RESEARCH PAPER

A Novel Improved Fermatean Fuzzy Optimization Technique and Its Application

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ABSTRACT

This paper addresses the dual challenge of optimizing the muti-objective Economic emission load dispatch (MOEELD) problem, which must fulfil load-minimizing generation cost and reduce emission from thermal power plants during power generation. Traditional optimization techniques that used in the past were unable to provide suitable, globally acceptable results due to their limitations. The goal of the MOEELD mathematical model is to reduce emissions and fuel cost at the same time. The Fermatean fuzzy optimization, a novel kind of fuzzy optimization technique, identifies the best optimal solutions for MOELD issues. Since the FFO technique is capable of dealing with a higher degree of uncertainty and ambiguity found in multi-objective optimization problems, it is especially well-suited for addressing the EELD problems. To illustrate the effectiveness and applicability of the suggested problems, we offered numerical examples. To demonstrate the effectiveness and suitability of the suggested approach, comparative numerical results have been provided. The proposed approach is tested using the data of six generating power unit systems along with power generation limits and power balance constraints.

Keywords: Fermatean Fuzzy Optimization; Multi-Objective Economic and Emission Dispatch Problem; Power System Optimization;

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1. INTRODUCTION

The ED problem [1] in the thermal power industry is an intriguing real-world issue that takes fuel cost minimization and into consideration as its relevant objective function. The significance of ED issues makes it important for researchers to make an effort to minimize the fuel cost objective function for community assistance. Interior point methods (1998) [2], evolutionary programming methods (2003) [3], Lagrangian relaxation methods (2013) [4] are various conventional techniques applied by researchers to the challenges associated with ED issues. Further, a variety of non-conventional approaches to address ED issues have been developed, such as, PSO(1995) [5] techniques, similarity crossover-based GA (2009) [6] technique , CI-grounded approaches like BFA (2010) [7], various GA approaches like the NSGA-II (2011) [8] model, the GSA (2012) [9] model, ABC (2014) [10] techniques, BA method (2014) [11], the GWO (2016) [12] model, etc., have been introduced to solve EED issues. To integrate the most advantageous aspects of various algorithms and thus attain enhanced performance compared to standalone approaches, researchers have created numerous hybrid methods by combining two or more algorithms to address EED issues. EED [13], a multi-objective optimization problem, has become crucial in real thermal power engineering, concurrently minimizing fuel cost and reducing emissions with power balance constraints and power generating limits as well. Abido et al. (2003) [14] developed a new SPEA approach for optimum. Güenç (2010) [15] introduced an innovative GA based on similarity edge for tackling the EED issue. Yang (2012) [16] introduced the FPA approach, which provides great versatility in tackling a variety of optimization issues since it can optimize both single-objective and multi-objective optimization problems. Silva et al. (2013) [17] suggested using an ISS technique to address the EED issues. They established that the ISS approach is a viable substitute for EED issues to provide high-quality solutions. Sayah et al. (2014) [18] applied the weighted sum technique in EED, which provides dimensional consistency along with a specific monetary value. Various PSO model, like modified PSO (2008) [19] models, quantum- behaved PSO (2009) [20] model, modulated PSO (2015) [21] approach, were developed. Sing et al. (2015) [22] developed a new PSO technique to solve the multi-objective EED problem. Tomar et al. (2019) [23] introduced the SMO technique, which is used to reduce the green- house effect by solving the optimal dispatch problem with cubic cost and emission functions. Chinnadurrai et al. (2020) [24] present an approach to dynamic EED with wind-power unpredictability through multi-objective crisscross optimization. The EED problem is solved in 2022 by Masud Dashtdar et al. [25] using HFA and GA techniques to converge to the best solution, and a local search strategy is put in place to enhance the quality of results. In 2023, Sing et al. [26] proposed a new variant of PSO methods for MOEELD problems with efficient outcomes. Abdulla et al. [27] in 2024 introduced a modified GWO-PSO technique to minimize both fuel cost and emission level. Mishra et al. [28], in 2024, introduced an advanced process to compare distinct EED models with load-shifting techniques.

In the past, the majority of research has ignored the decision-making process in multi-objective issues. The suggested study incorporates fuzzy lucidity and promotes decision-making to tackle the imprecise nature of MOEELD concerns, ensuring both cost effectiveness and reduced emissions in order to provide more beneficial and practical solutions.

1.2. Literature Review

FFP is a non-linear technique that is an extended form of PFP, developed to address multi- level, multi-objective issues. Zadeh (1965) [29] first Introduced a new decision-making concept, FS, to assist with uncertainty and ambiguity in real-world perimeters. Atanassov (1986) [30] expanded FS theory for integrating both MF and NMF in order to facilitate the illustration of uncertainty and indeterminacy. q-rung ortho-pairs, FSs are regarded by Yager (2014) [31] as PFSs when q=2. Bustince et al. (2016) [32] developed a chronological book about different FSs and their corresponding relations and stated that, anyway, IFSs are PFSs. Senapati and Yager (2020) [33] introduced FFSs and contrasted them with PFSs and IFSs. In q-rung ortho-pairs, FSs are considered as FFSs when q=3. In 2022, Lotfi et al. [34] introduced GWO and MHBMO, a novel approach to simulate the results that suggested the strategy is effective for both single-objective and multi-objective EED problems. Shafiee et al. [35] in 2023 proposed an improved DDAO algorithm that handle and minimizes the objectives of EED problems, indicating the profound exactness and convergence swiftness. In 2023, Hernando et al. [36] suggested an MPSO technique to find the optimum solution for both single and multi- objective EED problems. In 2024, M. Chandrashekhar et al. [37] proposed a novel method, the WOA technique, comparing it with PSO and ACO techniques, which focus on concurrently reducing emissions and minimizing fuel costs in the EED problem. Y. Gao et al. [38], in 2024, suggested the application of uniform assessment standards and the incorporation of a wide variety of case studies to improve the applicability of optimization methods in the EED industry. In 2025, T. Dong [39] suggested that the enhanced MDE algorithm can effectively equilibrate performance and operating costs, resulting in lower fuel costs and pollution emissions. In 2025, K. Chan et al. proposed an epsilon-based MOGA technique to solve EED problems through the reformulation of multi-objective problems into two different single- objective functions. The present research concentrates on dealing with supply and demand parameter uncertainty while balancing various targets, like reducing fuel costs and environmental emissions. The main objective of this study is to develop an FFPA that can provide effective, practical solutions for optimizing EED in thermal power generation systems in uncertain, multi-objective environments.

