is to review the effectiveness of CRA intervention on individuals with mathematical disability.

- To explore the effectiveness of CRA intervention
- It also attempts to highlight various methodologies used to understand the effectiveness of this interventional approach
- Finally, an effort is also made to identify gaps in CRA intervention among the existing literature

Methodology

This systematic review, a crucial endeavor in education and special education, aimed to synthesize empirical evidence on the effectiveness of Concrete Representational Abstract (CRA) intervention for individuals with mathematical disability. The review meticulously followed the guidelines provided by the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) statement to ensure comprehensive and transparent reporting.

Research Question

What is the effectiveness of Concrete Representational Abstract (CRA) intervention for individuals with a mathematical disability?

Search Strategy

A meticulously comprehensive search strategy was carried out to identify relevant peer-reviewed literature. The e-databases Web of Science (Core Collection) and PubMed were systematically searched for articles published between January 2008 and May 2025. Only articles published in English were considered.

The specific search strings used for each database are detailed below:

Web of Science (Core Collection) Search Strategy:

The search used the "Topic" (TS) field, which searches the title, abstract, author keywords, and Keywords Plus.

TS=("Concrete Representational Abstract" OR "CRA" OR "Concrete-Representational-Abstract" OR "Concrete Semicrete Abstract" OR "CSA" OR "Concrete-Pictorial-Abstract" OR "CPA" OR "Concrete to Abstract" OR "Representational Abstract" OR "Semi-Concrete Abstract" OR Manipulatives OR "visual representations") AND

TS=("mathematical disability" OR "mathematics disability" OR "math disability" OR "mathematical difficulties" OR "mathematics difficulties" OR "math difficulties" OR "dyscalculia" OR "mathematical learning disabilities" OR "mathematics learning disabilities" OR "math learning disabilities" OR ("learning disability" OR "learning difficulties") AND (mathematics OR math) OR "specific learning disorder with impairment in mathematics" OR "developmental dyscalculia" OR "low math achievement" OR ("struggling learners" AND (mathematics OR math)) OR ("at risk" AND (mathematics OR math))) AND

TS=(effectiveness OR efficacy OR impact OR outcome* OR interven* OR treatment OR program* OR instruction* OR "teaching method*" OR "instructional strateg*" OR "teach* method*" OR "instruct*" OR remediation OR support OR implement* OR trial* OR study* OR evaluation OR result*)

PubMed Search Strategy:

The search combined Medical Subject Headings (MeSH) terms with keyword searches in the title and abstract fields. ("Concrete Representational Abstract"[Title/Abstract] OR "CRA"[Title/Abstract] OR "Concrete-Representational-Abstract"[Title/Abstract] OR "Concrete Semicrete Abstract"[Title/Abstract] OR "CSA"[Title/Abstract] OR "Concrete-Pictorial Abstract"[Title/Abstract] OR "CPA"[Title/Abstract] OR "Concrete to Abstract"[Title/Abstract] OR "Representational Abstract"[Title/Abstract] OR "Semi-Concrete Abstract"[Title/Abstract] OR Manipulatives[Title/Abstract] OR "visual representations"[Title/Abstract])

AND

("Mathematics"[Mesh] OR "Learning Disabilities"[Mesh] OR "Mathematics Disorders"[Mesh] OR "Developmental Disabilities"[Mesh] OR "mathematical disability"[Title/Abstract] OR "mathematics disability"[Title/Abstract] OR "math disability"[Title/Abstract] OR "mathematical difficulties"[Title/Abstract] OR "mathematics difficulties"[Title/Abstract] OR "math difficulties"[Title/Abstract] OR "dyscalculia"[Title/Abstract] OR "mathematical learning disabilities"[Title/Abstract] OR "math learning disabilities"[Title/Abstract] OR "learning disability"[Title/Abstract]) AND

