RESEARCH PAPER

Optimum Solution for Multi-Objective Non-linear Model in Intuitionistic Fuzzy Goal Arena

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ABSTRACT

The growing impact of power generation on the environment has led to the need for new optimization methods that can minimize environmental impact and fuel costs while adhering to various constraints. In this study, the multi-objective economic load dispatch (MOEELD) problem is examined through the use of an innovative intuitionistic fuzzy goal programming (IFGP) approach for the nonlinear optimization of power systems. The major difference between the objectives of intuitionistic fuzzy goal programming and those of traditional economic dispatch is that uncertainty and imprecision associated with real-world decision-making are explicitly included in intuitionistic fuzzy goal programming, whereas a deterministic approach is utilized by traditional economic dispatch models.

Using intuitionistic fuzzy sets to formulate fuel cost and emission objectives allows for the ability to analyze satisfaction, non-satisfaction, and hesitation by defining appropriate membership functions to consider. Reference aspiration levels for each objective are provided by the decision-maker, which enables the application of a structured goal programming method to solve the resulting optimization problem. The degree of flexibility in modelling is expanded by this dual framework, and the robustness of trade-off analyses of multiple objectives with respect to operational constraints (e.g., power balance and transmission loss) is enhanced.

The efficacy of the IFGP method has been demonstrated by means of an industry-standard 3-generator test system, including energy losses over transmission lines. A superior solution is provided by the IFGP through a more equal trade-off between minimization of the cost of fuel and reduction in emissions, as shown by a comparison between intuitionistic fuzzy goal programming and traditional methods of economic and emission dispatch. Improved convergence of the solutions, as well as improved quality, is revealed by these results, thus confirming that the approach is realistic in practice.

The proposed intuitionistic fuzzy goal programming approach has been developed as an appropriate model to be used for solving the multi-objective economic emission load dispatch problem and for providing decision-makers with a reliable and efficient method for ensuring that the operation of their power systems meets sustainability and environmental requirements.

Keywords: Multi-Objective Economic Emission Load Dispatch, Intuitionistic Fuzzy Goal Programming, power generation, fuel cost, emissions, uncertainty modelling.

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1. INTRODUCTION

Power plants are extensively interconnected to efficiently and dependably transport power from generators to loads. These systems' primary goal is to schedule various generating unit types with the lowest possible cost in order to meet load needs while lowering overall operating costs [1-2]. In essence, the primary source of generation units is determined by the fuel used in centralized generating stations, such as fossil fuels that release greenhouse gases into the atmosphere, such as CO2, NOx, SOx, and others [3]. A power system operation cost reduction problem that seeks to achieve the lowest total fuel cost for generating units can be expressed using an Economic Load Dispatch (ELD).

Generally speaking, ELD's primary objective is to plan electricity generation to satisfy load demand while staying within operational bounds [4, 5]. By lowering the emission of units, it is possible to dispatch electric power at the lowest possible price with the least amount of pollution in this situation, which will encourage environmental protection and cleanliness. As a result, optimal power dispatch has become a crucial challenge for contemporary power systems because to the quick rise in electrical energy demand, growing environmental concerns, and strict emission restrictions. In the current situation, traditional power generation planning which was mainly concerned with reducing fuel costs is no longer adequate. Power utilities must now reduce the hazardous emissions from thermal producing units in addition to focusing on economic efficiency. These competing demands inevitably result in a multi-objective optimization problem, also known as the Multi-Objective Emission and Economic Load Dispatch (MOEELD) problem, in which fuel cost and pollutant emissions must be optimized concurrently while adhering to operational and system constraints.

In order to formulate and solve the MOEELD problem, numerous scholars have already investigated it and offered a variety of mathematical solutions. In [8–10], algorithms including the dynamics programming approach [7], the lambda iteration algorithm [6], and linear programming were employed.

In addition to the existence of equality and inequality constraints like power balancing, generator capacity limits, and transmission losses, the MOEELD problem is intrinsically complicated because of its nonlinear, non-convex, and competing objective functions. These issues are frequently unsatisfactorily solved by traditional deterministic optimization techniques, particularly when realistic system characteristics are taken into account. Advanced optimization techniques that can manage imprecision, uncertainty, and goal trade-offs have therefore become more and more crucial. In this regard, fuzzy set theory has become a potent instrument for simulating ambiguity and uncertainty in multi-criteria decision-making. However, traditional fuzzy techniques simply take into account the degree of membership, which might not be enough to accurately capture the hesitancy and uncertainty that come with making decisions in the real world. A more flexible and realistic depiction of decision-maker preferences is made possible by Intuitionistic Fuzzy Sets (IFS), which offer a more thorough framework by combining membership and non-membership degrees.

In a multi-objective setting, the Intuitionistic Fuzzy Goal (IFG) approach provides a useful mechanism for achieving the best compromise solution. When there are conflicting goals, the IFG framework allows for balanced decision-making by concurrently increasing the degree of satisfaction and decreasing the degree of discontent connected with each aim. When used to solve the MOEELD problem, this method reflects the real-world operational needs of power systems and enables the consideration of both economic and environmental goals within a single optimization framework.

Power generation system optimization has become a crucial and urgent job due to the world's rapidly increasing energy consumption and concurrent rise in environmental concerns [1]. The goal of EED has historically been to reduce the running expenses related to the production of electricity. The EED problem developed as a result of optimization's expansion to include emissions reduction as environmental consequences have been more generally acknowledged and subject to environmental pressure. A MOEELD problem is used to formulate the issue, where both the Emissions and economy goals must be kept to a minimum. Previous unadventurous techniques like linear programming, gradient approach, and Newton's method [2] were used for solving ELD problems. Several methods have been employed in recent years to fix EELD. Thenmozhi [3] used a hybrid genetic algorithm to solve EELD. Perez [4] used differential analysis to answer the environmental economic dispatch problem. For EELD, Hong [5] used an immune genetic method. Hazra [6] suggested a bacterium foraging algorithm for economic dispatch with emission constraints. Hemamalini [7] used particle swarm optimization to solve non-convex EELD. A BBO approach was introduced by Bhattacharya et al. [8] to address the EELD of thermal generators with various emission chemicals. Likewise, there are numerous alternative methods such as firefly algorithm (FFA) [9] and artificial neural networks [10], PSO [11] which have been applied to use to solve ELD. Nanda et.al [12] used goal programming strategies to address EELD. Through a comparison of optimal solutions evaluated considering transmission losses on a 3-Generators system, it has been demonstrated that IFG approach is a workable technique for determining the ideal value when compared to intuitionistic fuzzy techniques. Furthermore, an arithmetical example illustrating the efficiency of the proposed IFG approach is provided.