1.3 Contribution

- A FFA applies a linear membership function and non-membership function to solve the MOEELD problem.
- To addressed a MOEELD problem to demonstrate the effectiveness and efficiency proposed method.
- The outcomes are analyzed and compared based on how significantly they resemble the most effective solution.

2. MATHEMATICAL MODEL OF MOEELD PROBLEM

2.1 Economic Dispatch Model

The economic dispatch problem can be represented by minimizing the fuel cost of generating units while satisfying a set of equality and inequality constraints.

$$Minimize\ \ Cost(P) = \sum_{i=1}^{m}\left(a_i P_i^2 + b_i P_i + c_i\right) \tag{1}$$

where a_i, b_i and c_i are the cost coefficients for i^{th} unit, P_i is the power generated by unit i, MW and the number of generating units is n.

2.1.1. Power Balance Constraints

The total generated power should be equal to the total amount of the line loss and the load demand. It may be stated as

$$\sum_{i=1}^{m} P_i - (P_D + P_L) = 0 \tag{2}$$

where P_L is the total line loss and P_D is the total load demand.

Using the B-coefficient, the total system loss may be calculated as

$$P_L = \sum_{i=1}^{m}\sum_{j=1}^{m} P_i B_{ij} P_j + \sum_{i=1}^{m} B_{0i} P_i + B_{00} \tag{3}$$

where B_{ij}, B_{0i} and B_{00} are the transmission loss coefficients.

2.1.2. Inequality Constraints

The actual power generated by each generator mjst be kept within its lower and maximum operating limitations.

$P_i^{\min} \le P_i \le P_i^{\max}$ (i=1,2, 3…m) (4)

P_i is the generator's power output of i^{th} generator, where $P_i^{\max}$ and $P_i^{\min}$ is the generator's maximum and minimum generated power respectively.

2.2. Emission Dispatch Model

In order to make load demand, emission dispatch seeks to lower emissions from all generating units that burn fuels to produce energy. This is illustrated as

$$Minimize\ Em(P) = \sum_{i=1}^{m}\left(\rho_i P_i^2 + \sigma_i P_i + \varsigma_i\right) \quad (5)$$

where, by ρ_i, σ_i and ς_i are the emission coefficients for i^{th} unit, P_i is the power generated by unit i, MW and the number of generating units is n.

2.3. MOEELD problem Model

The MOEELD model with several objectives can be stated as follows:

$$Min\ \ Cost(P) = \sum_{i=1}^{m}\left(a_i P_i^2 + b_i P_i + c_i\right) \quad (6)$$

$$Min\ Em(P) = \sum_{i=1}^{m}\left(\rho_i P_i^2 + \sigma_i P_i + \varsigma_i\right)$$

Subject to $\sum_{i=1}^{m} P_i - (P_D + P_L) = 0;$

$$P_i^{\min} \le P_i \le P_i^{\max}.$$

where, $P = [P_1, P_2, ...P_m]^T$ are power generated, the no. of power plants is n, $Cost(P)$ is the power generation cost, $Em(P)$ is the emission during power generation, P_i, P_L and P_D is the power generated, transmission losses and power demand respectively. $P_i^{\min}$ is the lower limit, and $P_i^{\max}$ is the upper limit of generator output.

3. PREREQUISITE MATHEMATICS

3.1 Fuzzy Set [29]**:** Let $R = \{r_1, r_2, ..., r_n\}$ denote a universal set. Then the fuzzy subset $\tilde{A}$ of R is a set of order pairs $\tilde{A} = \{r, \eta_{\tilde{A}}(r) \mid r \in R\}$ where $\eta_{\tilde{A}} : R \to [0,1]$ define the degree of membership of the element $r \in R$, is being in $\tilde{A}$.

3.2 Intuitionistic Fuzzy Set [30]**:** Let a set R be fixed. An intuitionistic fuzzy set, or IFS $\tilde{A}^I$ in R, is an object of the form

$\tilde{A}^I = \left\{\left\langle r, \eta_{\tilde{A}^I}(r), \kappa_{\tilde{A}^I}(r)\right\rangle \mid r \in R\right\}$

where, $\eta_{\tilde{A}^I} : R \to [0,1]$ and $\kappa_{\tilde{A}^I} : R \to [0,1]$

Define the degree of membership and degree of non-membership, respectively, for every element $r \in R$, $0 \le \eta_{\tilde{A}^I} + \kappa_{\tilde{A}^I} \le 1$.

For all $r \in R$, $\omega_{\tilde{A}^I}(r)$ is the degree of hesitancy of $r \in R$, where $\omega_{\tilde{A}^I}(r) = 1 - \eta_{\tilde{A}^I}(r) - \kappa_{\tilde{A}^I}(r)$.

3.3 Pythagorean Fuzzy Set [31]**:** Let a set R be fixed. An intuitionistic fuzzy set, or IFS $\tilde{A}^{\wp}$ in R, is an object of the form

$\tilde{A}^{\wp} = \left\{\left\langle r, \eta_{\tilde{A}^{\wp}}(r), \kappa_{\tilde{A}^{\wp}}(r)\right\rangle \mid r \in R\right\}$

where, $\eta_{\tilde{A}^{\wp}} : R \to [0,1]$ and $\kappa_{\tilde{A}^{\wp}} : R \to [0,1]$

Define the degree of membership and degree of non-membership, respectively, for every element $r \in R$, $0 \le \eta_{\tilde{A}^{\wp}}^2 + \kappa_{\tilde{A}^{\wp}}^2 \le 1$.For all $r \in R$, $\omega_{\tilde{A}^{\wp}}(r)$ is the degree of hesitancy of $r \in R$, where $\omega_{\tilde{A}^{\wp}}(r) = \sqrt{1 - \eta_{\tilde{A}^{\wp}}^2(r) - \kappa_{\tilde{A}^{\wp}}^2(r)}$.

