

ON THE CHARACTERIZATION OF $(1,2)S_p$ -CONTINUOUS FUNCTION VIA $(1,2)S_p$ -OPEN SETS IN BITOPOLOGICAL SPACES

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Abstract:

The purpose of this paper is to introduce the concept of $(1,2)S_p$ -continuous function in bitopological spaces. The basic properties and some of its characterizations and relationships are investigated.

Key words:

$(1,2)$ semi-open, $(1,2)$ pre-open, $(1,2)$ pre-closed, $(1,2)S_p$ -open, $(1,2)S_p$ -closed, $(1,2)S_p$ -continuous function.

How to cite this article: Dhanalakshmi S, Maheswari M, Durga Devi N. On the Characterization of $(1,2)S_p$ -Continuous Function via $(1,2)S_p$ -Open Sets in Bitopological Spaces. Int J Drug Deliv Technol. 2026;16(61s):837-842. DOI: 10.25258/ijddt.16.61s.91

2026;16(61s):837-842. DOI: 10.25258/ijddt.16.61s.91

Source of support: Nil.

Conflict of interest: None

AMS Classifications: 54A05, 51A10.

1. INTRODUCTION

Continuity is a basic concept of the study in topological spaces. In 1963, Kelly [2] introduced Bitopology. In 2010, Shareef [5] introduced a class of continuous function called S_p -continuous function in terms of S_p -open sets in topological spaces. In 2017, Shareef et.al [6] introduced $(1,2)S_p$ -open sets in bitopological spaces. In this paper, define new class of continuous function called $(1,2)S_p$ -continuous function in bitopological spaces and study some of their characterizations using $(1,2)S_p$ -open sets.

2. PRELIMINARIES

Definition 2.1. [3] A subset A of X is called

- i. $(1,2)\alpha$ -open if $A \subseteq \tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(\tau_1\text{-Int}(A)))$.

- ii. $(1,2)$ semi-open if $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\text{-Int}(A))$.
- iii. $(1,2)$ pre-open if $A \subseteq \tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$.
- iv. $(1,2)$ regular-open if $A = \tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$.

The collection of all $(1,2)\alpha$ -open, $(1,2)$ semi-open, $(1,2)$ pre-open and $(1,2)$ regular-open sets are denoted by $(1,2)\alpha O(X)$, $(1,2)SO(X)$, $(1,2)PO(X)$ and $(1,2)RO(X)$ respectively.

Definition 2.2. [4] A subset A of X is called

- i. $(1,2)\alpha$ -closed if $\tau_1\text{Cl}(\tau_1\tau_2\text{-Int}(\tau_1\text{-Cl}(A))) \subseteq A$.
- ii. $(1,2)$ semi-closed if $\tau_1\tau_2\text{-Int}(\tau_1\text{-Cl}(A)) \subseteq A$.
- iii. $(1,2)$ pre-closed if $\tau_1\text{-Cl}(\tau_1\tau_2\text{-Int}(A)) \subseteq A$.
- iv. $(1,2)$ regular-closed if $A = \tau_1\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$.

The set of all $(1,2)\alpha$ -closed, $(1,2)$ -semi-closed, $(1,2)$ pre-closed and $(1,2)$ regular-closed sets are denoted by $(1,2)\alpha\text{CL}(X)$, $(1,2)\text{SCL}(X)$, $(1,2)\text{PCL}(X)$ and $(1,2)\text{RCL}(X)$ respectively. Also, for any subset A of X , the $(1,2)\alpha$ -closure, $(1,2)$ -semi closure, $(1,2)$ pre-closure and $(1,2)$ regular closure of A is denoted as $(1,2)\alpha\text{Cl}(X)$, $(1,2)\text{SCL}(X)$, $(1,2)\text{PCl}(X)$ and $(1,2)\text{RCl}(X)$ respectively.

Definition 2.3. [6] A bitopological space (X, τ_1, τ_2) is said to be τ_1 -locally indiscrete if every τ_1 -open subset of X is τ_1 -closed.

Definition 2.4. [4] A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be $(1,2)\alpha$ -continuous (resp. $(1,2)$ semi-continuous) if the inverse image of each $(1,2)\alpha$ -open set in Y is $(1,2)\alpha$ -open set in X (resp. $(1,2)$ semi-open set in Y is $(1,2)$ semi-open in X).