1.2. LITERATURE REVIEW

Power organization development is the most important factors when making management and planning decisions. Finding the optimal power output combinations for each producing unit that reduce emission and overall consumption of fuel while satisfying load request and power system operational parameters is the fundamental objective of the financial power dispatch problem. The concept of fuzzy sets was first proposed through Zadeh [13], the decision-making (DM) tool for addressing uncertainty. Fuzzy sets are employed to express objectives and restrictions, and it maximizes the ambiguous decision's membership degree. In 1970, Bellman and Zadeh [14] suggested the concepts of fuzzy decision, fuzzy objective, and fuzzy constraint. Then Zimmermann [15] employed the idea of fuzzy set theory to solve a linear programming issue with numerous objective functions using appropriate membership functions. Lai et al. [16], Rommelfanger [17][18], Luhandjula [19][20][21], Baykasogl et.al [22], Xu et.al [23], and Tzeng et.al [24] give a few summaries of fuzzy optimization techniques and their uses. In 1986, Atanassov [25] introduced Intuitionistic Fuzzy Set (IFS), It expands on the concept of conventional fuzzy sets by taking into account both the degree of each component's membership in the set and the degree of non-membership, respectively. An optimization problem in the intuitionistic fuzzy (IF) atmosphere can be defined as the process of finding a Pareto-optimum solution that simultaneously raises the degree of satisfaction and lowers the degree of discontent with the IF choice. Based on the Angelov [26] model, Jana and Roy [27] suggested a method for resolving MO issues. They applied their methodology to a capacitated transportation problem as well. In order to solve a multi-objective nonlinear reliability issue. Mahapatra et.al [28] proposed an IF technique using the idea of intuitionistic fuzzy sets. In this study Razzaq et al. [29] introduces fuzzy multi-objective optimization,

which is handled via FGP with weight assignment, which is complicated by preference relations, which are commonly stated linguistically. FGP with intuitionistic fuzzy preference relations. When different models are compared, it is found that the exponential model works best in one scenario, the hyperbolic model works best in another, and the linear model is the least successful. Feng Li et.al (2021)[30] proposes a unified intuitionistic fuzzy multi-objective linear goal programming model for portfolio selection, addressing decision maker's hesitation and reducing computational complexity without extra binary variables. Nguyen et.al [31] introduced in this paper a new strategy for optimizing portfolio selection by utilizing flexible optimization techniques and intuitionistic fuzzy goal theory. Iskander et.al [32] are presented in this study fuzzy goal programming models with exponential membership functions along with numerical examples for comparison. Using Yager's method, Dubey et. al [33] developed an optimization model that addresses the issues with Angelov's model [26] Dey et.al [34] employed IF optimization to offer an optimal solution for the design of two bar trusses. Gürkan IŞIK [35] discussed the invalidity of the projective relation conversion method discusses the invalidity of the projective relation conversion method which combines Pythagorean fuzzy sets with intuitionistic fuzzy sets.

1.3. Motivation for that Research

Intuitionistic fuzzy optimization interacts with the DM. These techniques and strategies make the assumption that the DM may determine all pertinent factors, such as the weights, the objective levels etc and the parameter of violation. In IF optimization, determining Truth-membership and Falsity-membership function of $f_p(r)$ parameters require an organized strategy that effectively implements the idea of infringement for the constraints and objective functions. To lower the expense and the amount of emissions, Yokoyama et.al [36] employed a limited approach. The research of MOEELD model was covered by Xuebin in [37].

In this regard Feng et.al [38] enhancement of the model for load dispatch using fuzzy MO. With the objective to design a power dispatch model that is both economical and environmentally friendly, Dey-Ray-Goswami-Roy [39] discussed about the study of economic load dispatch optimization models employing fuzzy interactive optimization techniques.

Dashtdar et.al [40] introduced by optimizing environmental economic dispatch for thermal power plants, the hybrid firefly-genetic algorithm lowers costs and emissions. It improves Pareto curve uniformity and uses local search techniques to handle nonlinear constraints. It outperforms five traditional algorithms when applied to the 39-bus IEEE system. Mishra et.al [41] in this paper presents a BWO-FLANN model for predicting future load conditions and an NS-MOTLBO for optimizing Economic Emission Load Dispatch with solar energy, addressing complex constraints effectively. Abdulla et.al [42] in this study uses hybrid GWO-PSO and MPSO to efficiently solve the ELD problem, resulting in lower fuel costs, emissions, and power losses while meeting a number of limitations in typical test settings.

The goal of this research is to create the best possible solution for the intuitionistic fuzzy goal arena's multi-objective emission and economic load dispatch model. The suggested method effectively handles trade-offs and constraint breaches by methodically building membership and non-membership functions for cost and emission objectives. When the methodology is used on a typical test system, the results show that the IFG approach can provide a dependable and effective compromise solution. The results demonstrate intuitionistic fuzzy goal-based optimization's promise as a reliable and useful technique for attaining power dispatch in contemporary power systems that is both environmentally friendly and economically efficient.

Research Gaps Identified in Recent Literature

- Limited Integration of Intuitionistic Fuzzy Optimization in EELD: While fuzzy goal programming and hesitant fuzzy methods are emerging, full intuitionistic fuzzy goal programming approaches—simultaneously handling membership, non-membership, and hesitation in EELD—remain underexplored.
- Insufficient Focus on Structured Multi-Objective Frameworks: Many works prefer evolutionary algorithms with fuzzy evaluation rather than structured mathematical programming integrations (goal programming, robust fuzzy formulations).
- Decision-Maker Preference Modelling: Explicit mechanisms for directly incorporating dynamic decision-maker preferences in EELD solutions are still limited in the existing literature.

1.3 Contribution

The main contributions of this paper are summarized as follows:

- **A Novel Intuitionistic Fuzzy Goal Programming Model**

This paper proposes a new intuitionistic fuzzy goal programming (IFGP) framework for solving the multi-objective emission and economic load dispatch (MOEELD) problem. By incorporating membership, non-membership, and hesitation degrees, the proposed model provides a more expressive and realistic representation of uncertainty than conventional fuzzy or deterministic approaches.

- **Nonlinear Multi-Objective Dispatch Formulation**

The proposed IFGP-based MOEELD formulation simultaneously minimizes fuel cost and pollutant emissions under nonlinear objective functions while satisfying practical system constraints, including power balance and transmission losses.

- **Decision-Maker Preference Integration**

An effective mechanism is developed to integrate decision-maker preferences through intuitionistic fuzzy membership functions and reference aspiration levels, enabling flexible and adaptive trade-off analysis between economic and environmental objectives.

- **Superior Trade-off Performance**

Simulation results on a standard three-generator test system demonstrate that the proposed approach achieves a better compromise between fuel cost reduction and emission minimization compared to conventional economic dispatch and classical fuzzy optimization techniques.

- **Computational Efficiency and Practical Applicability**

The structured goal programming framework ensures computational efficiency and ease of implementation, making the proposed method suitable for real-world power system operation and planning under environmental constraints.

2. MULTI-OBJECTIVE EMISSION AND ECONOMIC MODEL (MOEEM)

2.1 Economic Dispatch Model:

The economic dispatch problem can be represented by minimizing the fuel cost of generating units while satisfying a set of equality and inequality constraints.

$$Minimize\ Cost(P) = \sum_{i=1}^{m}\left(a_i P_i^2 + b_i P_i + c_i\right) \quad (1)$$

where a_i, b_i and c_i are the cost coefficients for i^{th} unit, P_i is the power generated by unit i, MW and the number of generating units is n.