3.4 Fermatean Fuzzy Set [33]**:** Let a set R be fixed. An intuitionistic fuzzy set, or IFS $\tilde{A}^{\mathrm{F}}$ in R, is an object of the form

$\tilde{A}^{\mathrm{F}} = \left\{\left\langle r, \eta_{\tilde{A}^{\mathrm{F}}}(r), \kappa_{\tilde{A}^{\mathrm{F}}}(r)\right\rangle \mid r \in R\right\}$

where, $\eta_{\tilde{A}^{\mathrm{F}}} : R \to [0,1]$ and $\kappa_{\tilde{A}^{\mathrm{F}}} : R \to [0,1]$.

Define the degree of membership and degree of non-membership, respectively, for every element $r \in R$, $0 \le \eta_{\tilde{A}^{\mathrm{F}}}^3 + \kappa_{\tilde{A}^{\mathrm{F}}}^3 \le 1$.

For all $r \in R$, $\omega_{\tilde{A}^{\mathrm{F}}}(r)$ is the degree of hesitancy of $r \in R$, where $\omega_{\tilde{A}^{\mathrm{F}}}(r) = \sqrt[3]{1 - \eta_{\tilde{A}^{\mathrm{F}}}^3(r) - \kappa_{\tilde{A}^{\mathrm{F}}}^3(r)}$.

3.5 Definition:

Let $\tilde{A}^{\mathrm{F}} = \left\langle \eta_{\tilde{A}^{\mathrm{F}}}, \kappa_{\tilde{A}^{\mathrm{F}}} \right\rangle$, $\tilde{A}^{\mathrm{F}_{11}} = \left\langle \eta_{\tilde{A}^{\mathrm{F}_1}}, \kappa_{\tilde{A}^{\mathrm{F}_1}} \right\rangle$, and $\tilde{A}^{\mathrm{F}_{21}} = \left\langle \eta_{\tilde{A}^{\mathrm{F}_2}}, \kappa_{\tilde{A}^{\mathrm{F}_2}} \right\rangle$ be three FFSs on universal set and $\lambda > 0$ be any scalar.

3.5.1 The Addition of FFSs [33]**:**

$$\begin{aligned}\tilde{A}^{\mathrm{F}_{11}} \oplus \tilde{A}^{\mathrm{F}_{21}} &= \left(\sqrt[3]{\eta_{\tilde{A}^{\mathrm{F}_{11}}}^3 + \eta_{\tilde{A}^{\mathrm{F}_{21}}}^3 - \eta_{\tilde{A}^{\mathrm{F}_{11}}}^3 \eta_{\tilde{A}^{\mathrm{F}_{21}}}^3}, \kappa_{\tilde{A}^{\mathrm{F}_{11}}} \kappa_{\tilde{A}^{\mathrm{F}_{21}}}\right) \\ &= \left(\sqrt[3]{\eta_{\tilde{A}^{\mathrm{F}_{21}}}^3 + \eta_{\tilde{A}^{\mathrm{F}_{11}}}^3 - \eta_{\tilde{A}^{\mathrm{F}_{21}}}^3 \eta_{\tilde{A}^{\mathrm{F}_{11}}}^3}, \kappa_{\tilde{A}^{\mathrm{F}_{21}}} \kappa_{\tilde{A}^{\mathrm{F}_{11}}}\right) \\ &= \tilde{A}^{\mathrm{F}_{21}} \oplus \tilde{A}^{\mathrm{F}_{11}}.\end{aligned}$$

3.5.2 The Multiplication of FFSs [33]**:**

$$\begin{aligned}\tilde{A}^{\mathrm{F}_{11}} \otimes \tilde{A}^{\mathrm{F}_{21}} &= \left(\eta_{\tilde{A}^{\mathrm{F}_{11}}} \eta_{\tilde{A}^{\mathrm{F}_{21}}}, \sqrt[3]{\kappa_{\tilde{A}^{\mathrm{F}_{11}}}^3 + \kappa_{\tilde{A}^{\mathrm{F}_{21}}}^3 - \kappa_{\tilde{A}^{\mathrm{F}_{11}}}^3 \kappa_{\tilde{A}^{\mathrm{F}_{21}}}^3}\right) \\ &= \left(\eta_{\tilde{A}^{\mathrm{F}_{21}}} \eta_{\tilde{A}^{\mathrm{F}_{11}}}, \sqrt[3]{\kappa_{\tilde{A}^{\mathrm{F}_{21}}}^3 + \kappa_{\tilde{A}^{\mathrm{F}_{11}}}^3 - \kappa_{\tilde{A}^{\mathrm{F}_{21}}}^3 \kappa_{\tilde{A}^{\mathrm{F}_{11}}}^3}\right) \\ &= \tilde{A}^{\mathrm{F}_{21}} \otimes \tilde{A}^{\mathrm{F}_{11}}.\end{aligned}$$

3.5.3 The Scalar Multiplication of FFSs [33]:

$$\lambda\tilde{A}^{F}=\left(\sqrt[3]{1-\left(1-\eta_{\tilde{A}^F}^{3}\right)^{\lambda}},\kappa_{\tilde{A}^F}^{\lambda}\right).$$

3.5.4 The Exponentiation of FFSs [33]:

$$\tilde{A}^{F^{\lambda}}=\left(\eta_{\tilde{A}^{F^{\lambda}}},\sqrt[3]{1-\left(1-\kappa_{\tilde{A}^F}^{3}\right)^{\lambda}}\right).$$

3.5.5 The Union of FFSs [33]:

$$\tilde{A}^{F_1}\cup\tilde{A}^{F_2}=\left(\max\left\{\eta_{\tilde{A}^{F_1}},\eta_{\tilde{A}^{F_2}}\right\},\min\left\{\kappa_{\tilde{A}^{F_1}},\kappa_{\tilde{A}^{F_2}}\right\}\right).$$

3.5.6 The Intersection of FFSs [33]:

$$\tilde{A}^{F_1}\cap\tilde{A}^{F_2}=\left(\min\left\{\eta_{\tilde{A}^{F_1}},\eta_{\tilde{A}^{F_2}}\right\},\max\left\{\kappa_{\tilde{A}^{F_1}},\kappa_{\tilde{A}^{F_2}}\right\}\right).$$

3.5.7 The Complement of FFSs [33]:

$$\tilde{A}^{F^{c}}=\left\langle\kappa_{\tilde{A}^F},\eta_{\tilde{A}^F}\right\rangle.$$

3.6 Accuracy Function[33]:

Let, $\tilde{A}^{F}=\left\langle\eta_{\tilde{A}^F},\kappa_{\tilde{A}^F}\right\rangle$ be an FFS, the accuracy function about $\tilde{A}^{F}$ is given as follows $acc\left(\tilde{A}^{F}\right)=\eta_{\tilde{A}^F}^{3}+\kappa_{\tilde{A}^F}^{3}$ where, $acc\ \tilde{A}^{F}\in[0,1]$ and $0\le acc\left(\tilde{A}^{F}\right)\le 1$.