(mathematics[Title/Abstract] OR math[Title/Abstract]) OR "learning difficulties"[Title/Abstract] AND (mathematics[Title/Abstract] OR math[Title/Abstract]) OR "specific learning disorder with impairment in mathematics"[Title/Abstract] OR "developmental dyscalculia"[Title/Abstract] OR "low math achievement"[Title/Abstract] OR "struggling learners"[Title/Abstract] AND (mathematics[Title/Abstract] OR math[Title/Abstract]) OR "at risk"[Title/Abstract] AND (mathematics[Title/Abstract] OR math[Title/Abstract]))

AND

("Treatment Outcome"[Mesh] OR "Program Evaluation"[Mesh] OR "Intervention Studies"[Mesh] OR "Teaching"[Mesh] OR "Instruction"[Mesh] OR "effectiveness"[Title/Abstract] OR "efficacy"[Title/Abstract] OR "impact"[Title/Abstract] OR "outcome"[Title/Abstract] OR "outcomes"[Title/Abstract] OR "intervention"[Title/Abstract] OR "interventions"[Title/Abstract] OR "treatment"[Title/Abstract] OR "program"[Title/Abstract] OR "programs"[Title/Abstract] OR "instruction"[Title/Abstract] OR "instructional strategy"[Title/Abstract] OR "instructional strategies"[Title/Abstract] OR "teaching method"[Title/Abstract] OR "teaching methods"[Title/Abstract] OR "remediation"[Title/Abstract] OR "support"[Title/Abstract] OR "implementation"[Title/Abstract] OR "trial"[Title/Abstract] OR "study"[Title/Abstract] OR "evaluation"[Title/Abstract] OR "result"[Title/Abstract] OR "results"[Title/Abstract])

Eligibility Criteria

The following inclusive and exclusion criteria were used to screen and select articles:

Inclusion Criteria.

1. Studies published in English between January 2008 and May 2025.
2. Original research articles published in peer-reviewed journals.
3. Studies from all geographical locations.
4. Original research studies that investigated the provision of Concrete Representational Abstract (CRA) intervention.
5. Participants included primary and secondary school students of both genders diagnosed with a mathematical disability or exhibiting mathematical difficulty.
6. Study designs included empirical approaches such as experimental designs (e.g., randomized controlled trials, quasi-experimental designs) or case studies.

Exclusion Criteria

1. Theoretical or conceptual papers.
2. Studies lacking sufficient detail regarding the CRA intervention provided make it challenging to assess its fidelity or components.
3. Studies that exclusively employed qualitative methodology.
4. Studies that utilized alternative or virtual interventions, such as virtual manipulatives or specialized software, for mathematical disability without explicitly detailing the application of the CRA sequence with concrete manipulatives.

Study Selection Process

The initial database searches yielded a total of 96 articles. Specifically, the Web of Science search produced 67 articles, and the PubMed search yielded 29 articles.

A systematic and rigorous screening process was meticulously implemented:

1. **Initial Filtering (Web of Science):** From the 67 articles retrieved from Web of Science, initial filters were applied within the database to exclude:
 - Review articles (n=4)
 - Proceeding papers (n=3)
 - Articles not published in English (n=2)After this initial filtering, 58 articles remained from the Web of Science.
2. **Duplicate Removal:** All articles retrieved from both databases were imported into a reference management software. Duplicates were systematically identified and removed, resulting in the exclusion of 9 duplicate articles. This process yielded a total of 78 unique articles for further screening.
3. **Abstract and Full-Text Screening:** The remaining 78 unique articles underwent a rigorous screening process based on the predefined inclusion and exclusion criteria. This involved:
 - **Title and Abstract Screening:** A team of researchers reviewed each article's title and abstract, ensuring a

comprehensive assessment of preliminary relevance. Articles were excluded if they did not meet the inclusion criteria. Any conflicts were resolved through open discussion and consensus, fostering a collaborative review process.