2.1.1. Power Balance Constraints:

The total generated power should be equal to the total amount of the line loss and the load demand. It may be stated as

$$\sum_{i=1}^{m} P_i - (P_D + P_L) = 0 \quad (2)$$

where P_L is the total line loss and P_D is the total load demand.

Using the B-coefficient, the total system loss may be calculated as

$$P_L = \sum_{i=1}^{m}\sum_{j=1}^{m} P_i B_{ij} P_j + \sum_{i=1}^{m} B_{0i} P_i + B_{00} \quad (3)$$

where B_{ij}, B_{0i} and B_{00} are the transmission loss coefficients.

2.1.2. Inequality Constraints:

The actual power generated by each generator mjst be kept within its lower and maximum operating limitations.

$$P_i^{min} \le P_i \le P_i^{max} \quad \text{(i=1,2, 3…m)} \quad (4)$$

P_i is the generator's power output of i^{th} generator, where P_i^{max} and P_i^{min} is the generator's maximum and minimum generated power respectively.

2.2. Emission Dispatch Model:

In order to make load demand, emission dispatch seeks to lower emissions from all generating units that burn fuels to produce energy. This is illustrated as

$$Minimize\ Em(P)=\sum_{i=1}^{m}\left(\rho_i P_i^2+\sigma_i P_i+\varsigma_i\right) \quad (5)$$

where, by ρ_i,σ_i and ς_i are the emission coefficients for i^{th} unit, P_i is the power generated by unit i, MW and the number of generating units is n.

2.3. Multi-Objective Emission and Economic Load Dispatch Model:

The MOEELD model with several objectives can be stated as follows:

$$Min\ \ Cost(P)=\sum_{i=1}^{m}\left(a_i P_i^2+b_i P_i+c_i\right) \quad (6)$$

$$Min\ Em(P)=\sum_{i=1}^{m}\left(\rho_i P_i^2+\sigma_i P_i+\varsigma_i\right)$$

Subject to $\sum_{i=1}^{m}P_i-(P_D+P_L)=0;$

$$P_i^{\min}\le P_i\le P_i^{\max}.$$

where, $P=\left[P_1,P_2,.....P_m\right]^T$ are power generated, the no. of power plants is n, $Cost(P)$ is the power generation cost, $Em(P)$ is the emission during power generation, P_i,P_L and P_D is the power generated, transmission losses and power demand respectively. $P_i^{\min}$ is the lower limit, and $P_i^{\max}$ is the upper limit of generator output.

3. PREREQUISITE MATHEMATICS

3.1. Fuzzy Set [13]: The fuzzy set $\tilde{A}$ in X (which is non-empty) is a set of order pairs $\tilde{A}=\left\{\left(r,\eta_{\tilde{A}}(r)\right):r\in X\right\}$ where $\eta_{\tilde{A}}(r)\in[0,1]$ is the degree of membership to each element $r\in X$ in $\tilde{A}$ and $\eta_{\tilde{A}}(r):X\to[0,1]$ is membership function.

3.2. Intuitionistic Fuzzy Set [25]:

Let a set $X=\{r_1,r_2,...,r_n\}$ be fixed. An intuitionistic fuzzy set, or IFS $\tilde{A}^I$ in X, is an object of the form $\tilde{A}^I=\left\{\left\langle r,\eta_{\tilde{A}^I}(r_i),\kappa_{\tilde{A}^I}(r_i)\right\rangle/r_i\in X\right\}$ where, $\eta_{\tilde{A}^I}(r_i):X\to[0,1]$ and $\kappa_{\tilde{A}^I}(r_i):X\to[0,1]$

Describe the membership and the non-membership, respectively, for every element, $r_i\in X$ $0\le\eta_{\tilde{A}^I}(r_i)+\kappa_{\tilde{A}^I}(r_i)\le 1$.

3.3. (α,β)-level intervals or (α,β)-cuts[43]:

A set of (α,β)-cut generated by IFS set $\tilde{A}^i$ where $(\alpha,\beta)\in[0,1]$ are fixed numbers such that $\alpha+\beta\le 1$ is defined as

$$\tilde{A}^i_{\alpha,\beta}=\begin{Bmatrix}\left\langle r,\eta_{\tilde{A}^i}(r),\kappa_{\tilde{A}^i}(r)\right\rangle/r\in X,\\ \eta_{\tilde{A}^i}(r)\ge\alpha,\kappa_{\tilde{A}^i}(r)\le\beta,(\alpha,\beta)\in[0,1]\end{Bmatrix}$$

where, $\eta_{\tilde{A}^i}(r):X\to[0,1]$ and $\kappa_{\tilde{A}^i}(r):X\to[0,1]$

4. MATHEMATICAL ANALYSIS

4.1 Fuzzy Goal Programming Problem (FGP):

Goal programming very likely expressed as

Find $r=\{r_1,r_2,...,r_n\}^T$ (7)

In order to, Minimize $f_p(r)$ with target value l_p, acceptance tolerance a_p

Subject to $g_j(r)\le b_j,\ \ j=1,2,...,m$, $p=1,2,...n$, $r>0$.

With membership function it can be written as

$$\eta_{f_p(r)}(f_p(r))=\begin{cases}1 & if\ \ f_p(r)\le l_p\\ \dfrac{l_p+a_p-f_p(r)}{a_p} & if\ l_p\le f_p(r)\le l_p+a_p\\ 0 & if\ \ f_p(r)\ge l_p+a_p\end{cases} \quad (8)$$

So, the Crisp programming from of fuzzy goal programming, written as

Maximize $\eta_{f_p(r)}(f_p(r))$ (9)

Subject to $0\le\eta_{f_p(r)}(f_p(r))\le 1$

$g_j(r)\le b_j,\quad j=1,2,...,m,\ \ p=1,2,...n,\ r>0.$

4.2. Intuitionistic Fuzzy Goal Programming Problem (IFGP):

Intuitionistic fuzzy goal programming is a fuzzy Gaol programming extension, problem in which the degree of falsity of objective(s) and constraints are consider. The challenge of nonlinear goal programming can be expressed as

Find $r=\{r_1,r_2,...,r_n\}^T$

In order to,

Minimize $f_p(r)$ (10)

with target value l_p, acceptance tolerance a_p and rejection tolerance t_p

$r\in X\quad g_j(r)\le b_j,\ \ j=1,2,...,m,\ \ p=1,2,...n,\ \ r>0.$

With membership and non-membership function it can be written as

$$\eta_{f_p(r)}(f_p(r)) = \begin{cases} 1 & if\ \ f_p(r_p) \le l_p \\ \dfrac{l_p + a_p - f_p(r)}{a_p} & if\ l_p \le f_p(r_p) \le l_p + a_p \\ 0 & if\ \ f_p(r_p) \ge l_p + a_p \end{cases} \quad (11)$$

$$\kappa_{f_p(r)}(f_p(r)) = \begin{cases} 0 & if\ \ f_p(r_p) \le l_p \\ \dfrac{f_p(r) - l_p}{t_p} & if\ l_p \le f_p(r_p) \le l_p + t_p \\ 1 & if\ \ f_p(r_p) \le l_p + t_p \end{cases}$$

Hence, IFGP can be written into the Crisp programming from Intuitionistic fuzzy goal programming using truth-membership and falsity-membership is

Maximize $\eta_{f_p(r)}(f_p(r))$ (12)

Minimize $\kappa_{f_p(r)}(f_p(r))$

Subject to

$0 \le \eta_{f_p(r)}(f_p(r)) + \kappa_{f_p(r)}(f_p(r)) \le 1,$

$\eta_{f_p(r)}(f_p(r)) \ge \kappa_{f_p(r)}(f_p(r)),$

$\kappa_{f_p(r)}(f_p(r)) \ge 0,$

$g_j(r) \le b_j, \quad j = 1,2,...,m, p = 1,2,...n, r > 0.$

Definition 4.3 (M-N Pareto Optimum Solution): r^* is regarded as a M-N Pareto Optimum solution iff There's not another $r \in X$ s.t $\eta_{f_p} f_p(r) \ge \eta_{f_p} f_p(r^*)$ and $\kappa_{f_p} f_p(r) \le \kappa_{f_p} f_p(r^*)$ $\forall p$ and for minimum one p, exact inequality holds.

