A Robust RK4 Shooting Scheme for MHD Copper-Oxide–Water Nanofluid Flow and Heat Transfer across a Porous Stretching Surface

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ABSTRACT

The present numerical model investigates the heat-mass transfer characteristics in magnetohydrodynamics (MHD) Copper oxide (CuO)- water (H_2O) nanofluid floating towards a porous stretching surface. The study considered the thermophoresis effect and Brownian motion used as main components in Buongiorno's nanofluid model. The fundamental coupled equations of momentum, energy and concentration are converted into a set of ordinary differential equations (ODEs) by using suitable similarities transformations and solved numerically by using Shooting technique coupled with fourth order Runge-Kutta technique. In order to utilize this technique, boundary value problem converted into initial value problem and then select appropriate initial gausses for unknown slope of velocity, temperature and concentration to understand the heat mass transfer behavior of nanofluid in this study. The effect of various emerging constraints like Brownian motion, Magnetic field, Lewis number and Thermophoresis on fluid's flow, temperature and concentration profile are analyzed and physical quantities like Nusselt number, Skin-friction coefficient, Sherwood number are calculated arithmetically by means of RK-4 (Runge-Kutta fourth order) technique coupling with Shooting scheme in MATLAB and represented graphically. Further, numerical outcomes of study are validated with classical study of nanofluid's flow and by using BVP4C solver in MATLAB. The degree of heat transfer enhancement is developed in copper oxide-water nanofluid as compared to normal base fluid. With rise in the numerical value of magnetic parameter, the concentration and thermal boundary-layer increase but velocity boundary layer decreases.

Keywords: Magnetohydrodynamic (MHD), Copper oxide – Water, Runge-Kutta Shooting, Buongiorno's nanofluid, Porous medium, Heat transfer.