- **Full-Text Review:** Articles deemed potentially relevant after abstract screening underwent full-text review. The full text of these articles was examined against the inclusion and exclusion criteria to determine eligibility. Discrepancies between reviewers were resolved through discussion or consultation with a third reviewer.

Twelve articles *met all* the inclusion criteria and were selected for detailed data extraction and synthesis.

Quality Appraisal

The quality of the 12 selected articles was critically appraised to ensure the methodological rigor and reliability of the included studies, given the nature of the included study designs (experimental, quasi-experimental, and case studies), appropriate critical appraisal tools from the Joanna Briggs Institute (JBI) were utilized. The JBI Critical Appraisal Tool for Quasi-Experimental Studies (for non-randomized experimental studies) and the JBI Critical Appraisal Tool for Case Reports (for case studies) will be employed.

Two independent reviewers assessed the methodological quality of each study. Any discrepancies in quality assessment were resolved through discussion and consensus among the reviewers.

Data Extraction and Synthesis

Data from the 12 selected articles were systematically extracted into a standardized, pre-piloted data extraction form. The extracted data will include:

1. **Study Characteristics:** Authors, publication year, specific study design (e.g., RCT, quasi-experimental, single-case design).
2. **Participant Characteristics:** Number of participants, age range, grade level, specific diagnostic criteria for mathematical disability/difficulty and relevant demographic information.
3. **Intervention Details:** Specific components of the CRA intervention (e.g., types of concrete manipulatives used, specific abstract representations), duration of the intervention (e.g., weeks, total hours), frequency of sessions, intensity, instructor qualifications, and whether it was delivered individually or in a group setting.
4. **Outcome Measures:** Specific mathematical skills assessed (e.g., number sense, operations, problem-solving), instruments used for measurement, pre- and post-intervention scores, and any follow-up data.
5. **Key Findings:** Relevant quantitative data and conclusions drawn by the study authors regarding the effectiveness of CRA.
6. **Limitations:** Limitations regarding the study sample, methodology, design, future scope etc. have been discussed

A narrative synthesis approach was employed to discuss the findings from the included studies. Given the heterogeneity in study designs, participant characteristics, intervention protocols, and outcome measures, a formal meta-analysis is not feasible. Findings were organized thematically, discussing common patterns of effectiveness, variations in intervention outcomes across different mathematical domains or age groups, and the methodological strengths and limitations of the included studies. The synthesis highlights consistencies and inconsistencies in the evidence base to comprehensively address the research question regarding the effectiveness of CRA intervention for individuals with a mathematical disability.

PRISMA 2020 Flow Diagram – Systematic Review on CRA Intervention
PRISMA Flowchart Summary

Identification	Records identified from: • Web of Science: 67 • PubMed: 29 Total records identified: 96
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	Records removed before screening: • Duplicates removed: 9 Records after duplicates removed: 87
Screening	Records screened (titles and abstracts): 87 Records excluded: 9 • Review articles: 4 • Proceedings papers: 3 • Non-English articles: 2 Records after initial filtering: 78
Eligibility	Full-text articles assessed for eligibility: 78 Full-text articles excluded: 66 • Did not meet inclusion criteria (e.g., insufficient CRA detail, qualitative-only, virtual manipulatives used)
Included	Studies included in qualitative synthesis (narrative review): 12 Quality appraisal conducted using JBI tools
Note: This flowchart is adapted based on the PRISMA 2020 guidelines.	