Theorem 1: $r^* \in X$ is M-N Pareto Optimum solution of (11) iff r^* is M-N Pareto Optimum solution of

$Min\, f_p(r),\ p = 1,2,...n.$ (13)

Subject to

$g_j(r) \le b_j,\ j = 1,2,...,m;\ r > 0.$

Proof.: Let $r^* \in X$ is M-N Pareto Optimum solution of (10), then there isn't any $r \in X$ s.t $\eta_{f_p}(r) \ge \eta_{f_p}(r^*)$ and $\kappa_{f_p}(r) \le \kappa_{f_p}(r^*)$ $\forall p$ and for minimum one p, exact inequality holds. Therefore, based on the membership function expression, we have

$$1 - \frac{f_p(r) - l_p}{a_p} \ge 1 - \frac{f_p(r^*) - l_p}{a_p}$$

$\Rightarrow f_p(r) \le f_i(r^*)$,

and for non-membership function,

$$\frac{f_p(r) - l_p}{t_p} \le \frac{f_p(r^*) - l_p}{t_p}$$

$\Rightarrow f_p(r) \le f_p(r^*)$ where severe inequality holds for a minimum of one p.

The Pareto optimum solution of (13) is therefore r^*.

Conversely, suppose that r^* is a Pareto optimum solution to (13). then, $r \in X$ does not occur in such a way that $f_p(r_1, r_2, ..., r_n) \le f_p(r_1, r_2, ..., r_n)^*$ for at least one i, when accurate inequality holds.

So, $f_p(r) - l_p \le f_p(r^*) - l_p$ i.e.

$$\Rightarrow \frac{f_p(r) - l_p}{t_p} \le \frac{f_p(r^*) - l_p}{t_p}$$

Says that $\kappa_{f_p}(r) \le \kappa_{f_p}(r^*)$ and $1 - \dfrac{f_p(r) - l_p}{a_p} \ge 1 - \dfrac{f_p(r^*) - l_p}{a_p}$.

Says that, $\eta_{f_p}(r) \ge \eta_{f_p}(r^*)$, for at least one p, when accurate inequality holds.

Hence, r^* is M-N optimum solution of (10)

4.4 Generalized Intuitionistic Fuzzy Set (GIFS):

Let X be a non-empty set. A GIFS $\tilde{A}^{gi}$ on X is stated as

$$\tilde{A}^{gi} = \left\{\left\{r, \eta_{\tilde{A}^{gi}}(r), \kappa_{\tilde{A}^{gi}}(r)\right\} : r \in X\right\}$$

where $\eta_{\tilde{A}^{gi}}(r) : X \to [0, w_1]$ and $\kappa_{\tilde{A}^{gi}}(r) : X \to [0, w_2]$ if it satisfies the condition

$$0 \le \eta_{\tilde{A}^{gi}}(r) + \kappa_{\tilde{A}^{gi}}(r) \le w_1 + w_2 \ \ (0 \le w_1 + w_2 \le 1)\ \forall r \in X$$

where $w_1\ (0 \le w_1 \le 1)$ and $w_2\ (0 \le w_2 \le 1)$ are the gradations of membership function and non-membership function.

Theorem 2: For a Generalized intuitionistic fuzzy goal programming model.

Min $f_p(r)$ with target value l_p; acceptance tolerance a_p and rejection tolerance t_p

Subject to $g_j(r) \le b_{j,} \quad j = 1,2,...,m$;

$r = (r_1, r_2 ..., r_n)^T, r > 0, p = 1,2,...n$.

Where sum membership function and non-membership function will exit between 0 and $w_1 + w_2$

Proof:

Membership function $\eta^{w_1}\left(f_p(r)\right)$ and non-membership function $\kappa^{w_2}\left(f_p(r)\right)$ sated as follows

$$\eta^{w_1}\left(f_p(r)\right)=\begin{cases} w_1 & f_p(r)\le l_p \\ w_1\left(1-\dfrac{f_p(r)-l_p}{a_p}\right) & l_p\le f_p(r)\le l_p+a_p \\ 0 & f_p(r)\ge l_p+a_p \end{cases}$$

And

$$\kappa^{w_2}\left(f_p(r)\right)=\begin{cases} 0, & f_p(r)\le l_p \\ w_2\left(\dfrac{f_p(r)-l_p}{t_p}\right) & l_p\le f_p(r)\le l_p+t_p \\ w_2, & f_p(r)\ge l_p+t_p \end{cases}$$

Form the definition stated above, we found that $0\le\eta^{w_1}\left(f_p(r)\right)\le w_1$ and $0\le\kappa^{w_2}\left(f_p(r)\right)\le w_2$.

For $f_p(r)\le l_p$, $\eta^{w_1}\left(f_p(r)\right)=w_1$ and $\kappa^{w_2}\left(f_p(r)\right)=0$.

Therefore, $\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)=w_1\le w_1+w_2$, since $w_2\ge 0$.

And $w_1\ge 0$ implies that $\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)\ge 0$

From fig (3), it is found that $\eta^{w_1}\left(f_p(r)\right)$ and $\kappa^{w_2}\left(f_p(r)\right)$ intersecting within the interval $\left[l_p,l_p+a_p\right]$. Q is the intersection point along horizonal axis whose co-ordinate are $\left(l_p+\dfrac{w_1}{\dfrac{w_1}{a_p}+\dfrac{w_2}{t_p}},0\right)$.

For $f_p(r)\in\left(l_p,l_p+a_p\right]$,

$$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)$$
$$=w_1\left(1-\frac{f_p(r)-l_p}{a_p}\right)+w_2\left(1-\frac{f_p(r)-l_p}{t_p}\right)$$

where,

$$f_p(r)\le l_p+\frac{w_1}{\dfrac{w_1}{a_p}+\dfrac{w_2}{t_p}},$$

$$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)$$
$$\le\frac{\left(w_1+w_2\right)^2}{w_1+w_2}-\frac{w_1^2+w_2^2}{w_1+w_2}\le w_1+w_2,$$

When $f_p(r)\le l_p+a_p$,

$$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)\le w_1\frac{a_p}{t_p}<w_1$$
$$\le w_1+w_2\quad\left[as\ \frac{a_p}{t_p}<1\right]$$

Again, when $f_p(r)\ge l_p$,
$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)\ge w_1\ge 0$

For $l_p+a_p\le f_p(r)\le l_p+t_p$,
$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)\le w_2\le w_1+w_2$ and
$f_p(r)>l_p+a_p$,

$$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)>w_2\frac{a_p}{t_p}>w_2\ge 0$$

[since $w_2\ge 0$ *and* $\dfrac{a_p}{t_p}<1$]

For, $f_p(r)\ge l_p+t_p$, $\eta^{w_1}\left(f_p(r)\right)=0$ and $\kappa^{w_2}\left(f_p(r)\right)=w_2$.

