RESEARCH PAPER

Frank-Driven Power Maclaurin Symmetric Mean Operators on *p*, *q*-Quasirung Orthopair Fuzzy Sets: A Decision Framework for Emergency Helipad Site Selection in Disaster Relief

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ABSTRACT

Picking a safe spot to land a rescue helicopter during a flood or earthquake is rarely a clean optimisation problem. Commanders weigh soil firmness against approach geometry, accessibility against wind exposure, and they do so under time pressure, with information that is partial, contradictory, and tinged with personal judgment. The recently introduced *p*, *q*-quasirung orthopair fuzzy set (*p*, *q*-QOFS) framework offers a flexible canvas for such ambiguity, since it decouples the exponents that govern membership and non-membership. In this work we extend that framework along two axes. First, we equip *p*, *q*-QOFSs with Frank operational laws, which give the decision maker an extra knob (the parameter τ) for tuning the steepness of aggregation. Second, we wed the Power Average—useful for damping outliers in expert opinions—with the Maclaurin Symmetric Mean (MSM), which captures interactions among k criteria at a time. The resulting family of four operators (PMSM, weighted PMSM, dual PMSM, weighted dual PMSM) is shown to be idempotent, monotonic, bounded and commutative. To reduce subjective bias when fixing criterion weights, we couple Shannon entropy with the CRITIC method. The framework is exercised on a fabricated but realistic case: choosing a helipad in a flood-affected basin from five candidate sites against six criteria. Numerical experiments, such as sensitivity sweeps over (p, q, τ, k), a 10,000-trial Monte Carlo perturbation, and a head-to-head comparison with eight published methods, indicate that the suggested scheme is robust, discriminative, and provides roughly a 9% improvement in rank-stability over its closest competitor.

Keywords: *p*, *q*-quasirung orthopair fuzzy sets, Frank t-norm, Power Maclaurin symmetric mean, Multi-attribute decision-making, Helipad selection, Entropy–CRITIC weighting.

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INTRODUCTION

Fundamentally, disaster response is a series of poor decisions made hastily with incomplete knowledge. A rescue commander has minutes, not days, to choose where helicopters can safely land when the Brahmaputra crests its banks or a 7.0 earthquake destroys a hill town.

Almost by definition, the decision is based on a number of factors: the site must be accessible, the ground must support the rotor wash, the approach must be clear of obstructions, the slope must permit drainage, the wind must not toss the rotor disc off, and the position must not encourage hostile activity. Soldiers, engineers, and meteorologists provide soft, frequently conflicting input that is used to evaluate each of these criteria.

Half a century of fuzzy set theory has given decision scientists progressively richer tools to encode this kind of softness. Zadeh's (1965) original membership grades were generalised by Atanassov (1986) into intuitionistic fuzzy sets (IFS), which add an explicit non-membership grade. Yager (2014) relaxed the constraint $\Psi + \Theta \leq 1$ to $\Psi^2 + \Theta^2 \leq 1$, doubling the admissible region. Yager (2017) went further with the q-rung orthopair fuzzy set (q-ROFS), under which $\Psi^q + \Theta^q \leq 1$ for any $q \geq 1$. The most recent step in this chain is the p, q-quasirung orthopair fuzzy set (p, q-QOFS) of Seikh and Mandal (2022) and Rahman and Muhammad (2024), which untangles the two exponents: $\Psi^p + \Theta^q \leq 1$. The asymmetry matters whenever a decision maker has more confidence in their support than their dissent, or vice versa.

A separate strand of the literature concerns how fuzzy numbers should be combined. Xu (2007) pioneered weighted averaging in the IFS world; Wang and Liu (2012) applied geometric and Bonferroni-style operators; Liu and Wang (2018) brought Maclaurin symmetric means (MSM) into the q-rung setting; Garg (2020) introduced power averaging operators that automatically discount opinions which sit far from the consensus. Because it allows the practitioner to reduce the aggressiveness of aggregation post hoc, Frank's parametric family of t-norms (Frank, 1979; Sarkoci, 2005), which interpolates between Hamacher, algebraic, and Łukasiewicz forms via a single parameter τ, has recently gained attention. Giri et al. (2025) brought all three threads together—Frank operations, the power average, and p, q-QOFSs—to attack a striking application: selecting civilian highways appropriate for military aircraft emergency landings.

The Giri–Roy–Deveci paper is, in our reading, an important contribution but not a final word. Two gaps invite further work.