3.7 Score Function[33]:

Let, $\tilde{A}^{F}=\left\langle\eta_{\tilde{A}^F},\kappa_{\tilde{A}^F}\right\rangle$ be an FFS, the score function about $\tilde{A}^{F}$ is given as follows $S_F\left(\tilde{A}^{F}\right)=\eta_{\tilde{A}^F}^{3}-\kappa_{\tilde{A}^F}^{3}$ where, $acc\ \tilde{A}^{F}\in[-1,1]$ and $-1\le S_F\left(\tilde{A}^{F}\right)\le 1$.

4. METHODOLOGY

4.1 Fuzzy Approach to solve the MONLP Problem:

Find $r=\left(r_1,r_2,\ldots r_p\right)^{T}$

To Min $\Psi_i(r)\quad (i=1,2,\ldots,p)$ (7)

with lower limit (minimum) be L_i and upper limit (maximum) be U_i

Subject to $\psi_j(r)\le b_j$, j=1, 2,..., m, $r>0$,

MF will be

$$\eta_{\Psi_i(r)}(\Psi_i(r))=\begin{cases}1 & if\ \Psi_i(r)\le L_i\\ \dfrac{U_i-\Psi_i(r)}{U_i-L_i} & if\ L_i\le\Psi_i(r)\le U_i\\ 0 & if\ \Psi_i(r)\ge U_i\end{cases}\quad (8)$$

The crisp programming from FP can be written as,

Min $\eta_{\Psi_i(r)}\left(\Psi_i(r)\right)$ (9)

Subject to $0\le\eta_{\Psi_i(r)}\left(\Psi_i(r)\right)\le 1$

$\psi_j(r)\le b_j$, j=1, 2,..., m, $r>0$,

Equation (9) can be written as,

$$Max\ s \quad (10)$$

$$such\ that\ \ \eta_{\Psi_i(r)}\left(\Psi_i(r)\right)\ge s,\quad r=1,2,\ldots,p$$

$$\psi_j(r)\le b_j,\ \ j=1,2,\ldots.m$$

$$r>0,\ s\in[0,1].$$

4.2 IFA to solve the MONLP problem:

Find $r=\left(r_1,r_2,\ldots r_n\right)^{T}$ (11)

To Min $\Psi_i(r)$

with upper limit (maximum) of acceptance be U_i^{Acc} and lower limit (minimum) acceptance be L_i^{Acc} . Where $U_i^{Acc}=\max\left\{\Psi_i(r^p)\right\}$, $L_i^{Acc}=\min\left\{\Psi_i(r^p)\right\}$ where $1\le i\le p$.

Subject to $\psi_j(r)\le b_j$, j=1, 2,..., m, $x>0$,

MF and NMF will be

$$\eta_{\Psi_i(r)}(\Psi_i(r))=\begin{cases}1 & if\ \Psi_i(r)\le L_i^{Ace}\\ \dfrac{U_i^{Ace}-\Psi_r(r)}{U_i^{Ace}-L_i^{Ace}} & if\ L_i^{Ace}\le\Psi_i(r)\le U_i^{Ace}\\ 0 & if\ \Psi_i(r)\ge U_i^{Ace}\end{cases},$$

$$\kappa_{\Psi_i(r)}(\Psi_i(r))=\begin{cases}0 & if\ \Psi_i(r)\le L_i^{\mathrm{Rej}}\\ \dfrac{\Psi_i(r)-L_i^{\mathrm{Rej}}}{U_i^{\mathrm{Rej}}-L_i^{\mathrm{Rej}}} & if\ L_i^{\mathrm{Rej}}\le\Psi_i(r)\le U_i^{\mathrm{Rej}}\\ 1 & if\ \Psi_i(r)\ge U_i^{\mathrm{Rej}}\end{cases}.$$

To find a appropriate crisp model, by with IFO to reach both membership and non-membership objectives described as below

$$\underset{\forall i}{Max}\ \left(\eta_i\left(\Psi_i(r)\right)\right) \quad (12)$$

$$\underset{\forall i}{Min}\ \left(\kappa_i\left(\Psi_i(r)\right)\right).$$

Subject to

$\eta_i(\Psi_i(r))+\kappa_i(\Psi_i(r))<1,$

$\eta_i(\Psi_i(r))\ge\kappa_i(\Psi_i(r)),$

$\eta_i(\Psi_i(r))\ge 0,$

$\psi_j(r)\le 0,$

$x>0,$

$i=1,2,\ldots,p;\quad j=1,2,\ldots,m$;

Equation (12) can be written as:

$$Max(\min(\eta_1,\eta_2,\ldots,\eta_p))-Min(\max(\kappa_1,\kappa_2,\ldots,\kappa_p)). \quad (13)$$

Subject to

$\eta_i(\Psi_i(r))+\kappa_i(\Psi_i(r))<1,$

$\eta_i(\Psi_i(r))\ge\kappa_i(v_i(r)),$

$\eta_i(\Psi_i(r)) \ge 0,$

$\psi_j(r) \le 0,$

$r > 0,$

$i = 1,2,...,p; \quad j = 1,2,...,m$.