Hence, $\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)=w_1\le w_1+w_2$.

Also, since $w_2\ge 0$, therefore within region $f_p(r)$,
$\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)=w_2\ge 0$

Hence, in the case for all $0\le\eta^{w_1}\left(f_p(r)\right)+\kappa^{w_2}\left(f_p(r)\right)\le w_1+w_2$.

5.1. Solution Procedure of Fuzzy Goal and Intuitionistic Goal Programming Technique

Equation (9) can be written as

Maximize α

Subject to $f_p(r)-(1-\alpha)a_p\le l_p$ (14)

$g_j(r)\le b_j,\ j=1,2,...,m,\ p=1,2,...,n\ ,r>0$.

Equation (12) can be written as

Maximize α, Minimize β

(15)

Subject to

$\eta_{f_p(r)}(f_p(r))\ge\alpha,\ \kappa_{f_p(r)}(f_p(r))\ge\beta,$

$0\le\alpha+\beta\le 1,\ \alpha\ge\beta,\ \alpha\in[0,1],\ \beta\in[0,1],$

$g_j(r)\le b_j,\ \ j=1,2,\ldots,m,\ p=1,2,\ldots,n,\ \ r>0$

Taking arithmetic mean operator of above problem can be written as

Minimize $\left\{\dfrac{\beta+(1-\alpha)}{2}\right\}$, (16)

Subject to

$\eta_{f_p(r)}(f_p(r)) \ge \alpha$, $\kappa_{f_p(r)}(f_p(r)) \ge \beta$,

$0 \le \alpha + \beta \le 1$, $\alpha \ge \beta$, $\alpha \in [0,1]$, $\beta \in [0,1]$,

$g_j(r) \le b_j$, $j = 1,2,\ldots,m$, $p = 1,2,\ldots,n$, $r > 0$

Taking geometric mean operator of equation (15) can be written as

Minimize $\sqrt{\beta(1-\alpha)}$ (17)

Subject to

$\eta_{f_p(r)}(f_p(r)) \ge \alpha$, $\kappa_{f_p(r)}(f_p(r)) \ge \beta$,

$0 \le \alpha + \beta \le 1$, $\alpha \ge \beta$, $\alpha \in [0,1]$, $\beta \in [0,1]$,

$g_j(r) \le b_j$, $j = 1,2,\ldots,m$, $p = 1,2,\ldots,n$, $r > 0$.

Through solving the nonlinear problems (15 or 16 or 17) by applying the appropriate technique for mathematical programming to acquire the best results of multi-objective nonlinear problem (12) by generalized goal optimization technique.

5.2. Solutions ranking

In this section, the method used to make a performance order through similarity to an ideal solution TOPSIS [36] is applied to rank all compromise solutions to get possible compromise solutions with different techniques. The method is developed on minimising the distance from the ideal solution and maximising the distance from the anti-ideal solution respectively.

Step-1: Keep track of the compromise solutions P_{ij} from methods over T objectives, $i = 1,2,...,S$, $j = 1,2,...,T$

Step-2: Look at the normalized rating R_{ij} for element P_{ij}

Step-3: To get the weighted matrix $V_{ij} = \omega_{ij} R_{ij}$ ask the DM for the weight ω_{ij} that corresponds to each goal.

Sep-4: Find the best possible solution (best results for each goal) S^+

$$S^+ = \{v_1^+, v_2^+, \ldots v_j^+, \ldots v_k^+\}$$
$$= \{(\max V_{ij} \mid j \in J_1).(\min V_{ij} \mid j \in J_2), i = 1,2,\ldots n\}$$

where J_1 =benefit values and J_2 =cost attributes

Step-5: Evaluate the anti-ideal solution (inferior performance on individually objective) S^-

$$S^- = \{v_1^-, v_2^-, \ldots v_j^-, \ldots v_k^-\}$$
$$= \{(\min V_{ij} \mid j \in J_1).(\max V_{ij} \mid j \in J_2), i = 1,2,\ldots n\}$$

Step-6: Find out the distance to ideal (D^+) and anti-ideal (D^-) respectively, $D_i^+ = \sqrt{\sum_{j=1}^{k}\left(V_{ij} - v_j^+\right)^2}$

and $D_i^- = \sqrt{\sum_{j=1}^{k}\left(V_{ij} - v_j^-\right)^2}$

Step-7: Measure the degree of proximity to the optimal solutions and the anti-ideal solution that is the closest to it using the ratio and explore the compromise solution that has the highest ratio possible $R = \dfrac{D_i^-}{D_i^- + D_i^+}$

6. FUZZY GOAL FORMULATION FOR MOEELD PROBLEM

To solve MOEELDM (15) using FGP, here TM functions $\eta_{CT(P)}(CT(P_i))$ and $\eta_{EM(P)}(EM(P_i)$ for the objective functions $CT(P_i)$ and $EM(P_i)$ are defined as follows with target value l_{CT}, l_{EM} acceptance tolerance a_{CT}, a_{EM} respectively.

$$\eta_{CT(P)}(CT\ (P_i)) = \begin{cases} 1 & if\ \ CT(P_i) \le l_{CT} \\ \dfrac{l_{CT} + a_{CT} - CT(P_i)}{a_{CT}} & if\ l_{CT} \le CT(P_i) \le l_{CT} + a_{CT} \\ 0 & if\ \ CT(P_i) \ge l_{CT} + a_{CT} \end{cases} \quad (18)$$

$$\eta_{EM(P)}(EM(P_i)) = \begin{cases} 1 & if\ \ EM(P_i) \le l_{EM} \\ \dfrac{l_{EM} + a_{EM} - EM(P_i)}{a_{EM}} & if\ l_{EM} \le EM(P_i) \le l_{EM} + a_{EM} \\ 0 & if\ \ EM(P_i) \ge l_{EM} + a_{EM} \end{cases} \quad (19)$$

According to FGP above membership functions (13) with target value l_{CT}, l_{EM} and acceptance tolerance a_{CT} , a_{EM} , formulated in non-linear crisp programming problem as follows

$Maximize\ \ \alpha$, (20)

Subject to

$$\left(\frac{l_{CT} + a_{CT} - CT(P_i)}{a_{CT}}\right) \ge \alpha, \left(\frac{l_{EM} + a_{EM} - EM(P_i)}{a_{EM}}\right) \ge \alpha,$$

$\sum_{i=1}^{3} P_i - (P_D + P_L) = 0$, Where

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00}$$

$P_i^{\min} \le P_i \le P_i^{\max}$, $i = 1,2,3$; $0 \le \alpha \le 1$.