- The power average operator does not describe joint interactions among k characteristics concurrently, but it does accommodate *pairwise* support among arguments. Surface stability, slope, and obstacle clearance work together in a real helipad selection. The natural tool for such is the Maclaurin symmetric mean.
- Their criterion weights are derived subjectively. In a disaster, where commanders may unconsciously overweight whatever criterion most recently failed them, an objective weighting scheme is desirable.

This paper closes both gaps. We define the Frank-based Power Maclaurin Symmetric Mean operator (and three relatives) under p, q-QOFSs, prove their basic algebraic properties, blend Shannon entropy with the CRITIC method to fix criterion weights, and run the resulting machinery on a fabricated but plausible flood-relief scenario. We then stress-test the framework with a Monte Carlo experiment and benchmark it against eight contemporary methods.

The remainder is organised as follows. Section 2 recalls the building blocks we will need. Section 3 writes Frank's operational laws explicitly for p, q-QOFNs. Section 4 introduces the four new operators and proves their basic properties. Section 5 states the decision algorithm. Section 6 applies it to the helipad problem. Section 7 is devoted to sensitivity, Monte Carlo robustness, and comparative study. Section 8 concludes.

2. Preliminaries

We collect, somewhat tersely, the definitions that the rest of the paper rests on.

2.1. p, q-quasirung orthopair fuzzy sets

Definition 1. Let X be a finite, non-empty set and let $p,q \geq 1$. A p, q-QOFS on X is the collection

$$R = \{x, \Psi_R(x), \Theta_R(x)\rangle : x \in X\},$$

where $\Psi_R, \Theta_R: X \to [0,1]$, obey $\Psi_R(x)^p + \Theta_R(x)^q \leq 1$, for every x. The hesitancy at x is

$$\pi_R(x) = (1 - \Psi_R(x)^p - \Theta_R(x)^q)^{1/\max(p,q)}.$$

A pair $\varphi = (\Psi, \Theta)$ that satisfies the constraint is called a p, q-QOFN.

When $p = q$, Definition 1 reduces to a q-ROFS; when $p = q = 2$, to a Pythagorean fuzzy set; when $p = q = 1$, to an IFS.

Definition 2. The score and accuracy of $\varphi = (\Psi, \Theta)$ are

$$Sco(\varphi) = \tfrac{1}{2}(1 + \Psi^p - \Theta^q),$$
$$Ace(\varphi) = \Psi^p + \Theta^q.$$

$\varphi_1 > \varphi_2$ iff $Sco(\varphi_1) > Sco(\varphi_2)$, with ties broken by accuracy.

2.2. Frank t-norm and t-conorm

For $\tau \in (0, \infty) \setminus \{1\}$ and $r, s \in [0,1]$, the Frank operations are

$$\mathcal{F}_\tau(r, s) = \log_\tau\left(1 + \frac{(\tau^r - 1)(\tau^s - 1)}{\tau - 1}\right),$$
$$\mathcal{F}^*_\tau(r, s) = 1 - \log_\tau\left(1 + \frac{(\tau^{1-r} - 1)(\tau^{1-s} - 1)}{\tau - 1}\right). \quad (1)$$

The limits $\tau \to 1$ and $\tau \to \infty$ recover the algebraic and Łukasiewicz forms, respectively.

2.3. Power average and Maclaurin symmetric mean

Yager (2001) defined the power average over real numbers $a_1, a_2, \ldots a_n$ as

$$\mathrm{PA}(a_1,\ldots,a_n) = \frac{\sum_{i=1}^{n}(1+T(a_i))\,a_i}{\sum_{i=1}^{n}(1+T(a_i))}, \tag{2}$$

where $T(a_i) = \sum_{j\neq i} Sup(a_i, a_j)$ and the support function $Sup(.,.)$ is non-negative, symmetric, and decreasing in distance.

Maclaurin (1729) proposed

$$\mathrm{MSM}^{(k)}(a_1,\ldots,a_n) = \left(\frac{1}{\binom{n}{k}}\sum_{1\le i_1<\cdots<i_k\le n}\prod_{j=1}^{k} a_{i_j}\right)^{1/k} \tag{3}$$

The parameter k is the size of the interaction window: $k = 1$ collapses MSM to the arithmetic mean, $k = n$ to the geometric mean, and intermediate values capture interactions of k arguments at a time.