If we take into account $s = \min(\eta_1, \eta_2, ..., \eta_p)$ and $t = \max(\kappa_1, \kappa_2, ..., \kappa_p)$, equation (13) can be rephrased as follows:

$$Max \ (s - t) \tag{14}$$

Subject to

$\eta_i(\Psi_i(r)) \ge s, \quad \kappa_i(\Psi_i(r)) \le t,$

$\psi_j(r) \le 0,$

$r > 0; \quad s + t \le 1,$

$s \in [0,1], \ t \in [0,1]; \ i = 1,2,...,p; \quad j = 1,2,...,m$

What should be used as a replacement for $\eta_i(\Psi_i(r)), \kappa_i(\Psi_i(r))$ for $i = 1,2,...,p$ becomes

$$Maximize \ (s - t) \tag{15}$$

Subject to

$\Psi_i(r) + s(U_i^{Ace} - L_i^{Ace}) \le U_i^{Ace},$

$\Psi_i(r) - t(U_i^{\text{Rej}} - L_i^{\text{Rej}}) \le L_i^{Ace},$

$\psi_j(r) \le 0,$

$s + t \le 1,$

$s \in [0,1], \ t \in [0,1]; \ i = 1,2,...,p; \quad j = 1,2,...,m$

4.3. FFO Approach to Solve MONLP Problem:

To find a appropriate crisp model, with FFO, to achieve both MF and NMF of objectives described as below

$$\underset{\forall i}{Max} \ \left(\eta_i\left(\Psi_i(r)\right)\right)^3 \tag{16}$$

$$\underset{\forall i}{Min} \ \left(\kappa_i\left(\Psi_i(r)\right)\right)^3$$

Subject to

$\eta_i(\Psi_i(r))^3 + \kappa_i(\Psi_i(r))^3 < 1,$

$\eta_i(\Psi_i(r))^3 \ge \kappa_i(\Psi_i(r))^3,$

$\eta_i(\Psi_i(r)) \ge 0,$

$\psi_j(r) \le 0,$

$x > 0,$

$i = 1,2,...,p; \quad j = 1,2,...,m$;

Equation (16) can be written as:

$$Max(\min(\eta_1, \eta_2, ..., \eta_p))^3 - Min(\max(\kappa_1, \kappa_2, ..., \kappa_p))^3 \tag{17}$$

Subject to

$\eta_i(\Psi_i(r))^3 + \kappa_i(\Psi_i(r))^3 < 1,$

$\eta_i(\Psi_i(r))^3 \ge \kappa_i(v_i(r))^3,$

$\eta_i(\Psi_i(r)) \ge 0,$

$\psi_j(r) \le 0,$

$r > 0,$

$i = 1,2,...,p; \quad j = 1,2,...,m$

If we take into account $s = \min(\eta_1, \eta_2, ..., \eta_p)$ and $t = \max(\kappa_1, \kappa_2, ..., \kappa_p)$, equation (17) can be rephrased as follows:

$$Max \ (s^3 - t^3) \tag{18}$$

Subject to

$\eta_i(\Psi_i(r))^3 \ge s^3, \quad \kappa_i(\Psi_i(r))^3 \le t^3,$

$\psi_j(r) \le 0,$

$r > 0; \quad s^3 + t^3 \le 1,$

$s \in [0,1], \ t \in [0,1]; \ i = 1,2,...,p; \quad j = 1,2,...,m$

What should be used as a replacement for $\eta_i(\Psi_i(r)), \kappa_i(\Psi_i(r))$ for $i = 1,2,...,p$ becomes

$$Maximize \ (s^3 - t^3) \tag{19}$$

Subject to

$\left(\dfrac{U_i^{Ace} - \Psi_i(r)}{U_i^{Ace} - L_i^{Ace}}\right)^3 \ge s^3,$

$\left(\dfrac{\Psi_i(r) - L_i^{Ace}}{U_i^{\text{Rej}} - L_i^{\text{Rej}}}\right)^3 \le t^3,$

$\psi_j(r) \le 0,$

$s^3 + t^3 \le 1,$

$s \in [0,1], \ t \in [0,1]; \ i = 1,2,...,p; \quad j = 1,2,...,m$

5. FFO TECHNIQUES FOR SOLVING THE MULTI-OBJECTIVE ECONOMIC EMISSION LOAD DISPATCH PROBLEM

To solve the MOEELD problem (6), pay -off matrix formulated as follows:

$$\begin{array}{c} \\ P^1 \\ P^2 \end{array} \begin{array}{c} \begin{array}{cc} Cost(P) & Em(P) \end{array} \\ \begin{pmatrix} Cost^*(P^1) & Em(P^1) \\ Cost(P^2) & Em^*(P^2) \end{pmatrix} \end{array}$$

From pay off matrix, the limits of the objective are U_1^{Ace} , L_1^{Ace} for $\text{Cost}(P)$ (where $L_1^{Ace} \le \text{Cost}(P) \le U_1^{Ace}$) and, L_1^{Rej} ,U_1^{Rej} (where $L_2^{\text{Rej}} \le \text{Cost}(P) \le U_2^{\text{Rej}}$, U_2^{Ace} , L_2^{Ace} for $Em(P)$ (where $L_2^{Ace} \le Em(P) \le U_2^{Ace}$) and L_2^{Rej} ,U_2^{Rej} where $L_2^{\text{Rej}} \le Em(P) \le U_2^{\text{Rej}}$, where $U_i^{Ace} = U_i^{\text{Rej}}$; $L_i^{\text{Rej}} = L_i^{Ace} + \omega_i$ for $i = 1,2.$ such that $0 < \omega_i < (U_i^{Ace} - L_i^{Ace})$ are identified.