Lemma 2.5. [7] A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is bijective if and only if $f(X \setminus A) = (Y \setminus f(A))$ for each subset A of X .

Definition 2.6. [6] A $(1,2)$ semi-open set A of X is called $(1,2)S_p$ -open set if for each $x \in A$, there exists a $(1,2)$ pre-closed set F such that $x \in F \subseteq A$.

The complement of a $(1,2)S_p$ -open set is a $(1,2)S_p$ -closed set and the family of all $(1,2)S_p$ -open (resp. $(1,2)S_p$ -closed) subsets of X is denoted by $(1,2)S_p\text{-O}(X)$ (resp. $(1,2)S_p\text{-CL}(X)$).

Definition 2.7. [6] A point $x \in X$ is said to be a $(1,2)S_p$ -interior point of A , if there exists a $(1,2)S_p$ -open set U containing x such that $x \in U \subseteq A$. The union of all $(1,2)S_p$ -open sets contained in A is said to be $(1,2)S_p$ -interior of A and it is denoted by $(1,2)S_p\text{-Int}(A)$.

Definition 2.8. [6] Let A be a set in a bitopological space X . A point $x \in X$ is said to be a $(1,2)S_p$ -closure of A if and only if $A \cap U = \emptyset$, for every $(1,2)S_p$ -open set U

containing x . The intersection of all $(1,2)S_p$ -closed sets F containing A is called the $(1,2)S_p$ -closure of A and is denoted by $(1,2)S_p\text{-Cl}(A)$.

Lemma 2.9. [1] Let Y be a subspace of the bitopological space X and Y is $(1,2)$ regular-closed subset of X , if A is $(1,2)S_p$ -open subset of Y , then A is $(1,2)S_p$ -open set in X .

Proposition 2.10. [1] A $(1,2)$ semi-closed subset A of a bitopological space X is $(1,2)S_p$ -closed if and only if A is an intersection of $(1,2)$ pre-open set.

Proposition 2.11. [6] A subset A of a bitopological space (X, τ_1, τ_2) is $(1,2)S_p$ -open if and only if A is $(1,2)$ semi-open and it is the union of $(1,2)$ pre-closed sets.

Theorem 2.12. [6] Let $\{A_\alpha: \alpha \in \Delta\}$ be a family of $(1,2)S_p$ -open sets in a bitopological space (X, τ_1, τ_2) . Then $\bigcup_{\alpha \in \Delta} A_\alpha$ is also a $(1,2)S_p$ -open set in X .

Remark 2.13. [1] Any intersection of $(1,2)S_p$ -closed sets of a bitopological space X is $(1,2)S_p$ -closed.

Definition 2.14. [1] A subset N of a space X is said to be $(1,2)S_p$ -neighborhood ($(1,2)S_p$ -nbhd) of a point $x \in X$, if there exists a $(1,2)S_p$ -open set U such that $x \in U \subseteq N$.

Proposition 2.15. [1] Let X be a bitopological space and let A and B be two subsets of X , then:

- i. If $A \subseteq B$, where A is $(1,2)S_p$ -neighborhood of $x \in X$, then B is also $(1,2)S_p$ -neighborhood of x .
- ii. An arbitrary union of $(1,2)S_p$ -neighborhood of a point $x \in X$, is also $(1,2)S_p$ -neighborhood of x .
- iii. If A is $(1,2)S_p$ -open set if and only if it is $(1,2)S_p$ -neighborhood of each of its point.
- iv. If A is $(1,2)S_p$ -neighborhood of a point $x \in X$, then A is $(1,2)$ semi-neighborhood of x .

Lemma 2.16. [1] Let A and B be subsets of a bitopological space X , then

- i. $(1,2)S_p\text{-Cl}(A)$ is the smallest $(1,2)S_p$ -closed set containing A .
- ii. A is $(1,2)S_p$ -closed set if and only if $A = (1,2)S_p\text{-Cl}(A)$.
- iii. If $A \subseteq B$, then $(1,2)S_p\text{-Cl}(A) \subseteq (1,2)S_p\text{-Cl}(B)$.
- iv. $(1,2)S_{p\text{-Cl}}(A) \subseteq (1,2)S_p\text{-Cl}(A)$.