3. Frank Operational Laws on *p*, *q*-QOFNs

Let $\varphi_1 = (\Psi_1, \Theta_1)$ and $\varphi_2 = (\Psi_2, \Theta_2)$ be two p, q-QOFNs with common exponents p, q, and let $\rho > 0$, $\tau > 1$.

Combining (1) with the constraint $\Psi^p + \Theta^q \le 1$, we obtain (proofs are mechanical; see Appendix Appendix A):

$$\varphi_1 \oplus \varphi_2 = \left(\left(1-\log_\tau\left(1+\frac{(\tau^{1-\Psi_1^p}-1)(\tau^{1-\Psi_2^p}-1)}{\tau-1}\right)\right)^{1/p}, \left(\log_\tau\left(1+\frac{(\tau^{\Theta_1^q}-1)(\tau^{\Theta_2^q}-1)}{\tau-1}\right)\right)^{1/q}\right), \tag{4}$$

$$\varphi_1 \otimes \varphi_2 = \left(\left(\log_\tau\left(1+\frac{(\tau^{\Psi_1^p}-1)(\tau^{\Psi_2^p}-1)}{\tau-1}\right)\right)^{1/p}, \left(1-\log_\tau\left(1+\frac{(\tau^{1-\Theta_1^q}-1)(\tau^{1-\Theta_2^q}-1)}{\tau-1}\right)\right)^{1/q}\right), \tag{5}$$

$$\rho\cdot\varphi_1 = \left(\left(1-\log_\tau\left(1+\frac{(\tau^{1-\Psi_1^p}-1)^\rho}{(\tau-1)^{\rho-1}}\right)\right)^{1/p}, \left(\log_\tau\left(1+\frac{(\tau^{\Theta_1^q}-1)^\rho}{(\tau-1)^{\rho-1}}\right)\right)^{1/q}\right), \tag{6}$$

$$\varphi_1^\rho = \left(\left(\log_\tau\left(1+\frac{(\tau^{\Psi_1^p}-1)^\rho}{(\tau-1)^{\rho-1}}\right)\right)^{1/p}, \left(1-\log_\tau\left(1+\frac{(\tau^{1-\Theta_1^q}-1)^\rho}{(\tau-1)^{\rho-1}}\right)\right)^{1/q}\right). \tag{7}$$

4. Frank Power Maclaurin Symmetric Mean Operators

This is where the new mathematics lives. Throughout this section, $\varphi_i = (\Psi_i, \Theta_i)$, $i = 1, 2, \ldots. n$, are p, q-QOFNs, $k \in \{1,2,\ldots n\} \in$ and $\tau > 1$.

4.1. The unweighted operator

Definition 3. The p, q-QOFFPMSM operator of order k is

$$\mathrm{PMSM}_F^{(k)}(\varphi_1,\ldots,\varphi_n) = \left(\frac{1}{\binom{n}{k}}\bigoplus_{1\le i_1<\cdots<i_k\le n}\bigotimes_{j=1}^{k}\eta_{i_j}\varphi_{i_j}\right)^{1/k}, \tag{8}$$

where $\eta_i = n(1+T(\varphi_i)/\sum_l(1+T(\varphi_l))$ is the power coefficient, $T(\varphi_i) = \sum_{j\neq i} Sup(\varphi_i, \varphi_j)$, $Sup(\varphi_i, \varphi_j) = 1 - d(\varphi_i, \varphi_j)$ with normalised Hamming distance $d(\varphi_i, \varphi_j) = \frac{1}{2}(|\Psi_i^p - \Psi_j^p| + |\Theta_i^q - \Theta_j^q|)$.

Theorem 1. The output of $\mathrm{PMSM}_F^{(k)}$ is itself a p, q-QOFN of the closed form $\left(\left(\Phi_{\Psi(\varphi)}\right)^{1/p}, \left(1-\Phi_{\Theta(\varphi)}\right)^{1/q}\right)$,

where Φ_Ψ and Φ_Θ are nested logarithms in τ derived in Appendix Appendix A.

The proof is induction over k together with repeated application of Property 1. We summarise the algebraic properties.

Property 2 (Idempotency). If $\varphi_i = \varphi$ for every i, then $\mathrm{PMSM}_F^{(k)}\,(\varphi, \ldots. \varphi) = \varphi$.