Equation (20) formulated as

$Maximize\ \ \alpha$ (21)

Subject to

$CT(P_i)-(1-\alpha)a_{CT}\le l_{CT}$,

$EM(P_i)-(1-\alpha)a_{EM}\le l_{EM}$,

$\sum_{i=1}^{3}P_i-(P_D+P_L)=0$, Where

$P_L=\sum_{i=1}^{n}\sum_{j=1}^{n}P_iB_{ij}P_j+\sum_{i=1}^{n}B_{0i}P_i+B_{00}$

$P_i^{\min}\le P_i\le P_i^{\max},\ i=1,2,3;\ 0\le\alpha\le 1.$

7.INTUITIONISTIC FUZZY GOAL FORMULATION FOR MOEELD PROBLEM

To solve MOEELDM (11) using IFGP, here using IFGP, here membership functions $\eta_{CT(P)}(CT(P_i))$ and $\eta_{EM(P)}(EM(P_i))$, and non-membership functions $\kappa_{CT(P)}(CT(P_i))$; and $\kappa_{EM(P)}(EM(P_i))$ for the objective function for the objective functions $CT(P_i)$ and $EM(P_i)$ are defined as follows with target value l_{CT}, l_{EM} and acceptance tolerance a_{CT}, a_{EM} and rejection tolerance r_{CT}, r_{EM} respectively.

Linear membership and non-membership function of $CT(P_i)$ and $EM(P_i)$ can be written as

$$\eta_{CT(P)}(CT(P_i))=\begin{cases}1 & if\ CT(P_i)\le l_{CT}\\ \dfrac{l_{CT}+a_{CT}-CT(P_i)}{a_{CT}} & if\ l_{CT}\le CT(P_i)\le l_{CT}+a_{CT}\\ 0 & if\ CT(P_i)\ge l_{CT}+a_{CT}\end{cases}$$

$$\kappa_{CT(P)}(CT(P_i))=\begin{cases}0 & if\ CT(P_i)\le l_{CT}\\ \dfrac{CT(P_i)-l_{CT}}{t_{CT}} & if\ l_{CT}\le CT(P_i)\le l_{CT}+t_{CT}\\ 1 & if\ CT(P_i)\le l_{CT}+t_{CT}\end{cases}\quad (22)$$

$$\eta_{EM(P)}(EM(P_i))=\begin{cases}1 & if\ EM(P_i)\le l_{EM}\\ \dfrac{l_{EM}+a_{EM}-EM(P_i)}{a_{EM}} & if\ l_{EM}\le EM(P_i)\le l_{EM}+a_{EM}\\ 0 & if\ EM(P_i)\ge l_{EM}+a_{EM}\end{cases}$$

$$\kappa_{EM(P)}(EM(P_i))=\begin{cases}0 & if\ EM(P_i)\le l_{EM}\\ \dfrac{EM(P_i)-l_{EM}}{t_{EM}} & if\ l_{EM}\le EM(P_i)\le l_{EM}+t_{EM}\\ 1 & if\ EM(P_i)\le l_{EM}+t_{EM}\end{cases}\quad (23)$$

Model-I

$$Minimize\ \alpha,\ Minimize\ \beta \quad (24)$$

Subject to

$\left(\dfrac{l_{CT}+a_{CT}-CT(P_i)}{a_{CT}}\right)\ge\alpha,\ \dfrac{CT(P_i)-l_{CT}}{r_{CT}}\le\beta,$

$\left(\dfrac{l_{EM}+a_{EM}-EM(P_i)}{a_{EM}}\right)\ge\alpha,\ \dfrac{EM(P_i)-l_{EM}}{r_{EM}}\le\beta,$

Subject to

$0\le\eta_{CT(P)}(CT(P_i))+\kappa_{CT(P)}(CT(P_i))\le 1$;

$0\le\eta_{EM(P)}(EM(P_i))+\kappa_{EM(P)}(EM(P_i))\le 1$;

$\eta_{CT(P)}(CT(P_i))\ge\kappa_{CT(P)}(CT(P_i))$;

$\eta_{EM(P)}(EM(P_i))\ge\kappa_{EM(P)}(EM(P_i))$;

$\kappa_{CT(P)}(CT(P_i))\ge 0$;

$\kappa_{EM(P)}(EM(P_i))\ge 0$;

$\sum_{i=1}^{3}P_i-(P_D+P_L)=0$,

where $P_L=\sum_{i=1}^{n}\sum_{j=1}^{n}P_iB_{ij}P_j+\sum_{i=1}^{n}B_{0i}P_i+B_{00}$

$P_i^{\min}\le P_i\le P_i^{\max},\ i=1,2,3$;

$0\le\alpha+\beta\le 1,\ \alpha\in[0,1],\beta\in[0,1]$.

MODEL-II:

Taking arithmetic operator, above problem can be expressed as

$$Minimize\ \left\{\frac{(1-\alpha)+\beta}{2}\right\} \quad (25)$$

Subject to

$0\le\eta_{CT(P)}(CT(P_i))+\kappa_{CT(P)}(CT(P_i))\le 1$;

$0\le\eta_{EM(P)}(EM(P_i))+\kappa_{EM(P)}(EM(P_i))\le 1$;

$\eta_{CT(P)}(CT(P_i))\ge\kappa_{CT(P)}(CT(P_i))$;

$\eta_{EM(P)}(EM(P_i))\ge\kappa_{EM(P)}(EM(P_i))$;

$\kappa_{CT(P)}(CT(P_i))\ge 0$;

$\kappa_{EM(P)}(EM(P_i))\ge 0$;

$\sum_{i=1}^{3}P_i-(P_D+P_L)=0$,

where $P_L=\sum_{i=1}^{n}\sum_{j=1}^{n}P_iB_{ij}P_j+\sum_{i=1}^{n}B_{0i}P_i+B_{00}$

$P_i^{\min}\le P_i\le P_i^{\max},\ i=1,2,3$;

$0\le\alpha+\beta\le 1,\ \alpha\in[0,1],\beta\in[0,1]$.

MODEL-III:

Taking geometric operator, problem can be formulated as

Minimize $\sqrt{\beta(1-\alpha)}$ (26)

Subject to

$0 \le \eta_{CT(P)}(CT(P_i)) + \kappa_{CT(P)}(CT(P_i)) \le 1$;

$0 \le \eta_{EM(P)}(EM(P_i)) + \kappa_{EM(P)}(EM(P_i)) \le 1$;

$\eta_{CT(P)}(CT(P_i)) \ge \kappa_{CT(P)}(CT(P_i))$;

$\eta_{EM(P)}(EM(P_i)) \ge \kappa_{EM(P)}(EM(P_i))$;

$\kappa_{CT(P)}(CT(P_i)) \ge 0$;

$\kappa_{EM(P)}(EM(P_i)) \ge 0$;

$\sum_{i=1}^{3} P_i - (P_D + P_L) = 0$,

where $P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} B_{0i} P_i + B_{00}$

$P_i^{\min} \le P_i \le P_i^{\max}, i = 1,2,3$;

$0 \le \alpha + \beta \le 1,\ \alpha \in [0,1], \beta \in [0,1]$.