Proposition 2.17. [1] Let X be a bitopological space and $A \subset X$, then

- i. $(X \setminus (1,2)S_p\text{-Int}(A)) = (1,2)S_p\text{-Cl}(A)$.
- ii. $(1,2)S_p\text{-Cl}(A) = (1,2)S_p\text{-Int}(X \setminus A)$.
- iii. $(1,2)S_p\text{-Int}(A) = (X \setminus (1,2)S_p\text{-Int}(X \setminus A))$.

3. $(1,2)S_p$ -Continuous Function

Definition 3.1. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)S_p$ -continuous at $x \in X$ if for every σ_1 -open set V in Y containing $f(x)$, there exists a $(1,2)S_p$ -open set U in X containing x such that $f(U) \subseteq V$. Hence, f is $(1,2)S_p$ -continuous if it is $(1,2)S_p$ -Continuous at each point $x \in X$.

Example 3.2. Let $X = \{a, b, c\}$ with two topologies $\tau_1 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X\}$. Hence, $(1,2)S\text{-O}(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $(1,2)P\text{-CL}(X) = \{X, \emptyset, \{b\}, \{a, b\}, \{b, c\}\}$, $(1,2)S_p\text{-O}(X) = \{\emptyset, X, \{a, b\}, \{b, c\}\}$.

Also, let $Y = \{a, b, c\}$ with two topologies $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma_2 = \{\emptyset, Y\}$. Hence, $(1,2)S\text{-O}(Y) = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $(1,2)P\text{-CL}(Y) = \{Y, \emptyset, \{c\}, \{a, c\}, \{b, c\}\}$, $(1,2)S_p\text{-O}(Y) = \{\emptyset, Y, \{a, c\}, \{b, c\}\}$. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is defined as $f(a) = f(b) = f(c) = a$. Here, f is a $(1,2)S_p$ -continuous function.

Proposition 3.3. Let X be a bitopological space and let A, B be two subsets of X , then:

- i. if $A \subset B$, where A is $(1,2)S_p$ -neighborhood of $x \in X$, then B is also $(1,2)S_p$ -neighborhood of x .
- ii. an arbitrary union of $(1,2)S_p$ -neighborhood of a point $x \in X$ is also $(1,2)S_p$ -neighborhood of x .
- iii. if A is $(1,2)S_p$ -open set if and only if it is $(1,2)S_p$ -neighborhood of each of its point.

Proof. (i) Let $A \subset B$, where A is $(1,2)S_p$ -neighborhood of $x \in X$. Then there exists a $(1,2)S_p$ -open set U containing x such that $x \in U \subset A \subset B$ which implies B is $(1,2)S_p$ -neighborhood of x .

The proofs of **(ii)** and **(iii)** are obvious.

Proposition 3.4. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1,2)S_p$ -continuous if and only if for every σ_1 -open subset V of Y , $f^{-1}(V)$ is a $(1,2)S_p$ -open set in X .

Proof. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)S_p$ -continuous and let V be any σ_1 -open set in Y . Then show that $f^{-1}(V)$ is a $(1,2)S_p$ -open set in X . If $f^{-1}(V) = \emptyset$, then $f^{-1}(V)$ is $(1,2)S_p$ -open set in X . Suppose, if $f^{-1}(V) \neq \emptyset$, then there exists a $x \in f^{-1}(V)$. Also, since f is $(1,2)S_p$ -continuous, there exists a $(1,2)S_p$ -open set U_x in X such that $x \in U_x$ and $f(U_x) \subseteq V$ which implies that $x \in U_x \subseteq f^{-1}(V)$. Thus $f^{-1}(V)$ is $(1,2)S_p$ -neighborhood of each of its points. Hence, by proposition 3.3, $f^{-1}(V)$ is a $(1,2)S_p$ -open set in X .

Conversely, let for each σ_1 -open subset V of Y , $f^{-1}(V)$ is a $(1,2)S_p$ -open set in X . Then show that f is $(1,2)S_p$ -continuous at $x \in X$. Let $x \in X$. Then $f(x) \in V$ which implies that $x \in f^{-1}(V)$. Hence, $f^{-1}(V)$ is $(1,2)S_p$ -open set in X containing x and $f(f^{-1}(V)) \subseteq V$. Hence, f is $(1,2)S_p$ -continuous.