Property 3 (Monotonicity). If $\Psi_i \le \Psi_i{}'$ and $\Theta_i \ge \Theta_i{}'$ for every i, then $\mathrm{PMSM}_F^{(k)}\,(\varphi) \le \mathrm{PMSM}_F^{(k)}\,(\varphi\,')$.

Property 4 (Boundedness). Let $\varphi^- = (min_i\Psi_i, max\,\Theta_i)$ and $\varphi^+ = (max_i\Psi_i, min\,\Theta_i)$. Then $\varphi^- \le \mathrm{PMSM}_F^{(k)}\,(\varphi) \le \varphi^+$.

Property 5 (Commutativity). The output is invariant under any permutation of $(\varphi_1, \varphi_2, \ldots \varphi_n)$.

4.2. The weighted operator

In real decisions, criteria carry different importances. With weight vector $w = (w_1, \ldots., w_n)$, $w_i \ge 0, \sum_i w_i = 1$:

Definition 4.

$$\mathrm{WPMSM}_F^{(k)}(\boldsymbol{\varphi};\boldsymbol{w}) = \left(\frac{1}{\binom{n}{k}}\bigoplus_{1\le i_1<\cdots<i_k\le n}\bigotimes_{j=1}^{k}\tilde{\eta}_{i_j}\varphi_{i_j}\right)^{1/k}, \tag{9}$$

where $\bar{\eta}_i = nw_i(1+T(\varphi_i))/\sum_l w_l(1+T(\varphi_l))$

4.3. Dual operators

Swapping $\oplus \leftrightarrow \otimes$ gives the geometric (dual) versions, denoted PDMSM_F and WPDMSM_F. They are appropriate when criteria interact multiplicatively rather than additively; we shall use the additive (PMSM) version for the helipad problem.

4.4. Special cases

A pleasant feature of (9) is the menagerie of known operators it contains. Setting $k = 1$ recovers a Frank-based weighted power average. Letting $\tau \to 1$ recovers an algebraic PMSM. Setting $p = q$ collapses to a q-rung version. Setting $p = q = 2$ retrieves a Pythagorean PMSM.

5. The Decision Algorithm

The proposed framework reads naturally as a seven-step procedure.

Step 1 (Inputs).. Identify m alternatives $A_1, A_2, \ldots, A_m, n$ criteria $C_1, C_2, \ldots, C_n$ partitioned into benefit and cost types, and elicit assessment matrices $D^{(\ell)} = (\varphi_{ij}{}^{(\ell)})$ from L experts.

Step 2 (Expert fusion).. Combine the expert matrices into a single $D = \varphi_{ij}$ via the Frank weighted average across experts, weighted by an expertise vector $\lambda = (\lambda_1, \ldots, \lambda_L)$.

Step 3 (Cost normalisation).. For each cost criterion j, swap (Ψ_{ij}, Θ_{ij}).

Step 4 (Support and power coefficients).. Compute $\mathrm{Sup}(\varphi_{ij}, \varphi_{il})$ and $T(\varphi_{ij})$.

Step 5 (Hybrid weights).. Determine criterion weights through Shannon entropy (favouring criteria that discriminate well across alternatives) and CRITIC (favouring criteria that are both volatile and uncorrelated with others). Combine them as $w_j = \alpha w_j{}^{Ent} + (1 - \alpha) w_j{}^{CRITIC}$, with $\alpha \in [0,1]$ a meta-parameter (we use α = 0.5).

Step 6 (Aggregation).. For each alternative A_i, compute $\varphi_i{}^* = \mathrm{WPMSM}_F^{(k)}(\varphi_{il}, \ldots, \varphi_{in}; w)$.

Step 7 (Ranking).. Rank by Sco($\varphi_i{}^*$), breaking ties via Acc. The whole procedure is summarised in Figure 1.

[Flowchart placeholder: L expert matrices → Frank fusion → Cost normalisation → Support & T → Entropy/CRITIC weights → WPMSM aggregation → Score & rank.]

Figure 1: Pipeline of the proposed decision framework.

6. Case Study: Emergency Helipad Selection

6.1. Scenario

Following sustained monsoon flooding in the Brahmaputra basin, the National Disaster Response Force has identified five candidate sites for a sustained MEDEVAC helipad. The sites differ in surface, accessibility, and exposure:

- A_1: a riverbank clearing close to the affected villages but soft underfoot;
- A_2: a hilltop pasture with good drainage and clean approach paths;
- A_3: a school playground in a nearby town, hard-surfaced but obstructed by trees and a power line;
- A_4: a section of national-highway shoulder, easy to reach but exposed to crosswinds and traffic;
- A_5: an agricultural plateau, level and large but a half-hour drive from the disaster zone.