A crisp nonlinear programming problem for MOEELD problem (6) is structured by taking into account MF and NMF,

$$\begin{aligned} &Max(\min(\eta_{\cos t}(Cost(P)), \eta_{Em}(Em(P))))^3 \\ &\quad - Min(\max(\kappa_{\cos t}(Cost(P)), \kappa_{Em}(Em(P))))^3 \end{aligned} \tag{20}$$

Subject to

$$\eta_{COST}(Cost(P))^3 + \kappa_{COST}(Cost(P))^3 < 1,$$
$$\eta_{Em}(Em(P))^3 + \kappa_{Em}(Em(P))^3 < 1,$$
$$\eta_{COST}(Cost(P))^3 \ge \kappa_{COST}(Cost(P))^3,$$
$$\eta_{Em}(Em(P))^3 \ge \kappa_{Em}(Em(P))^3,$$
$$\kappa_{COST}(Cost(P)) \ge 0, \kappa_{Em}(Em(P)) \ge 0,$$
$$\sum_{i=1}^{m} P_i - (P_D + P_L) = 0$$
$$P_i^{\min} \le P_i \le P_i^{\max} \; ; P > 0,$$

The problem stated above can be expressed as,

$$Max\ (s^3 - t^3) \tag{21}$$
$$subject\ to$$
$$\eta_{COST}(Cost(P))^3 \ge s^3, \;\; \kappa_{COST}(Cost(P))^3 \le t^3;$$
$$\eta_{Em}(Em(P))^3 \ge s^3, \;\; \kappa_{Em}(Em(P))^3 \le t^3;$$
$$\sum_{i=1}^{m} P_i - (P_D + P_L) = 0$$
$$P_i^{\min} \le P_i \le P_i^{\max} \; ; s + t \le 1,$$
$$P > 0, s \in [0,1], t \in [0,1];$$

Applying an appropriate mathematical programming approach, optimize the model (21) to find the optimal solution that incorporates the objective function, which is the thermal power plant's expenditure and emissions, leading to a pareto optimality.

6. NUMERICAL SOLUTION OF A POWER GENERATION SYSTEM OF SIX GENERATORS' MOEELD MODEL

A system of 6-genetarors power plant is consider here. In order to fulfil the power balancing and generation limit requirements, the goals are (i) to minimize the fuel $\mathrm{Cost}(P)$ and (ii) to simultaneously minimize the emission level $\mathrm{Em}(P)$.

The generator power output is one of the design factors in this model.

The problem of multi-objective optimization is defined as:

$$\text{Minimize } Cost(P) = \sum_{i=1}^{6}\left(a_i P_i^2 + b_i P_i + c_i\right) \tag{22}$$

$$\text{Minimize } Em(P) = \sum_{i=1}^{6}\left(\rho_i P_i^2 + \sigma_i P_i + \varsigma_i\right)$$

$$\text{Subject to } P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = P_D + P_L$$

$$P_i^{\min} \le P_i \le P_i^{\max}; i = 1, 2, ..., 6;$$

$$\text{where } P_L = \sum_{i=1}^{6}\sum_{j=1}^{6} P_i B_{ij} P_j + \sum_{i=1}^{6} B_{0i} P_i + B_{00}$$

Cost and Emission coefficients of generator for IEEE 30 bus system is provided in table-1

$$B_{ij} = 10^{-6}\begin{bmatrix} 140 & 17 & 15 & 19 & 26 & 22 \\ 17 & 60 & 13 & 16 & 15 & 20 \\ 15 & 13 & 65 & 17 & 24 & 19 \\ 19 & 16 & 17 & 71 & 30 & 25 \\ 26 & 15 & 24 & 30 & 69 & 32 \\ 22 & 20 & 19 & 25 & 32 & 85 \end{bmatrix}$$

where B_{0i} *and* B_{00} are considered zero.

Table2: Pay-off matrix for 6 generating unit plants with different load demands.

$\underline{POWER\ DEMAND = 700MW}$

$$\begin{array}{c} \\ P^1 \\ P^2 \end{array} \begin{array}{cc} Cost(P) & Em(P) \\ \left(\begin{array}{c} 38101.03 \\ 36912.20 \end{array}\right. & \left.\begin{array}{c} 501.0120 \\ 434.1306 \end{array}\right) \end{array}$$

$\underline{POWER\ DEMAND = 800MW}$

$$\begin{array}{c} \\ P^1 \\ P^2 \end{array} \begin{array}{cc} Cost(P) & Em(P) \\ \left(\begin{array}{c} 43719.05 \\ 41896.70 \end{array}\right. & \left.\begin{array}{c} 648.9866 \\ 548.7063 \end{array}\right) \end{array}$$

$\underline{POWER\ DEMAND = 1000MW}$

$$\begin{array}{c} \\ P^1 \\ P^2 \end{array} \begin{array}{cc} Cost(P) & Em(P) \\ \left(\begin{array}{c} 55456.53 \\ 52361.27 \end{array}\right. & \left.\begin{array}{c} 1022.4820 \\ 837.7675 \end{array}\right) \end{array}$$

1) <u>Power Demand=700MW</u>

The pay-off matrix is,

$$\begin{array}{c} \\ P^1 \\ P^2 \end{array} \begin{array}{cc} Cost(P) & Em(P) \\ \left(\begin{array}{c} 38101.03 \\ 36912.20 \end{array}\right. & \left.\begin{array}{c} 501.0120 \\ 434.1306 \end{array}\right) \end{array}$$

Here $Cost_U^{Acc} = 38101.03 = Cost_U^{Rej}$; $Cost_L^{Acc} = 36912.20$; $Cost_L^{Rej} = Cost_L^{Acc} + \omega_1$;

where $0 < \omega_1 < (38101.03 - 36912.20)$;

$Em_U^{Acc} = 501.0120 = Em_U^{Rej}$; $Em_L^{Acc} = 434.1306$ $Em_L^{Rej} = Em_L^{Acc} + \omega_2$,

Where $0 < \omega_2 < (501.0120 - 434.1306)$;

The objective functions of $Cost(P_1, P_2, P_3, P_4, P_5, P_6)$ are represented by the MF and NMF as follows

$$\eta_C(Cost(P)) = \begin{cases} 1 & if\ Cost(P) \le 36912.20 \\ \dfrac{38101.03 - Cost(P)}{1188.83} & if\ 36912.20 \le Cost(P) \le 38101.03 \\ 0 & if\ Cost(P) \ge 38101.03 \end{cases}$$

$$\kappa_C(Cost(P)) = \begin{cases} 0 & if\ Cost(P) \le 36912.20 + \omega_1 \\ \dfrac{Cost(P) - (36912.20 + \omega_1)}{1188.83 - \omega_1} & if\ 36912.20 + \omega_1 \le Cost(P) \le 38101.03 \\ 1 & if\ Cost(P) \ge 38101.03 \end{cases}$$