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Introduction: The fluid that comprises nanoparticles stirring in the ordinary fluid termed as"Nanofluid" have ahigh heat transfer degree between the heat transfer surface and mediumas compared to base fluids: water, motor -oil and ethylene -glycol, having low thermal conductivity. Thus, these superior types of fluids are used torecover the heat transfer coefficient. The term, "nanofluid" proposed by Choi and Eastman [1] and the discovery of nanofluid led to extensive research used in many branches of technology and industry.Makinde and Aziz [2] verified that under convective boundary conditions heat transfer enhance significantly by an inclusion of nanofluids. In the coming years,thegrowth of nanotechnology is going to bring an incredible and multi-dimensional changes in mode of our life. Recently many scholarsgave attention on this subject due to its prominent applications in engineering and its related areas.By using Cu–water nanofluid, influence on heat transfer rate due to nanoparticle shape is studied byEllahi et al. [3]. Naramgari and Sulochana[4]examined the flow of radiative MHD nanofluid past an exponentially stretching sheet and find the dual solutions using a Runge–Kutta shooting method, prominence numerical stability. The MHDflow of CuO–water nanofluid towards a stretching sheet investigated byMohammadein et al. [5] andobserved thermal boundary layer thickness increase and velocity decrease thatwith rise in the magnetic field strength velocity. Banji Jafar et al. [6] combined porous media effects with MHD nanofluid flow and concluded that permeability parameters strongly effect heat transfer and momentum characteristics. Elgazery[7] usedShooting scheme coupled with Runge–Kuttafourth method to explain the movement of nanofluid over a permeable stretching surface and observed the robustness and accuracy of themethod. In the nanotechnologies, nanofluids developed as an advanced SFNs ((spinel ferrite nanomaterials) to reduce antibiotic resistance and play the role of antibiotic agent. According to researcher, water base nanofluid have low thermal conductivity as compared to sodium -alginate based$Cu - Fe_3O_4$ $hybrid$ nanofluid. Boundary layer flow of MHD nanofluid and heat transfer characteristics proposed by Tiwari-Das in his nanofluid model which was a fascinating approach in the field of nanoparticle mechanics. The coupled heat and mass transfer process used many chemical reactions and it is possible to experience chemical reactions involve in this process as damage to crop due freezing, drying and dispersal of temperatures in the field of agricultural and energy transfer in a wet cooling tower. Kameswaranet al. [8]explained MHD nanofluid flow with viscous dissipation and chemical reaction influence andobtained the analytic solution using Kummer function without observing Soret effect. Sheikholeslami et al. [9] used GMDH-type neutrons to study magnetic effect on flow of Cu-water nanofluid.Study on non-Newtonian fluid flwing through acone under MHD connective heat-mass transfer was presented by Raju et al [10].Mathematical model for continued radiative -heat exchange and Cu-water nanofluid flow with a strong magnetic field through permeable disk produced by Shamshuddin et al. [11]. Hayat et al. [12] examined theMHDeffect under partial slip conditions in Cu-water nano-fluid flow in a rotating disk.Study done by many researchers of fluid mechanics to comprehend and validate the effects of MHD flow of nanofluids.Rashidi et al. [13] examined the analytic and numerical solution of water based viscous. Ravindra Kumar et al. [15] numerically calculated MHD nanofluid flow using RK fourth order with shooting method in a porous medium and highlighted its effectiveness in solving extremely nonlinear boundary value problems. Collectively, theseresearchers found that MHD effects, nanoparticle volume fraction and porous medium playimportant rolesin heat-mass transfer rates. However, partial studies focus on thecombined application of RK4 shooting method for CuO–water nanofluids towards a porous stretching surface, motivating the present research. Simulation of MHD CuO-H_2O nanofluid flow and heat transfer under the Lorentz- forces was explained by Sheikholeslami et al. [16].The idea behind this paper is to interpret heat and mass transfer characteristic MHD CuO-H_2O nanofluid flow pasta stretching porous surface. The fundamental partial equation continuity, momentum, thermal energy and of nanoparticle concentration are changed into coupled nonlinear ODEs by means of appropriate similarity transformation. Then,using RK-4 shooting technique,these ordinary differential equations solvednumerically in MATLAB and presented graphically effect of involved constraints on velocity, temperature and concentration profile. Also, effect of various embedded constraints on fluid flow, Skin- friction, temperature profile, Nusselt-number and Sherwood- number have been determined.

Nomenclature:c_p : specific heat
g: acceleration due to gravity
u: velocity-component in x-direction
v: velocity -component in y-direction
α: Coefficient of thermal − diffusivity
ν: Kinematic − viscosity
ρ: Density of base fluid
k: Thermal − conductivity
η: Similarity variable
ϕ: Dimensionaless concentration
θ: Dimentionaless temperature
ψ: Stream function
Pr: Prandtl number ($^{\nu}/_{\alpha}$)
Sc: Schmidth Number
Sh: Sherwood Number
Cf: Skin -Friction coefficient
Nu: Nusselt Number
Nb: Brownian Motion Parameter
Le: Lewis Number
M: Magnetic field Parameter
K: Porous medium Parameter
Nt: Thermophoresis Parameter

Mathematical -Analysis:

Consider aMHD steady two-dimensional copper-oxide-water nanofluid flowing with velocity u along x-axis past a porous stretching surface and v be the velocity of fluid in perpendicular direction. Magnetic field of strength B is applied perpendicularto the plate. Rectangular Cartesian coordinates O (x, y) with the plate placed at y=0 and reference origin O (x,y) is taken as reference where both and y axis meet. Take Reynold Number is supposed to be very small therefore eternal magnetic field (applied magnetic field) is very large as compared the induced magnetic field. Initially, fluid is taken as rest and motion is created by stretching the sheet directly proportional to distance from the origin. There is a corresponding motion in the fluid as sheet is stretched starching the **Thegoverning equations of nanofluid fluid under boundary conditions using from Buongiorno convective model givenbelow**:

Equation of Continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{-----------------(1)}$$