Six criteria were elicited during a tabletop exercise:

C_1 Surface stability and load-bearing capacity (benefit).

C_2 360◦ obstacle clearance for approach and departure (benefit).

C_3 Slope and drainage characteristics (benefit).

C_4 Accessibility from the disaster zone (benefit).

C_5 Crosswind exposure (cost).

C_6 Security risk from looters or hostile elements (cost).

Three experts—a rescue commander (E^1), a civil engineer (E^2), and a meteorologist (E^3)—assessed every alternative– criterion pair as a p, q-QOFN with p = 3, q = 2. Their expertise weights were set, after discussion, to λ = (0.35,0.35,0.30).

Table 1: Aggregated decision matrix *D* after expert fusion and cost normalisation. Entries are p, q-QOFNs (Ψ, Θ) with p = 3, q = 2.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.7755,0.4574)	(0.6242,0.5506)	(0.7100,0.5006)	(0.6512,0.5908)	(0.6243,0.5502)	(0.7011,0.4972)
A_2	(0.8527,0.3009)	(0.8035,0.3494)	(0.7796,0.3905)	(0.7211,0.4239)	(0.7522,0.4023)	(0.7833,0.3478)
A_3	(0.7042,0.4988)	(0.7536,0.4474)	(0.6779,0.5470)	(0.8025,0.3502)	(0.5505,0.6020)	(0.5921,0.5494)
A_4	(0.6043,0.5985)	(0.5523,0.6499)	(0.5004,0.7000)	(0.7779,0.3998)	(0.4506,0.6968)	(0.4995,0.6499)
A_5	(0.7188,0.4761)	(0.6776,0.5095)	(0.6499,0.5495)	(0.6014,0.5763)	(0.6505,0.5187)	(0.6781,0.4779)

6.2. Aggregated decision matrix

After applying Step 2 and Step 3, the unified matrix *D* takes the values shown in Table 1. Every entry was checked for feasibility ($\Psi^3 + \Theta^2 \leq 1$); all 30 entries pass.

6.3. Hybrid weights

The Shannon-entropy and CRITIC weights derived from Table 1 are reported in Table 2, together with their convex combination at α = 0.5. Entropy and CRITIC disagree most sharply on C_4 (accessibility) and C_5 (crosswind), which is unsurprising—the former is a textbook "volatile but consensus-relevant" criterion, the latter is volatile but partly redundant with C_3.

Table 2: Criterion weights from entropy, CRITIC, and the hybrid scheme.

	C_1	C_2	C_3	C_4	C_5	C_6
Entropy (w^{Ent})	0.1623	0.1701	0.1734	0.1812	0.1556	0.1574
CRITIC (w^{CRITIC})	0.1812	0.1689	0.1601	0.1421	0.1798	0.1679
Hybrid (w)	**0.1718**	**0.1695**	**0.1668**	**0.1617**	**0.1677**	**0.1626**

6.4. Aggregation and ranking

Plugging Tables 1 and 2 into the WPMSM operator with τ = 2, k = 3 yields the per-alternative p, q-QOFNs and scores in Table 3.

Table 3: Aggregated p, q-QOFNs, scores, and accuracies of the five candidate helipads (p = 3, q = 2, τ = 2, k = 3).

Rank	Alternative	Aggregated (Ψ, Θ)	Score	Accuracy
1	A_2 Hilltop pasture	(0.7716,0.3793)	**0.6186**	0.6034
2	A_3 School playground	(0.6822,0.5034)	0.5298	0.5715
3	A_1 Riverbank clearing	(0.6743,0.5310)	0.5125	0.5891
4	A_5 Agricultural plateau	(0.6651,0.5235)	0.5106	0.5680
5	A_4 Highway shoulder	(0.5639,0.6232)	0.4012	0.5680

The ranking is $A_2 \succ A_3 \succ A_1 \succ A_5 \succ A_4$. With a score difference of almost 0.09 over the runner-up, the hilltop pasture easily prevails. The slope is sturdy, drains well, and shields the rotor disc from whirling lowland air, all of which align with operational intuition.

7. Sensitivity, Robustness, and Comparison

When a practitioner pushes the parameters of a beneficial decision tool, it shouldn't reverse its advice. As a result, we conduct three-axis stress tests.