Results

Table 1: The table below shows the selected articles with details of title, authors, methodology, findings and limitations

Sl. No	Author (Year)	Title	Methods	Findings	Limitations
1	Emily Bouck, Jiyeon Park, Barb Nickell (2017) [5]	Using the Concrete-Representational-Abstract Approach to Support Students with Intellectual Disability to Solve Change-Making Problems	Sample size: 4 students with IDD Age/Class: Middle school students, age 12 to 13 years/ 6th grade Research Design: Multiple probe across participant design Intervention: 5 to 8 baseline sessions, 9 to 11 intervention sessions and 2 maintenance sessions	The CRA instructional strategy was effective in teaching all participants to correctly solve problems	- Lack of generalization phase - Multiple disabilities participant is a limitation - Could use notes also, not only coins - Intervention can be provided at a less noise place and no observers - Instead of experimenter, teacher can teach CRA - Functional based mathematics and generalization
2	Shudong Zhang, Shuyuan Yu, Jing Xiao, Yunting Liu, Ting Jiang (2021) [36]	The Effects of Concrete-Representational-Abstract Sequence Instruction on Fractions for Chinese Elementary Students with Mathematics Learning Disabilities	Sample: Four, 4 th grade students, with LD Research design: Multiple probe across students single case design Intervention: CRA	Concrete-Representational-Abstract Sequence Instruction helped to improve ability to solve fractions in Chinese Elementary Students with Mathematics Learning Disabilities	- No control group - Not comparative study - Improvement could also be due to traditional learning in school - Details of intervention not provided
3	Sarah J. Watt & William J. Therrien (2016) [32]	Examining a Preteaching Framework to Improve Fraction Computation Outcomes Among Struggling Learners	Sample: 32 students with math difficulties, 6 th grade, from 2 elementary schools Research design: Pre – post experimental design with control and experimental group Intervention: Preteaching was done, before learning the core unit, with three essential prerequisite skills using CRA	Using CRA instructional sequence for preteaching can help in improving learning fractions in students who are struggling with mathematics	- Efficiency of Preteaching on maintenance is not observed - Details of the intervention not provided

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4	Vanessa Hinton, Margaret M Flores (2019) ^[19]	The Effects of the Concrete-Representational-Abstract Sequence for Students at Risk for Mathematics Failure	Sample: 2 students, 3rd grade Research Design: Multiple probes across behaviours design Intervention: 25 mins, 4 days per week, 12 week	Functional relation found between CRA intervention, rounding, regrouping and fraction concepts for 2 students	<ul style="list-style-type: none"> - Implementation by researcher rather than teacher - Small sample, lack of generalization - Follow up was not there to check functional use
5	Mohammad Mousa Salem AL-salahat (2022) ^[1]	The Effect of Using Concrete-Representational-Abstract Sequence in Teaching the Perimeter of Geometric Shapes for Students with Learning Disabilities	Sample size: 8 students with difficulties in mathematics, four from 4th grade, four from 5 th grade, aged 10 to 11 years Research design: quasi-experimental research design for one treatment group with a pre-posttest Intervention: 2 to 3 lessons per week, 30 to 45 minutes	CRA intervention helped in calculating perimeter of geometric shapes and solving word problems in perimeter. The learned skills was maintained post intervention for 3 weeks	<ul style="list-style-type: none"> - Limited sample size, Lack of generalizability - Maintenance period is short - No control group - Confounding factors not identified
6	Margaret M Flores, Vanessa Hinton, Shaunita D, Strozier (2014) ^[13]	Teaching Subtraction and Multiplication with Regrouping Using the Concrete-Representational-Abstract Sequence and Strategic Instruction Model	Sample: 6 students 4th grade Research Design: Single group experimental design Intervention: 10 sessions, 30 minutes, 2 days a week	Results showed that there were significant gains after intervention in these children who displayed difficulty. Students fluently computed multiplication	<ul style="list-style-type: none"> - Limited sample size - No comparison with other intervention - No follow up - No generalisation of skills assessed
7	Margaret M Flores (2010) ^[12]	Using the Concrete-Representational-Abstract Sequence to Teach Subtraction With Regrouping to Students at Risk for Failure	Sample: 6, 3 rd grade students Research design: Multiple probe across students design Intervention: 3 days per week, 30 minutes session	Functional relation between CRA instruction and subtraction with regrouping across all students was found	<ul style="list-style-type: none"> - Small sample, unable to generalize - No comparison with other methods - CRA conducted by researcher and not class teacher - No programmed module for future application
8	Dustin B. Mancl, Susan P. Miller, and Meghan Kennedy (2012) ^[19]	Using the Concrete-Representational-Abstract Sequence with Integrated Strategy Instruction to Teach Subtraction with Regrouping to Students with Learning Disabilities	Sample: 5 students, 4 th and 5 th grade Research design: A multiple-probe-across-participants design Intervention: No. of sessions not given	The students benefited from Concrete-Representational-Abstract Sequence with Integrated Strategy Instruction	<ul style="list-style-type: none"> - CRA was delivered in special setting, it needs to be applied to classroom setting - Maintenance probes were not given - Some students had exposure to the content taught, eliminating any confounding variables could be done - Details of intervention not provided
9	Emily C. Bouck, Jordan Shurr, Laura Bassette, Jiyoung Park, and Abbie Whorley (2018) ^[6]	Adding It Up: Comparing Concrete and App-Based Manipulatives to Support Students With Disabilities With Adding Fractions	Sample: 3 students, 2 students with mild IDD and 1 student with LD Research Design: adapted alternating treatment design Intervention: No. of sessions not given	Students were successful in both methods of intervention	<ul style="list-style-type: none"> - Small sample size - Inclusion of students with different types of disabilities. - Same problems were given across different manipulatives - Details of intervention not provided
10	Margaret M. Flores, Vanessa M. Hinton, Margaret O. Podemski, Madeline	Teaching Place Value Using the Concrete-Representational-Abstract Integrated Sequence of Instruction	Sample: 5 students, 3 rd grade, 2 students had LD, 3 students were at risk for LD Research Design: multi-probe single-case research design	There was a functional relation between CRA-I and place value skills, and students varied in their maintenance of skills 6 weeks after instruction as well as in generalization.	<ul style="list-style-type: none"> - Limited sample size - Lack of comparison - Small period to assess maintenance - Confounding factors not identified