Along x-axis momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\frac{\partial^2 u}{\partial y^2}-\frac{\mu_{nf}}{\rho_{nf}K}\,u-\frac{\sigma_{nf}}{\rho_{nf}}B_0^2\,u \quad \text{--------------(2)}$$

where K is the Darcy permeability and Brinkman viscous diffusion is represented by the u_{yy} term

Temperature equation for fluid moment:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{n_f}\frac{\partial^2 T}{\partial y^2}+\tau D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{\tau D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \quad \text{-----------------(3)}$$

Nanoparticle concentration (from Buongiorno convective model):

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \quad \text{-----------------(4)}$$

Using Boundary conditions:

$u=U_w=ax\,;\ \ v=V_w\,;\ \ T=T_w\,;\ \ C=C_w$ as $y\to0$;

$u\to0\,;\ \ T\to T_\infty\,;\ \ C\to C_\infty$ a $y\to\infty$

where $V_w(t)$ allows suction/injection; C_w *and* T_w are the nanoparticle concentration and temperature at the wall.

Similarity transformation:

Using the similarity variable and the stream function : ψ such that $u=\frac{\partial\psi}{\partial y},\ v=-\frac{\partial\psi}{\partial x}$ *and* using the base kinematic viscosity $\nu_f=\mu_f/\rho_f$

using similarity variable η and stream function ψ as:

$$\eta=y\sqrt{\frac{a}{\nu_f}},\qquad \psi=\sqrt{a\,\nu_f}\,x\,f(\eta)$$

and define the dimensionless temperature and concentration:

$$\theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)},\qquad \varphi(\eta)=\frac{(C-C_\infty)}{(C_w-C_\infty)}$$

Here ν_f is the base-fluid kinematic viscosity

$$u=\frac{\partial\psi}{\partial y}=\sqrt{a\,\nu_f}\,x.\sqrt{\frac{a}{\nu_f}}\,.f'(\eta),\qquad v=-\frac{\partial\psi}{\partial x}=-\sqrt{a\,\nu_f}.f(\eta)$$

Above similarities transformations, reduce the governing equations into the following nonlinear coupled ordinary- differential equations as :

Transformed Momentum equation:

$$f'''+ff''-(f')^2-\left(M+\frac{1}{K_p}\right)f'=0 \text{ ---------------------------(5)}$$

Transformed Energy equation:

$$\theta''+[f\,\theta'+Nb\,\theta'\phi'+Nt\,(\theta')^2].Pr=0 \quad \text{---(6)}$$

Transformed Concentration equation:

$$\phi''+Le\,f\phi'+\frac{Nt}{Nb}\,\theta''=0 \text{ ---(7)}$$

Transformed Boundary conditions

$f(0)=S;f'(0)=1;\theta(0)=1;\phi(0)=1$ *as* $\eta=0$ --------------------------------(8)

$f'(\eta)\to0;\theta(\eta)\to0;\phi(\eta)\to0$ *as* $\eta\to\infty$ --------------------------------(9)

Where $S=\frac{-V_w(t)}{\sqrt{a\nu_f}}$

Non-dimensional terms involved in the equations (4-7) are given below:

$M=\frac{\sigma_{nf}B_0^2}{\rho_{nf}a}$ (Magnetic term) ; $K_p=\frac{Ka}{\nu_f}$ (Porous- medium term)

$Pr=\frac{\nu_f}{\alpha_f}$ (Prandtl Number); $Le=\frac{\nu_f}{D_B}$ (Lewis Number),

$Nb=\frac{\tau D_B(C_w-C_\infty)}{\nu_f}$ (*Brownian motion term*); $Nt=\frac{\tau D_T(T_w-T_\infty)}{\nu_f T_\infty}$ (Thermophoresis term)