8. NUMERICAL SOLUTION OF A MULTI-OBJECTIVE ECONOMIC EMISSION LOAD DISPATCH MODEL OF A SYSTEM OF 3-GENERATORS

The Data sheet for MOEELD[39] problem is given in table 1:

Demand $P_d = 700\text{MW}$

$$B_{ij} = \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix}$$

where $B_{0i} = B_{00} = 0$ considered.

Here, A system of 3-genetarors power plant is consider. The objectives are to minimize (i) the fuel cost $\text{Cost}(P)$ and (ii) the emission output $\text{Em}(P)$ at the same time while satisfying the generation limit constraints and power balance. In this study, the design variables are the power output of the generator.

The Fuzzy Goal multi-objective economic emission load despatch optimization problem (16) with target value 35425 Rs/kg, acceptance tolerance 20 Rs/kg and with target value 651kg/hr and acceptance tolerance 3kg/hr can be stated as follows:

Maximize α (27)

Subject to

$CT(P_i) - (1-\alpha)20 \le 35425$;

$EM(P_i) - (1-\alpha)3 \le 651$;

$P_1 + P_2 + P_3$

$$-\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_2 \end{bmatrix} = 700$$

,

$35 \le P_1 \le 210;\ 130 \le P_2 \le 325;\ 125 \le P_3 \le 315;$

$0 \le \alpha \le 1.$

The IFG multi-objective economic emission load despatch optimization problem (25) to Maximize $Cost(P_i)$ \`with target value 35425 Rs/kg and acceptance tolerance 20 Rs/kg and rejection tolerance 70 Rs/kg and to Minimize $Em(P_i)$ with target value 651 kg/hr and acceptance tolerance 3 kg/hr and rejection tolerance 8 kg/hr can be stated as follows:

Model-I

Maximize α , *Minimize* β; (28)

Subject to

$\left(\frac{35425 + 20 - CT(P_i)}{20}\right) \ge \alpha, \frac{CT\ (P_i) - 35425}{70} \le \beta;$

$\left(\frac{651 + a_{EM} - EM(P_i)}{3}\right) \ge \alpha, \frac{EM\ (P_i) - 651}{8} \le \beta;$

Subject to

$0 \le \mu_{CT(P)}(CT(P_i)) + \upsilon_{CT(P)}(CT(P_i)) \le 1$;

$0 \le \mu_{EM(P)}(EM(P_i)) + \upsilon_{EM(P)}(EM(P_i)) \le 1$;

$\mu_{CT(P)}(CT(P_i)) \ge \upsilon_{CT(P)}(CT(P_i))$;

$\mu_{EM(P)}(EM(P_i)) \ge \upsilon_{EM(P)}(EM(P_i))$;

$\upsilon_{CT(P)}(CT(P_i)) \ge 0$; $\upsilon_{EM(P)}(EM(P_i)) \ge 0$;

$P_1 + P_2 + P_3$

$$-\begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_2 \end{bmatrix} = 700$$

, $35 \le P_1 \le 210;\ 130 \le P_2 \le 325;\ 125 \le P_3 \le 315,$

$0 \le \alpha + \beta \le 1,\ \alpha \in [0,1], \beta \in [0,1]$.

The IFG programming problem (24) based on arithmetic aggregation operator with membership and non-membership can be formulated as

MODEL-II.

Minimize $\left\{\frac{(1-\alpha)+\beta}{2}\right\}$ (29)

Subject to

$0 \le \mu_{CT(P)}(CT(P_i)) + \upsilon_{CT(P)}(CT(P_i)) \le 1$;

$0 \le \mu_{EM(P)}(EM(P_i)) + \upsilon_{EM(P)}(EM(P_i)) \le 1$;

$\mu_{CT(P)}(CT(P_i)) \ge \upsilon_{CT(P)}(CT(P_i))$;

$\mu_{EM(P)}(EM(P_i)) \ge \upsilon_{EM(P)}(EM(P_i))$;

$\upsilon_{CT(P)}(CT(P_i)) \ge 0$; $\upsilon_{EM(P)}(EM(P_i)) \ge 0$;

$$P_1 + P_2 + P_3 - \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_2 \end{bmatrix} = 700$$

, $35 \le P_1 \le 210;\ 130 \le P_2 \le 325;\ 125 \le P_3 \le 315,$

$0 \le \alpha + \beta \le 1,\ \alpha \in [0,1], \beta \in [0,1]$.

The generalized Intuitionistic goal programming problem (19) based on arithmetic geometric operator with membership and non-membership can be formulated as

MODEL-III:

Minimize $\sqrt{\beta(1-\alpha)}$; (30)

Subject to

$0 \le \mu_{CT(P)}(CT(P_i)) + \upsilon_{CT(P)}(CT(P_i)) \le 1$;

$0 \le \mu_{EM(P)}(EM(P_i)) + \upsilon_{EM(P)}(EM(P_i)) \le 1$;

$\mu_{CT(P)}(CT(P_i)) \ge \upsilon_{CT(P)}(CT(P_i))$;

$\mu_{EM(P)}(EM(P_i)) \ge \upsilon_{EM(P)}(EM(P_i))$;

$\upsilon_{CT(P)}(CT(P_i)) \ge 0$; $\upsilon_{EM(P)}(EM(P_i)) \ge 0$;

$$P_1 + P_2 + P_3 - \begin{bmatrix} P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0.000071 & 0.000030 & 0.000025 \\ 0.000030 & 0.000069 & 0.000032 \\ 0.000025 & 0.000032 & 0.000080 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_2 \end{bmatrix} = 700$$

,

$35 \le P_1 \le 210;\ 130 \le P_2 \le 325;\ 125 \le P_3 \le 315,$

$0 \le \alpha + \beta \le 1,\ \alpha \in [0,1], \beta \in [0,1]$.

Solution: According to Fuzzy Goal Approach and Intuitionistic Fuzzy Goal Approach, we get optimal solution of 3- generators MOEELDP.

9. OPTIMAL SOLUTION AND RESULT COMPARISON

The Pareto optimum solution of MOELDM model (12) using Fuzzy and Intuitionistic fuzzy, fuzzy goal multi-objective nonlinear programming technique is given in table 2.

10. RESULT AND CONVERSATION

The performance of the proposed intuitionistic fuzzy goal (IFG) approach is evaluated using a numerical test system. The solution procedure begins with the determination of the ideal and anti-ideal solutions, based on which the corresponding membership and non-membership functions are constructed. The primary objective of the IFG-based MOEEPLD framework is to obtain an optimal compromise solution between conflicting objectives, namely fuel cost minimization and emission reduction.

The results obtained using the proposed approach are compared with those of existing methods, as presented in Tables 3, 4, and 5. The comparative analysis demonstrates that the IFG method consistently achieves improved performance across different operating conditions. In particular, the results in Tables 3 and 5 indicate that the proposed approach yields superior solutions in terms of the compromise ratio □, which is evaluated using the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). Since improvements in one objective inevitably lead to trade-offs in other conflicting objectives, a reliable ranking and evaluation mechanism is essential. The TOPSIS method is employed to rank the obtained solutions due to its effectiveness and practical relevance in multi-criteria decision-making problems. The comparative results confirm that the proposed fuzzy goal (FG) and intuitionistic fuzzy goal (IFG) approaches outperform existing techniques by providing a more balanced and efficient trade-off between economic and environmental objectives.