	G. Burdette (2025) ^[14]		Intervention: Tier 3 interventions through a multi-tiered system of supports		
11	Shaunita Stroizer, Vanessa Hinton, Margaret Flores, LaTonya Terry (2015) ^[31]	An investigation of the effects of CRA Instruction and students with Autism Spectrum Disorder	Sample: 3 elementary students with ASD Research Design: Multiple baseline across behaviour design Intervention: 4 weeks, CRA	Students with mathematical difficulties have benefitted from CRA intervention strategy	- Limited sample size - Lack of generalizability - No control group - Confounding factors not identified
12	Tricia K. Strickland, P. Maccini (2013) ^[30]	Exploration of Quadratic Expressions through Multiple Representations for Students with Mathematics Difficulties	Sample: 5 students with mathematical difficulties, 3 had LD Research Design: A multiple-probe design Intervention: (CRA I) strategy and the use of a graphic organizer called the Box Method	Students significantly improved their algebraic accuracy and maintained their skills over time	- Limited sample size - Lack of generalizability - No control group - Confounding factors not identified

Discussion

Effectiveness of CRA Approach

Research studies have proved that CRA approach has helped in improving the mathematical skills of individuals with mathematical disability across the years. Some of the skills include addition, subtraction, multiplication, division, number sense, computational skills, conceptual understanding, rounding, regrouping, change making problems, word problem solving, fractions and perimeter of geometric shapes (Witzel 2005; Mancl *et al.*, 2012; Flores *et al.*, 2014; Bouck *et al.*, 2017; Eastburn, 2020; Morano *et al.*, 2020; Zhang *et al.*, 2021; Yildirim, 2022; Salahat, 2022; Shen, 2023)^[34, 1, 19, 13, 5, 9, 21, 36, 35, 29].