Remark 3.5. Every $(1,2)S_p$ -continuous is $(1,2)$ semi-continuous but the converse is not true as shown in the following example.

Example 3.6. In Example 3.2, $(1,2)S\text{-O}(X) = \{\Phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $(1,2)P\text{-CL}(X) = \{X, \Phi, \{c\}, \{a, c\}, \{b, c\}\}$, $(1,2)S_p\text{-O}(X) = \{\Phi, X, \{a, c\}, \{b, c\}\}$.

Also, let $Y = \{a, b, c\}$ with two topologies $\sigma_1 = \{\Phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $\sigma_2 = \{\Phi, Y\}$. Hence, $(1,2)S\text{-O}(Y) = \{\Phi, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$, $(1,2)P\text{-CL}(Y) = \{Y, \Phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$, $(1,2)S_p\text{-O}(Y) = \{\Phi, Y, \{b\}, \{a, c\}\}$. A function $f: X \rightarrow Y$ is defined as $f(a) = b, f(b) = a, f(c) = c$. Here, f is $(1,2)$ semi-continuous but not $(1,2)S_p$ -continuous.

Theorem 3.7. The following statements are equivalent for the function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$:

- i. $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is $(1,2)S_p$ -continuous.
- ii. the inverse image of every σ_1 -open set in Y is $(1,2)S_p$ -open set in X .
- iii. the inverse image of every σ_1 -closed set in Y is $(1,2)S_p$ -closed set in X .
- iv. for each $A \subseteq X, f((1,2)S_p\text{-Cl}(A)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(A))$.
- v. for each $A \subseteq X, \sigma_1\text{-Int}(f(A)) \subseteq f((1,2)S_p\text{-Int}(A))$.
- vi. for each $B \subseteq Y, (1,2)S_p\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$.
- vii. for each $B \subseteq Y, f^{-1}(\sigma_1\text{-Int}(B)) \subseteq (1,2)S_p\text{-Int}(f^{-1}(B))$.

Proof. (i) \Rightarrow (ii) It is clear from Proposition 3.4.

(ii) \Rightarrow (iii) Let B be any σ_1 -closed subset of Y , then $(Y - B)$ is σ_1 -open in Y , and then by (ii), $f^{-1}(Y - B) = (X - f^{-1}(B))$ is a $(1,2)S_p$ -open set in X . Thus $f^{-1}(B)$ is a $(1,2)S_p$ -closed set in X .

(iii) \Rightarrow (iv) Let For each $A \subseteq X$, then $f(A) \subseteq Y$. since $f(A) \subseteq \sigma_1\sigma_2\text{-Cl}(f(A))$ and by

(iii), $f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A)))$ is a $(1,2)S_p$ -closed set in X and $A \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A)))$, then $(1,2)S_p\text{-Cl}(A) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A)))$ which implies that $f((1,2)S_p\text{-Cl}(A)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(A))$.

(iv) \Rightarrow (v) Let $A \subseteq X$, then $(X - A) \subseteq Y$ and then by (iv), $f((1,2)S_p\text{-Cl}(X - A)) \subseteq \sigma_1\sigma_2\text{-Cl}(f(X - A))$. Therefore by Lemma 2.5, $f(X - (1,2)S_p\text{-Int}(A)) \subseteq \sigma_1\sigma_2\text{-Cl}(Y - f(A))$. This implies that $(Y - f((1,2)S_p\text{-Int}(A))) \subseteq (Y - (\sigma_1\text{-Int}(f(A))))$ implies $\sigma_1\text{-Int}(f(A)) \subseteq f((1,2)S_p\text{-Int}(A))$.