And the quantities which are important for engineering point of view in above study;Nu : local Nusselt number,C_f: local coefficient of skin friction; Sh: local Sherwood number. After using the transformations, it found that, the skin-friction coefficient proportional to $f''(0)$,the local -Nusselt number directly proportional to $\theta'(0)$ and the local Sherwood number directly proportional to $\varphi'(0)$.The model discusses through this paper involve shooting scheme coupled with Runge-Kutta method of fourth order in MATLAB. Since,shooting scheme apply only on first order initial value problem so, we convert boundary value problem into initial value problem and reduced higher order ordinary differential equations into ordinary differential equations of first order. Then, Runge Kutta method of fourth order is used to find the solution of initial value problem assuming seven initial guesses at lower boundary point. Starting with the initial approximation, the iterative shooting procedure was applied with a convergence tolerance of 10^{-6}. The residual errors related with the boundary conditions at $\eta \to \infty$ remained gradually reduced during the iteration process. It was detected that the numerical method converged quickly, and the vital tolerance attained after five iterations. Usingthe value of involved parameters M=1.00, Pr= 6.20, Nb=0.30, Nt=0.20, Kp=1.00, Le=2.00, we get the numerical value of following physical-quantities-

Skin friction coefficient (Cf) = 0.487582, Local Nusselt number (Nu) =0.712430, Local Sherwood number (Sh) = 1.075072

The figure (I) represents Flow (Velocity), Temperature and concentration profile for selected value of substantial parameters involved in Differential equations

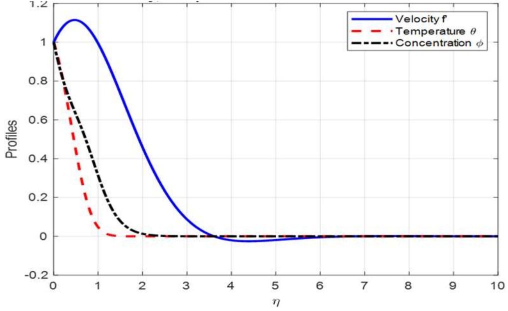
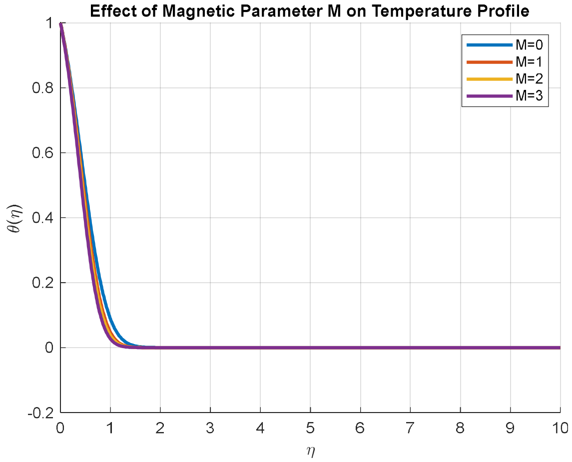


figure. (1)

Magnetic constraint (M) Effect: The flow, temperature, and nanoparticle concentration- profiles for fixed value of other parameters discussed in details. The computations were performed for fixed values Kp=1 .00, Pr=6.20, Nb=0.2, Nt=0.2, and Le=2.It is noticed that increasing the magnetic constraint M significantly reduces the velocity profile $f'(\eta)$ because due to applied perpendicular magnetic field there is a Lorentz force, which acts reverse to the direction of fluid motion. Momentum boundary layer is suppressed by this opposing force due to which motion of fluid slowdown. This behavior of fluid motion reliable with old boundary layer theory of MHD fluid flow. The skin friction coefficient,rises with increasing numerical value of magneticterm M. This is due to increased shear stress produced by magnetic restraining. The temperature profile $\theta(\eta)$ increases with increasing M. As the magnetic field strengthens, the repellant Lorentz force converts kinetic -energy into thermal -energy through Joule heating. This enhances internal energy within the boundary layer therefore thermal boundary layer become thicker. Higher temperature reduced local Nusselt number, The Nusselt number decreases with increase in the numerical value of M, indicating a reduction in heat transfer rate. Thus, the thermaltransfer at the surface decreases with increasing magnetic term M. As shown in figure (4), concentration profile $\phi(\eta)$ shows a minor increase with increasing M. Since velocity is reduced by the magnetic field, convective mass transport weakens. This leads to thicker concentration boundary layers and enhanced nanoparticle accumulation near the surface. The Sherwood number slightly decreases with increasing M, showing a reduction in mass transfer,