Studies have shown that apart from individuals with mathematical disabilities and difficulties, CRA Approach has found to be effective in individuals with conditions such as intellectual disability, autism spectrum disorder, developmental disabilities (Flores *et al.*, 2014)^[13] emotional and behavioural disorders (Peltier, 2017)^[25]. It is also noticeable that CRA Approach is effective in not only individuals who have difficulty understanding mathematics but also among average and above average performing students (Witzel, 2005)^[34].

Impressively, CRA Approach has shown effective results in children from 1st grade to secondary school students. Nevertheless, CRA can promote a positive impact on children's learning at a very young age. Early math foundation leads to long term academic success. Preteaching framework based on CRA Approach can help in developing mathematical skills (Watt & Therrien, 2016)^[32]. Therefore, across years various research has proven that CRA can be effective in helping individuals learn mathematical skills without barrier of age, conditions and abilities.

Methodologies adopted for CRA intervention

Distinct methodologies have been adopted in terms of research design and intervention in applying CRA approach to teach mathematical skills. Single group experimental design, two group experimental design, experimental design

with control and experimental group, multiple probes across behaviours design, adapted alternating treatment design and quasi-experimental research design have been adopted. (Flores *et al.*, 2014; Bouck *et al.*, 2017; Peltier *et al.*, 2017; Bouck *et al.*, 2018; Hinton *et al.*, 2019; Eastburn, 2020; Salahat, 2022; Yildirim, 2022; Nugroho and Jailani, 2019)^[13, 15, 9, 35, 1, 25, 23]. It has been observed that multiple probe across behaviour design has been used in most of the studies, which could be due to flexibility of multiple assessments, reduced data saturation and fatigue of participants and to focus on intervention effectiveness, majorly on establishing functional relationship between the intervention and its effectiveness.

One of the important features that has come up through various studies is that CRA Approach for mathematics can be given as an individual approach or clubbed with other approaches or given as supplementary approach. The results of comparative studies between conventional approach and CRA approach has shown that, performance of students who learn through CRA Approach is significantly higher than students who received conventional approach of learning (Nugroho & Jailani, 2019)^[23].

It has been observed that as minimum as 10 CRA sessions for a duration of 25 minutes has also provided significant improvement with respect to mathematical skills of students with mathematical difficulties (Flores *et al.*, 2014; Shen, 2023)^[13, 29]. However, the number of sessions and duration of sessions is varied across different research. While the minimum number of sessions is 10 sessions, the maximum number of sessions across studies seems to be around 50 sessions (Yildirim, 2022)^[35]. Unexpectedly, students have benefited from the lowest session duration of 25 minutes (Shen, 2023)^[29] and highest session duration of 3 hours a day (Flores *et al.*, 2014)^[13] However it would be ideal to adapt a duration, considering their grasping ability, attention capacity, current skill level and providing appropriate breaks for optimal comprehension.

Recent studies have shifted from using hands on manipulatives in 'concrete phase' to virtual manipulatives using assistive technology. It has been observed that students with disabilities, and students at risk, both have

benefitted from this in four stages: target skill acquisition, fluency, maintenance and generalization (Bassette & Bouck, 2023) ^[4]. However, a study conducted on students with intellectual disability showed that the required skill was acquired but students were struggling in maintenance of the skill.

Research Gap in CRA Intervention

Despite the emerging research on CRA, it has been applied to only fewer mathematical concepts, it can be applied to wide varieties of mathematical concepts at varied levels. Rather than theoretical aspects, learning functional mathematics could be beneficial for individuals with disabilities. There is lack of research concentrating on teaching functional mathematics.

After learning the concepts, generalization of the skill to day-to-day life settings and applying the same skills at different situations needs to be studied through follow up. Fewer research have examined the long term effects of CRA intervention. Follow up studies are crucial because it is easy to learn a skill, however maintaining the skill requires through practice. A measure can be used to assess the time of mastery for different skills.