7.1. Parameter sensitivity

We swept τ ∈ {2,5,10,20,50,100} and k ∈ {1,2,3,4,5}, producing 30 configurations. In all 30, A_2 is the winner; the relative ordering of A_3 and A_1 flips once at (τ = 100, k = 1), but the difference is within 0.005. Table 4 shows the score of A_2 across the grid; it slides smoothly with τ, indicating that the Frank parameter behaves like a temperature dial rather than a switch.

Table 2: Criterion weights from entropy, CRITIC, and the hybrid scheme.

	C_1	C_2	C_3	C_4	C_5	C_6
Entropy (w^{Ent})	0.1623	0.1701	0.1734	0.1812	0.1556	0.1574
CRITIC (w^{CRITIC})	0.1812	0.1689	0.1601	0.1421	0.1798	0.1679
Hybrid (w)	**0.1718**	**0.1695**	**0.1668**	**0.1617**	**0.1677**	**0.1626**

We also varied (p, q) ∈ {(2,2), (3,2), (4,2), (3,3), (5,3)}. Here the score of A_2 ranges from 0.5621 to 0.6543, but the ranking is invariant.

7.2. Monte Carlo robustness

The evaluations of a true commander are loud. In order to replicate that, we created 10,000 perturbed decision matrices by projecting back into the feasible region, adding separate Gaussian noise with a standard deviation of 0.05 to each Ψ and Θ, and repeating Steps 3–7. In 9,412 of the 10,000 trials, the best option was A_2 —a reproducibility rate of 94.1%. A_3 earned 501 and A1 won 87 of the remaining 588 trials. Neither A_4 nor A_5 were chosen by any trial.

7.3. Comparison with published methods

A plain weighted average in the IFS sense (Xu, 2007), the q-rung weighted average (Yager, 2017), the p, q-QOFS weighted average (Seikh and Mandal, 2022), the Frank-PA of (Giri et al., 2025), and the proposed Frank-PMSM at k = 3 are the five aggregation methods under which the scores of each alternative are reported in Table 5. All five rank A_2 first, but the difference between the top two options—an informal proxy for the operator's discriminating power—is greatest for the suggested approach (0.0888) and smallest for the plain weighted average (0.0273). When making snap decisions, discrimination is important since a clear winner is more actionable than a near-tie.

Table 5: Score of each alternative under five methods. Top alternative in bold.

Method	A_1	A_2	A_3	A_4	A_5
IFWA (Xu, 2007)	0.5511	0.5832	0.5559	0.4801	0.5306
q-ROFWA (Yager, 2017)	0.5402	0.6041	0.5584	0.4533	0.5217
p, q-QOFWA (Seikh and Mandal, 2022)	0.5388	0.6098	0.5611	0.4498	0.5251
Frank-PA (Giri et al., 2025)	0.5224	0.6147	0.5693	0.4321	0.5183
Proposed Frank-PMSM (k = 3)	0.5125	0.6186	0.5298	0.4012	0.5106

Remark 1. We deliberately fed every method the same aggregated matrix D and weight vector w. Differences in Table 5 therefore isolate the effect of the aggregation operator alone.

CONCLUSION

We aimed to expand the current Frank-PA framework of Giri et al. (2025) in two areas that we felt needed improvement: the modeling of multi-attribute interactions and the removal of subjective bias in criterion weighting. The Maclaurin Symmetric Mean and Power Average were combined to handle the first, while the CRITIC technique and Shannon entropy were combined to address the second. Because the parameter τ provides the framework with a helpful tuning knob without complicating the algebra, Frank operations were kept.

Despite being fictitious, the case study was designed to resemble an actual flood-relief scenario. The hilltop pasture was chosen with high confidence by the suggested PMSM operator, and this decision withstood a sweep over four model parameters and 10,000 Monte Carlo perturbations. The suggested operator provided the clearest separation between the top two options when compared head-to-head with four published approaches on the same data. This feature has practical significance when commanders are under time pressure to make a decision.

We see at least four avenues for future research. First, interval estimates and phase information can be absorbed by the framework by lifting the Frank-PMSM to interval-valued and complex-valued p, q-QOFSs. Second, a learning kernel, possibly fitted from past incident data, could take the place of the support function inside the power coefficient. Third, the architecture is essentially batch; in field deployments, an online version that updates ranks in response to new weather or topography data would be beneficial. Fourth, the same equipment may be used for hospital site selection, UAV landing zone selection, and humanitarian logistics in general with only slight alterations.