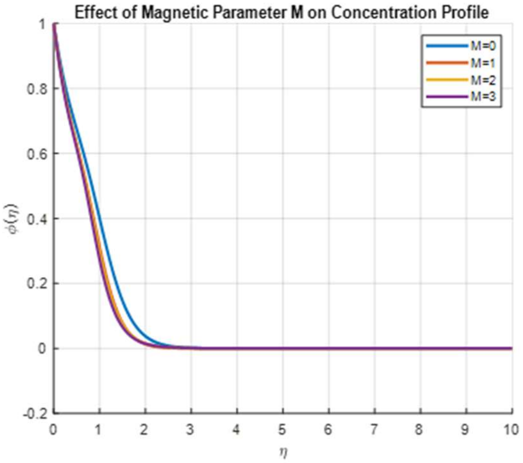
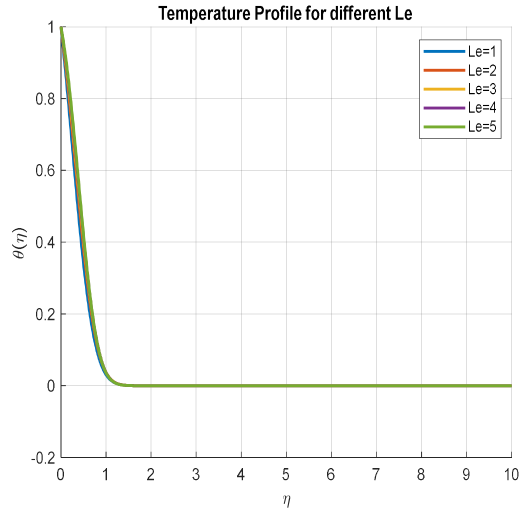


figure (2) figure (3)

Lewis Number (Nb) Effect:

From table (2), it has been observed that there is almost no change in the value of skin -friction coefficient C_f. Due to weak coupling between concentration and thermalfield, small variation observed in theNusselt number as shown in table. While, the value of Sherwood number increase with increase in the value of Lewis number because with rise in the value of Lewis parameter the corresponding concentration boundary layer becomesthinner and mass diffusivity decrease hence, mass transfer rate increase with Lewis's parameter.



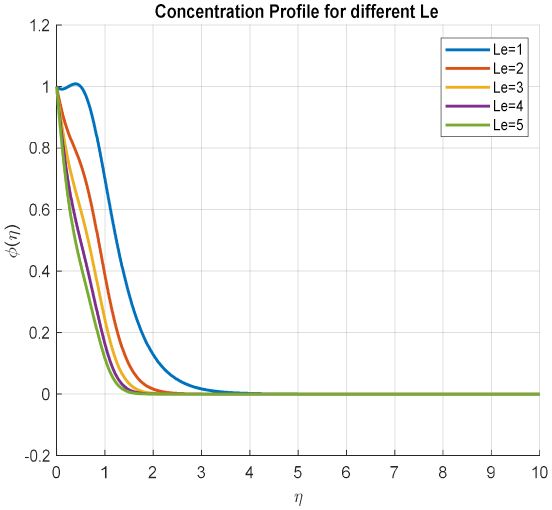
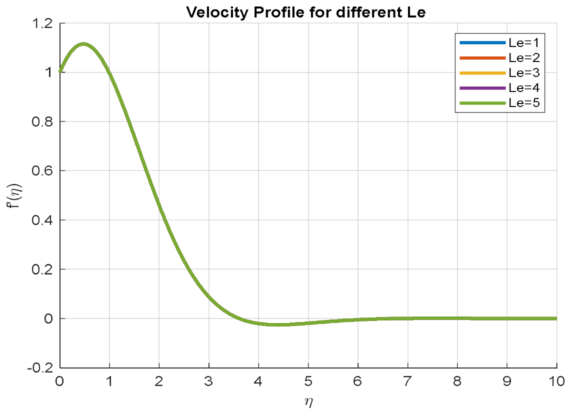