The objective functions of $Em(P_1, P_2, P_3, P_4, P_5, P_6)$ are represented by the MF and NMF as follows

$$\eta_{Em}(Em(P)) = \begin{cases} 1 & if\ Em(P) \le 434.1306 \\ \dfrac{501.0120 - Em(P)}{66.8814} & if\ 434.1306 \le Em(P) \le 501.0120 \\ 0 & if\ Em(P) \ge 501.0120 \end{cases},$$

$$\kappa_{Em}(Em(P)) = \begin{cases} 0 & if\ Em(P) \le 434.1306 + \omega_2 \\ \dfrac{Em(P) - (434.1306 + \omega_2)}{66.8814 - \omega_2} & if\ 434.1306 + \omega_2 \le Em(P) \le 501.0120 \\ 1 & if\ Em(P) \ge 501.0120 \end{cases}$$

Power Demand=800MW

The pay-off matrix is,

$$\begin{matrix} & Cost(P) & Em(P) \\ P^1 & 43719.05 & 648.9866 \\ P^2 & 41896.70 & 548.7063 \end{matrix}$$

Here $Cost_U^{Acc} = 43719.05 = Cost_U^{Rej}$; $Cost_L^{Acc} = 41896.7$; $Cost_L^{Rej} = Cost_L^{Acc} + \omega_1'$;

where $0 < \omega_1' < (43719.05 - 41896.70)$;

$Em_U^{Acc} = 648.9866 = Em_U^{Rej}$; $Em_L^{Acc} = 548.7063$

$Em_L^{Rej} = Em_L^{Acc} + \omega_2'$,

Where $0 < \omega_2' < (648.9866 - 548.7063)$;

The objective functions of $Cost(P_1, P_2, P_3, P_4, P_5, P_6)$ are represented by the MF and NMF as follows

$$\eta_C(Cost(P)) = \begin{cases} 1 & if\ Cost(P) \le 41896.70 \\ \dfrac{43719.05 - Cost(P)}{1822.35} & if\ 41896.70 \le Cost(P) \le 43719.05 \\ 0 & if\ Cost(P) \ge 43719.05 \end{cases},$$

$$\kappa_C(Cost(P)) = \begin{cases} 0 & if\ Cost(P) \le 41896.70 + \omega_1' \\ \dfrac{Cost(P) - (41896.70 + \omega_1')}{1822.35 - \omega_1'} & if\ 41896.70 + \omega_1' \le Cost(P) \le 43719.05 \\ 1 & if\ Cost(P) \ge 43719.05 \end{cases}$$

The objective functions of $Em(P_1, P_2, P_3, P_4, P_5, P_6)$ are represented by the MF and NMF as follows

$$\eta_{Em}(Em(P)) = \begin{cases} 1 & if\ Em(P) \le 548.7063 \\ \dfrac{648.9866 - Em(P)}{100.2803} & if\ 548.7063 \le Em(P) \le 648.9866 \\ 0 & if\ Em(P) \ge 648.9866 \end{cases},$$

$$\kappa_{Em}(Em(P)) = \begin{cases} 0 & if\ Em(P) \le 548.7063 + \omega_2' \\ \dfrac{Em(P) - (548.7063 + \omega_2')}{100.2803 - \omega_2'} & if\ 548.7063 + \omega_2' \le Em(P) \le 648.9866 \\ 1 & if\ Em(P) \ge 648.9866 \end{cases}.$$

2) Power Demand=1000MW

The pay-off matrix is,

$$\begin{matrix} & Cost(P) & Em(P) \\ P^1 & 55456.53 & 1022.4820 \\ P^2 & 52361.27 & 837.7675 \end{matrix}$$

Here $Cost_U^{Acc} = 55456.53 = Cost_U^{Rej}$; $Cost_L^{Acc} = 52361.27$; $Cost_L^{Rej} = Cost_L^{Acc} + \omega_1''$;

where $0 < \omega_1'' < (55456.53 - 52361.27)$;

$Em_U^{Acc} = 1022.4820 = Em_U^{Rej}$; $Em_L^{Acc} = 837.7675$

$Em_L^{Rej} = Em_L^{Acc} + \omega_2''$,

Where $0 < \omega_2'' < (1022.4820 - 837.7675)$;

The objective functions of $Cost(P_1, P_2, P_3, P_4, P_5, P_6)$ are represented by the MF and NMF as follows

$$\eta_C(Cost(P)) = \begin{cases} 1 & if\ Cost(P) \le 52361.27 \\ \dfrac{55456.53 - Cost(P)}{3095.26} & if\ 52361.27 \le Cost(P) \le 55456.53 \\ 0 & if\ Cost(P) \ge 55456.53 \end{cases},$$

$$\kappa_C\left(Cost(P)\right)$$
$$=\begin{cases} 0 & if\ Cost(P) \le 52361.27+\omega_1'' \\ \dfrac{Cost(P)-\left(52361.27+\omega_1''\right)}{3095.26-\omega_1''} & if\ 52361.27+\omega_1'' \le Cost(P) \le 55456.53 \\ 1 & if\ Cost(P) \ge 55456.53 \end{cases}$$

The objective functions of $Em(P_1, P_2, P_3, P_4, P_5, P_6)$ are represented by the MF and NMF as follows

$$\eta_{Em}\left(Em(P)\right)$$
$$=\begin{cases} 1 & if\ Em(P) \le 837.7675 \\ \dfrac{1022.4820-Em(P)}{184.7145} & if\ 837.7675 \le Em(P) \le 1022.4820 \\ 0 & if\ Em(P) \ge 1022.4820 \end{cases},$$

$$\kappa_{Em}\left(Em(P)\right)$$
$$=\begin{cases} 0 & if\ Em(P) \le 837.7675+\omega_2'' \\ \dfrac{Em(P)-\left(837.7675+\omega_2''\right)}{184.7145-\omega_2''} & if\ 837.7675+\omega_2'' \le Em(P) \le 1022.4820 \\ 1 & if\ Em(P) \ge 1022.4820 \end{cases}$$

7. RESULT COMPARISON

The results are shown in table 2, table 3 and table 4 respectively with different load demand in comparison with different methods.