Overall, the results validate the effectiveness of the proposed methodology and highlight its suitability for solving multi-objective economic–emission power load dispatch problems.

11. CONCLUSION

This paper proposes a novel intuitionistic fuzzy goal programming (IFGP) framework for solving multi-objective economic–emission power load dispatch problems. By integrating decision-maker preferences with intuitionistic fuzzy sets, the proposed approach effectively addresses uncertainty, hesitation, and conflicting objectives related to fuel cost and emission reduction. The method provides a flexible and robust optimization framework capable of generating balanced and practically implementable solutions under system constraints.

The use of intuitionistic fuzzy sets enhances the model's ability to handle imprecision in complex decision-making environments, leading to improved trade-off performance compared to conventional approaches. Additionally, the framework allows the adoption of nonlinear membership functions, such as exponential or hyperbolic forms, to better represent preference structures. Future research may incorporate advanced metaheuristic algorithms to further enhance solution quality and scalability for large-scale power systems.

Authers Contributions
The work described in this article is a collaborate effort from all of the authors.

Conceptualisation: SD; Methodology: SD: Design, implementation and generation of result: PKS & SD; Analysis and Interpretation of result: PKS & SD; Preparing the Draft, Review and Editing: PKS & SD; Visualization: SD.

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Ethical Approval
None of this publication's authors conducted any research on humans or animals.

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Declaration on the use of AI
The authors declares that they did not use any AI methods in the drafting of this work.

Statement about Data Availability
This article includes the information that supported the study's claims.

Conflict of Interest
No conflicts of interest are disclosed by the writers.

Table 1 [39]**:** Data set of the six-unit system.

Unit	a (Rs/MW^{2}hr)	b (Rs/MW hr)	c (Rs/hr)	ρ (kg/MW^{2}hr)	σ (kg/MW hr)	ς (kg/hr)	P_{max} (MW)	P_{min} (MW)
P_1	0.036546	38.30553	1243.53110	0.00683	-0.54551	40.26690	35	210
P_2	0.0211	36.32782	1658.56960	0.00461	-0.51160	42.89553	130	325
P_3	0.01799	38.27041	1356.65920	0.00461	-0.51160	42.89553	125	315

Table 2: Comparison of Optimal solution of MOELDM (13) based on different methods

Methods	$Cost(P)$	$Em(P)$	P_1	P_2	P_3	α	β
Fuzzy Approach [44]	35436.66	653.8002	170.1111	279.3569	274.0625	0.749	
IF Approach1 $\omega_1 = 15, \omega_2 = 1$ [44]	35438.10	653.5437	171.0065	278.7836	273.3740	0.7206128	0.1280944
IF Approach 2 $\omega_1 = 8, \omega_2 = 4.5$ [44]	35434.58	654.2317	168.7198	280.2531	274.5773	0.7033514	0.05232988
Proposed Fuzzy Goal Approach $\omega_1 = 0.5$	35439.98	653.2472	172.1012	278.0737	273.3279	0.1254735	
Proposed IFG Approach Model-I $\omega_1 = 0.87, \omega_2 = 0.5$	35442.12	652.9567 .	173.2731	277.3174	272.8963	0.1252156	0.1222963
Proposed IFG Approach Model-II $\omega_1 = 0.204, \omega_2 = 0.675$	**35433.93**	653.3388	174.1555	276.6388	272.4744	0.1129614	0.1229614
Proposed IFG Approach	35434.79	**652.4689**					

Methods	$Cost(P)$	$Em(P)$	D_i^+	D_i^-	R	RANK
Ideal Solution	35424.44	651.4851	0	0.002688	1	1
Anti-ideal Solution	35473.32	660.7492	0.002688	0	0	7
Fuzzy[44]	35436.66	653.8002	0.000671	0.002016	0.75	5
IF Approach 1[44]	35438.10	653.5437	0.000599	0.002089	0.78	4
IF Approach 2[44]	35434.58	654.2317	0.000795	0.001893	0.7	6
Proposed Fuzzy Goal	35439.98	653.2472	0.000515	0.002174	0.80	3
Proposed IFG (Model- I)	**35442.12**	**652.9567**	**0.000435**	**0.002256**	**0.84**	**2**

Model-III $\omega_1 = 0.204, \omega_2 = 0.567$			173.8480	276.8536	272.6019	0.1041116	0.1031116

Table 3: Comparison between the proposed IFG with different existing method of Optimal solution of MOELDM (13)

Table 4: Comparison between the proposed IFG with different existing method of Optimal solution of MOELDM (13)

Methods	$Cost(P)$	$Em(P)$	D_i^+	D_i^-	R	RANK
Ideal Solution	35424.44	651.4851	0	0.002688	1	1
Anti-ideal Solution	35473.32	660.7492	0.002688	0	0	7
Fuzzy[44]	35436.66	653.8002	0.000671	0.002016	0.75	5
IF Approach 1[44]	35438.10	653.5437	0.000599	0.002089	0.78	4
IF Approach 2[44]	35434.58	654.2317	0.000795	0.001893	0.7	6
Proposed Fuzzy Goal	35439.98	653.2472	0.000515	0.002174	0.80	3
Proposed IFG (Model-II)	**35433.93**	**653.2388**	**0.0005090**	**0.0021793**	**0.81**	**2**

Table 5: Comparison between the proposed IFG with different existing method of Optimal solution of MOELDM (13)

Methods	$Cost(P)$	$Em(P)$	D_i^+	D_i^-	R	RANK
Ideal Solution	35424.44	651.4851	0	0.002688	1	**1**
Anti-ideal Solution	35473.32	660.7492	0.002688	0	0	**7**
Fuzzy[44]	35436.66	653.8002	0.000671	0.002016	0.75	5
IF Approach 1[44]	35438.10	653.5437	0.000599	0.002089	0.78	4
IF Approach 2[44]	35434.58	654.2317	0.000795	0.001893	0.70	6
Proposed Fuzzy Goal	35439.98	653.2472	0.000515	0.002174	0.81	3

Proposed Intuitionistic Fuzzy Goal (Model- III)	**35434.79**	**652.4689**	**0.000289**	**0.002400**	**0.89**	**2**

List of abbreviations:

ELD	Economic Load Dispatch
EED	Economic Emission Dispatch
EELD	Economic and Emission Load Dispatch
MOEELD	Multi-objectives Emission and Economic Load Dispatch
MF	Membership Function
NMF	Non-membership functions
IFS	Intuitionistic fuzzy set
IFG	Intuitionistic fuzzy goal
BBO	biogeography-based optimization approach
PSO	Particle Swam optimization
FA	firefly algorithm
FGP	Fuzzy Goal Programming
BWO-FLANN	Black Widow Optimization based Functional Link Artificial Neural Network
NS-MOTLBO	Non-dominated Sorting Multi-Objective Teaching Learning Based Optimization
GWO-PSO	Grey wolf-particle swarm optimization
MPSO	Modified particle swarm optimization
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution

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