Most of the studies have lesser sample size due to the difficulty in screening and obtaining the population. However, a study with larger sample size, would accurately represent the population. Also, it is observed that most of the studies have used single group experimental design. Due to this it is impossible to generalize the effect on larger population. To minimise the errors, having a control group ensures internal validity, establishes causality, reduces confounding variables and produces a baseline for comparison.

It has been observed that CRA is effective in helping students acquire skills related to mathematics, however there is a lack of experimental studies with larger sample sizes and across various skills. Therefore, directions for future research have been discussed to explore these areas.

Future research

Study should be replicated in a larger setting with a bigger sample size with both control and experimental groups. Future research can concentrate on assessing long-term effects of application of this model in helping students with disabilities (Eastburn, 2020) ^[9].

From teaching a concept through CRA, and understanding it, we need to assess the generalisability to day-to-day life functioning as well. The intervention model also needs to be applied on diverse variety of students which will help them to develop their proficiency in mathematical skills. This would act as a step in adopting inclusive education in classroom for students (Shen, 2023) ^[29].

Government authorities and special educators can organise workshops for teachers regarding recognising and helping students with disabilities through Concrete Representational Abstract Approach (Nneka, 2019) ^[22].

Teachers can make use of CRA approach in teaching mathematics. It has shown promising results on not only individuals who have difficulties in mathematics, but also

who have progressive scores. This could increase student interaction and reinforce gaining knowledge. Using hands on experiences as warm up and slowly guiding to blackboard can provide effective transition. This helps in providing effective connections to conceptual knowledge and application of skills (Witzel, 2005) ^[34]

It is crucial to note that an understanding of math sense begins at a very young age. Very often parents play a major role in enriching this experience. Parents can involve the child in 'concrete' activities which includes skills of counting, regrouping or computing at a very young age. Eg: Discussing the number of bowls to put for dinner, folding napkins in different shapes, measure ingredients etc. Providing positive reinforcement when the child shows perseverance in maths skills creates supportive environment and opportunities for learning.

Comparative studies can be conducted, comparing the effectiveness between conventional learning and CRA model on both typically growing children and children with various special needs (Nugroho & Jailani, 2019) ^[23].

The techniques used in the CRA sessions does not have any standardised measures that have been validated. Earliest attempts have been made by Mercer and Miller (1992) to come up with activities for different phases of CRA approach learning. However future studies can concentrate on developing a standardised manual for developing different skills with different techniques of CRA.

It is interesting to observe that student variables can also affect performance in mathematics. Hence observing student personality traits with individuals with mathematics disability and difficulty; and examining its impact in performance in mathematics can also explain certain aspects that needs to be considered while teaching mathematics (Witzel, 2005) ^[34]

Surprisingly, no experimental research studies could be found in India, regarding CRA intervention application on students with mathematical disabilities or any other special needs, hence this paves as a way towards future research and reporting of the same. Students with various special needs can benefit from this intervention model in Indian setting.

Limitation of the current study

The author did not have access to certain paid journals, hence only the abstract could be retrieved but the full study was not analysed. Having access to various indexed databases can provide a robust review of literature. Automation apps were not used as part of search strategy, as it could have helped to cover all journals and articles compared to human search and get a more systematised review. All the aspects of the PRISMA guidelines checklist could not be met. However, being one of the first Indian study on review of CRA Approach, the study has met the major guidelines and there is further scope for improvement.

Conclusion

The present study aims to explore the effectiveness of CRA intervention on students with mathematical disabilities and discuss various methodologies and highlight the existing

research gaps in this field which can pave way to enhanced future research. The paper has ventured to provide a conceptual understanding of CRA intervention and application of the same. However considering the scarcity of CRA intervention in India, it can be the starting point to researchers, scholars and teachers learning new information, developing emerging theories and apply the models to create an inclusive educational setting catering diverse variety of learners. It is never too late to learn, sometimes it is time for teachers to learn

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