8. CONCLUSION

In order to solve a variety of traditional industrial engineering mathematical models like operations research, food chain, logistics and EED planning, this study introduced the FFP approach. This paper intends to demonstrate how the FFP approach might be applied to a challenging issue of minimizing pollution and cost in a power system. The flexibility of this approach allows for several uncertainties like fuzzy, intuitionistic, Pythagorean and FFSs.

Here, three different load demand numerical are considered, which show the efficiency of the proposed approach. The outcomes of numerical examples highlight the inspiration of the FFP approach for decision-making. To find the ideal solution, decision-makers are encouraged to use the FFP approach within the fermatean environment. This study presents an effective tool for organizations seeking more reliable and adaptive approaches to MOEELD issues by implementing FFSs into the solution framework, opening up new ways of addressing more challenging and ambiguous circumstances in real-world applications.

9. LIMITATIONS AND FUTURE WORK

The possible loss of information during processing fuzzy data into crisp values and the significant amount of computing data required to solve large-scale issues are two downsides of the FFP approach [42]. The accuracy and score function, which may change depending on certain applications, makes the strategy effective. These constraints should be addressed by study by creating additional information using advanced optimization methods for different domains.

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This article includes the data that supported the study's entitlements.

Conflict of Interest
No conflicts of interest are disclosed by the authors.

Table1 [40]**:** Data set of the six-unit system.

Unit	a $(\$/MW^2hr)$	b $(\$/MWhr)$	c $(\$/hr)$	ρ (lb/MW^2hr)	σ $(lb/MWhr)$	ς (lb/hr)	P_{max} (MW)	P_{min} (MW)
P_1	0.15247	38.553975	756.79886	0.00419	00.32767	13.8593	10	125
P_2	0.10587	46.15916	451.32513	0.00419	00.32767	13.8593	10	150
P_3	0.002803	40.39655	1049.9977	0.00683	-0.54551	40.2669	35	225

P_4	0.03546	38.30553	1243.5311	0.00683	-0.54551	40.2669	35	210
P_5	0.02111	36.32782	1658.5596	0.00461	-0.51116	42.8955	130	325
P_6	0.011799	38.27041	1356.6592	0.00461	--0.51116	42.8955	125	315

Table 2: Evaluation of the MOEELD model (23) optimal solution based on several techniques

Power demand (PD)=700				
OUTPUT POWER (MW)	**WEO** [41]	**FA** [41]	**BA** [41]	**PROPOSED METHOD**
P1	62.0893	62.1127	62.1032	50.43771
P2	61.6638	61.6689	61.6698	45.15606
P3	119.9716	119.9746	119.9712	123.4302
P4	119.4758	119.4606	119.4756	122.7693
P5	178.1915	178.1913	178.1929	190.3793
P6	175.6549	175.648	175.6432	185.4281
FUEL COST($/h)	37500.17	37500.93	37500.84	**37195.4**
EMISSION (tons/h)	439.6	439.61	439.61	450.7609

Table 3: Evaluation of the MOEELD model (23) optimal solution based on several techniques

Power demand (PD)=800				
OUTPUT POWER (MW)	**WEO** [41]	**FA** [41]	**BA** [41]	**PROPOSED METHOD**
P1	76.5712	76.5733	76.5756	60.41874
P2	79.2635	79.2629	79.2678	58.31546
P3	135.2372	135.2268	135.225	140.6849
P4	134.1532	134.1554	134.153	138.8303
P5	199.7063	199.7071	199.71	214.6627
P6	197.2528	197.2628	197.256	210.0184
FUEL COST($/h)	42784.22	42784.41	42784.52	**42295.15**
EMISSION (tons/h)	557.19	557.2	557.2	575.52

Table 4: Evaluation of the MOEELD model (23) optimal solution based on several techniques

Power demand (PD)=1000				
OUTPUT POWER (MW)	**WEO** [41]	**FA** [41]	**BA** [41]	**PROPOSED METHOD**
P1	107.1622	107.1685	107.1631	78.20694
P2	116.5485	116.5498	116.5483	82.32248
P3	165.6537	165.655	165.6599	176.8075
P4	163.4009	163.4014	163.4001	171.9967
P5	242.0415	242.038	242.0355	265.3557
P6	239.7982	239.7979	239.8036	261.3035
FUEL COST($/h)	54123.83	54124.28	54124.12	**53160.46**
EMISSION (tons/h)	851.52	851.53	851.53	885.46

List of abbreviations:

ABC	Artificial Bee Colony
ACO	Ant Colony Optimization
BA	Bat Algorithm
BFA	Bacterial Foraging Algorithm
DDAO	Dynamic Differential Annealed Optimization
EED	Economic Emission Dispatch
EELD	Economic and Emission Load Dispatch
FA	Firefly Algorithm
FFO	Fermatean Fuzzy Optimization
FFPA	Fermatean Fuzzy Programming Approach
FPA	Flower Pollination Algorithm
FP	Fuzzy Programming
FS	Fuzzy Set
GA	Genetic Algorithm
GSA	Gravitational Search Algorithm
GWO	Grey wolf Optimization
GWO-PSO	Grey wolf-particle swarm optimization
HFA	Hybrid Firefly Algorithm
IFA	Intuitionistic fuzzy Approach
IFS	Intuitionistic fuzzy set
IFO	Intuitionistic fuzzy optimization
ISS	Improved Scatter Search
MDE	Multi-Objective Differential Evolution
MF	Membership Function
MHBMO	Modified Honey Bee Mating Optimization
MO	Multi-objectives
MOEELD	Multi-Objective Economic and Emission Load Dispatch
MONLP	Multi-objective Nonlinear Programming
MOGA	Multi-Objective Genetic Algorithm
MPSO	Modified Particle Swarm Optimization
NMF	Non-membership functions
NSGA-II	Non-dominated Sorting Genetic algorithm-II
PFP	Pythagorean Fuzzy Programming
PSO	Particle Swam optimization
SMO	Spider-Monkey Optimization
SPEA	Strength Pareto Evolutionary Approaches
WEO	Water Evaporation Optimization
WOA	Whale Optimization Algorithm

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