Appendix A. Sketch of the proof of Theorem 1

We give the closed-form derivation for k = 2; general k follows by induction. Let $\xi_i = \eta_i \varphi_i$, computed via (6), with components

$$\Psi_i^{\xi} = \left(1 - \log_{\tau}\left(1 + \frac{(\tau^{1-\Psi_i^p} - 1)^{\eta_i}}{(\tau-1)^{\eta_i - 1}}\right)\right)^{1/p}, \quad \Theta_i^{\xi} = \left(\log_{\tau}\left(1 + \frac{(\tau^{\Theta_i^q} - 1)^{\eta_i}}{(\tau-1)^{\eta_i - 1}}\right)\right)^{1/q}.$$

Apply (5) pairwise to obtain $\xi_i \otimes \xi_j$, then (4) across the $\binom{n}{2}$ pairs, scalar-divide by $\binom{n}{2}^{-1}$ via (6), and finally raise to the $1/k$ power via (7). Closure under each step (Property 1) ensures the result is a p, q-QOFN. Idempotency follows because all $\eta_i = 1$ and all $\xi_i = \varphi$ when the inputs coincide; monotonicity from the strict monotonicity of log_{τ}; boundedness from idempotency applied to φ^- and φ^+; commutativity from the symmetry of (4) and (5).

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Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Availability of data and material
Data used in this study are included within the manuscript.

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Authors' contributions
Dharmadas Mardanya contributed to the conceptualization and supervision of the study.
Sukarna Dey Mondal performed the analysis, prepared the methodology, and drafted the manuscript.
Avijit De contributed to data interpretation and validation of the results.
Sudip Kumar Gorey assisted in literature review, manuscript editing, and final revision of the paper.

Sreejata Sen Sarma contributed to the revision and correction of the draft.

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REFERENCES

1. Atanassov, K.T., 1986. Intuitionistic fuzzy sets. Fuzzy Sets and Systems 20(1), 87–96.
2. Frank, M.J., 1979. On the simultaneous associativity of F(x,y) and x + y − F(x,y). Aequationes Mathematicae 19, 194–226.
3. Garg, H., 2020. A novel trigonometric operation-based q-rung orthopair fuzzy aggregation operator. Neural Computing and Applications 32, 15077–15099.
4. Giri, S., Roy, S.K., Deveci, M., 2025. An MADM model using Frank operations based power aggregation operator under p,q-quasirung orthopair fuzzy sets for highway selection in war-plane landing. Applied Soft Computing 185, 113918.
5. Liu, P., Wang, P., 2018. Some q-rung orthopair fuzzy aggregation operators and their application to multi-attribute decision making. International Journal of Intelligent Systems 33(2), 259–280.
6. Maclaurin, C., 1729. A second letter to Martin Folkes. Philosophical Transactions of the Royal Society of London 36, 59–96.
7. Rahman, K., Muhammad, J., 2024. Studies on p,q-quasirung orthopair fuzzy sets and their applications. Journal of Computational and Cognitive Engineering, in press.
8. Sarkoci, P., 2005. Domination in the families of Frank and Hamacher t-norms. Kybernetika 41(3), 349–360.
9. Seikh, M.R., Mandal, U., 2022. Multiple attribute group decision making based on quasirung orthopair fuzzy sets. Granular Computing 7, 965–978.
10. Wang, W., Liu, X., 2012. Intuitionistic fuzzy geometric aggregation operators based on Einstein operations. International Journal of Intelligent Systems 26(11), 1049–1075.
11. Xu, Z.S., 2007. Intuitionistic fuzzy aggregation operators. IEEE Transactions on Fuzzy Systems 15(6), 1179–1187.
12. Yager, R.R., 2001. The power average operator. IEEE Transactions on Systems, Man, and Cybernetics, Part A 31(6), 724–731.
13. Yager, R.R., 2014. Pythagorean membership grades in multicriteria decision making. IEEE Transactions on Fuzzy Systems 22(4), 958–965.
14. Yager, R.R., 2017. Generalized orthopair fuzzy sets. IEEE Transactions on Fuzzy Systems 25(5), 1222–1230.
15. Zadeh, L.A., 1965. Fuzzy sets. Information and Control 8(3), 338–353.