figure. (5)
figure. (6)



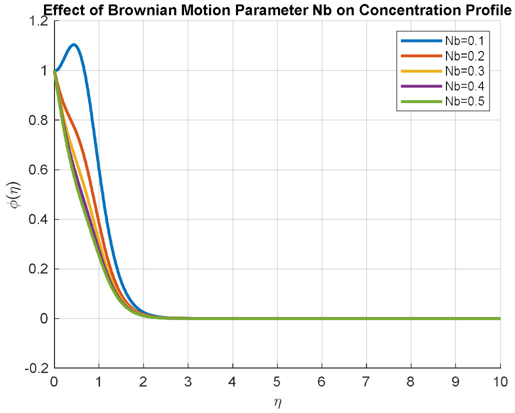
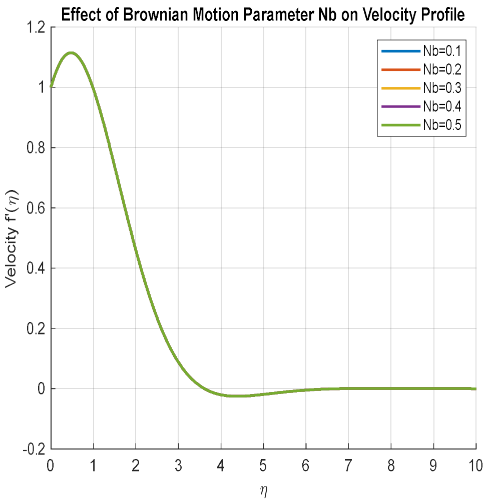


figure. (7)
figure. (8)

Brownian Motion (Nb) Effect:

Nanofluids concentration and thermal field effects significantly with the changein valueof Brownian motion (Nb). since Nb term is not present in the momentum equation of fluid flow therefore velocity profile remains unchanged with change in the value of Nb but thermal energy transport increasesdue to nanoparticle motion with increase in the Brownian motion parameterhence, temperature profile significantly increase with Nb. In contrast, Nusselt number decreases because there is diffusion of nanoparticles away from surface. Thus, heat and mass transfer rate effected by Brownian parameter (Nb) as shown in table and figures.



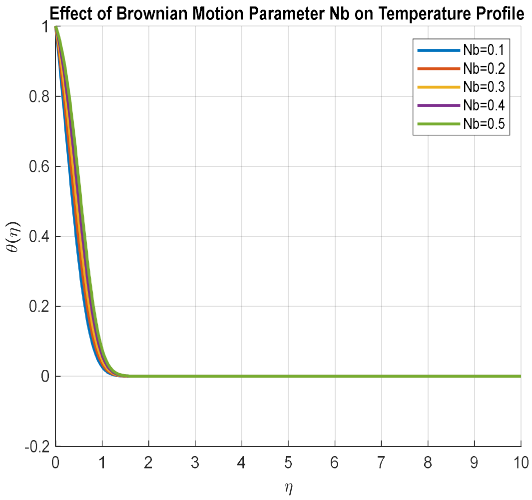


figure. (9)

figure. (10)

Thermophoresis parameter (Nt) Effect:

Since momentum equation does not contain thermophoresis parameter term Nt, therefore velocity profile remains unchanged with variation in the value of Nt. Then again, as value of Ntincreasethere is a thermophoretic transport of nanoparticle enhances both thermal and concentration boundary layer. As a result of which temperature and concentration profile increase, this lead to decrease in Nusselt and Sherwood numbers. Hence, thermal and mass transfer amount reduced with growth in the value of thermophoresis parameter Nt.

Table for values of skin friction coefficient (Cf), Sherwood number (Sh), Nusselt number for fixed value of Pr=6.2; Kp=1.0

Sr.No	Magnetic Term (M)	Lewis-Number (Le)	Brownian-Motion (Nb)	Thermophoresis Value (Nt)	Skin-Friction Coefficients (Cf)	Sherwood Number (Sh)	Nusselt Number (Nu)
1	0.00	2.0	0.2	0.2	-0.32706	0.92086	0.68695
2	1.00				0.48758	1.07507	0.71243
3	2.00				1.02245	1.09898	0.74067
4	3.00				1.67397	1.1433	0.76873
1	1.0	1.00	0.2	0.2	0.48758	0.17086	1.14527
2		2.00			0.48758	0.84696	0.93107
3		3.00			0.48758	1.27503	0.82036
4		4.00			0.48758	1.59802	0.75051
5		5.00			0.48758	1.86253	0.70170
1	1.0	2.0	0.10	0.2	0.48758	0.07514	1.19385
2			0.20		0.48758	0.84696	0.93107
3			0.30		0.48758	1.07507	0.71243
4			0.40		0.48758	1.16912	0.53476
5			0.50		0.48758	1.21145	0.39382

1	1.0	2.0	0.2	0.10	0.48758	0.93164	1.10514
2				0.20	0.48758	0.84696	0.93107
3				0.30	0.48758	0.86613	0.78584
4				0.40	0.48758	0.95582	0.66526
5				0.50	0.48758	1.09054	0.56548

Validation of Result: The model discussed in this paper exactly matches with popular Khan and Pop model (2010) in absence of magnetic parameter (M=0) when porous term (Kp) approaches infinity.

Comparison Table(using M=0, Pr=6.200, Nb=0.300, Le=2, Nt=0.2 and Kp→ ∞)

Result outcome	**Present study (RK4 Shooting method)**	**Present study (BVP4C solver)**	**Khan & Pop [22]**	**Rana &Bhargava [23]**
$C_f = f''(0)$	-1.00063	-1.0008	-1.0003	-1.0006
$Nu = -\theta'(0)$	0.681356	0.68158	0.6814	0.681356
$Sh = -\phi'(0)$	0.706392	0.706457	0.7065	0.706458

The outcome of present study obtained are validated by using classical popular model of Khan&Pop (2010) and Flow and heat transfer of a nanofluid flowing over a nonlinearly stretching: Numerical study in absence of magnetic parameter and limiting case of porous medium. The calculated results of present study show good agreement with classical studies showing the physical consistency and accuracy of present study. Again, boundary equations of this study aresolvedby using MATLAB BVP4C solver and numerical value of Skin friction coefficient (Cf), Nusselt number (Nu), Sherwood number (Sh)are found in close agreement with the values calculated using the RK4 coupling with shooting scheme, thereby confirming the numerical accuracy of the present paper.

Conclusion:

The transformed ordinary differential equation involved in MHD nanofluid flow past a stretching surface with porous medium were solved by means of shooting technique coupledby Runge-Kutta method of fourth order. The effect of dimensionless parameter like-Magnetic constraint term (M), Lewis number (Le.), Brownian motion (Nb.), Thermophoresis parameter (Nt) on Velocity, Temperature profile, concentration profile are observed and represented graphically. in result section of this paper. Consequently, effect of this parameter on Skin-Friction coefficient, Nusseltnumber, Sherwood number calculated mathematically to analyze heat and mass transfer characteristics at surface.Thus, the studyestablishes the complex nature of magneto-thermal mass transport in nanofluid and help in highlight the benefits of such modelling of fluid flow using multiple physical mechanism.

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