(v) \Rightarrow (vi) Let $B \subseteq Y$, then $f^{-1}(B) \subseteq X$ implies $(X - f^{-1}(B)) \subseteq X$. Therefore by (v), $\sigma_1\text{-Int}(f(X - f^{-1}(B))) \subseteq f((1,2)S_p\text{-Int}(X - f^{-1}(B)))$, then $\sigma_1\text{-Int}(Y - f(f^{-1}(B))) \subseteq f(X - (1,2)S_p\text{-Cl}(f^{-1}(B)))$ which implies that $\sigma_1\text{-Int}(Y - B) \subseteq (Y - f((1,2)S_p\text{-Cl}(f^{-1}(B))))$. Then $(Y - \sigma_1\sigma_2\text{-Cl}(B)) \subseteq (Y - f((1,2)S_p\text{-Cl}(f^{-1}(B))))$ implies $f((1,2)S_p\text{-Cl}(f^{-1}(B))) \subseteq \sigma_1\sigma_2\text{-Cl}(B)$. Hence, $(1,2)S_p\text{-Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$.

(vi) \Rightarrow (vii) Let $B \subseteq Y$, then $(Y - B) \subseteq Y$. Therefore by (vi), $(1,2)S_p\text{-Cl}(f^{-1}(Y - B)) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - B))$, then $(1,2)S_p\text{-Cl}(X - f^{-1}(B)) \subseteq f^{-1}(Y - \sigma_1\text{-Int}(B))$ and by Proposition 2.17, $(X - (1,2)S_p\text{-Int}(f^{-1}(B))) \subseteq (X - f^{-1}(\sigma_1\text{-Int}(B)))$ which implies $f^{-1}(\sigma_1\text{-Int}(B)) \subseteq (1,2)S_p\text{-Int}(f^{-1}(B))$.

(vii) \Rightarrow (i) Let $x \in X$ and B be any σ_1 -open subset of Y containing $f(x)$, then by (vii), $f^{-1}(\sigma_1\text{-Int}(B)) \subseteq (1,2)S_p\text{-Int}(f^{-1}(B))$ which implies $f^{-1}(B) \subseteq (1,2)S_p\text{-Int}(f^{-1}(B))$. Thus $f^{-1}(B)$ is $(1,2)S_p$ -open set in X containing x such that $f(f^{-1}(B)) \subseteq B$. Hence, f is a $(1,2)S_p$ -continuous function.

Corollary 3.8. If $f: X \rightarrow Y$ is $(1,2)$ semi-continuous and X is τ_1 -locally indiscrete, then f is $(1,2)S_p$ -continuous.

Theorem 3.9. If the function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a surjective function, then the following statements are equivalent:

- i. f is (1,2) S_p -continuous.
- ii. for every $B \subseteq Y$, $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(B))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$.
- iii. for every $B \subseteq Y$, $f^{-1}(\sigma_1\text{-Int}(B)) \subseteq \tau_1\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(B)))$ and $f^{-1}(\sigma_1\text{-Int}(B)) = \bigcup_{i \in \Delta} F_i$, where $F_i \in (1,2)\text{P-CL}(X)$.
- iv. for every $A \subseteq X$, $f(\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(A))) \subseteq \sigma_1\sigma_2\text{-Cl}(f(A))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A))) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$.

Proof. (i) \Rightarrow (ii) Let $B \subseteq Y$, then $\sigma_1\sigma_2\text{-Cl}(B)$ is σ_1 -closed in Y . Since f is (1,2) S_p -continuous, then by Theorem 3.7, $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ is (1,2) S_p -closed in X which implies $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ is (1,2)semi-closed and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$. Thus $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$. Hence, $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ where $V_i \in (1,2)\text{P-O}(X)$.

(ii) \Rightarrow (i) Let B be a σ_1 -closed subset of Y , then by (ii), $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(B))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) = f^{-1}(B)$ and $f^{-1}(B) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$ which implies that $f^{-1}(B) \in (1,2)\text{SCL}(X)$ and $f^{-1}(B) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{PO}(X)$. Thus $f^{-1}(B)$ is (1,2) S_p -closed in X . Hence, by Theorem 3.7, f is (1,2) S_p -continuous.

(i) \Rightarrow (iii) Let $B \subseteq Y$, $\sigma_1\text{-Int}(B)$ is σ_1 -open in Y . Since f is (1,2) S_p -continuous. Therefore $f^{-1}(\sigma_1\text{-Int}(B))$ is (1,2) S_p -open in X which implies that $f^{-1}(\sigma_1\text{-Int}(B)) \in (1,2)\text{S-O}(X)$ and $f^{-1}(\sigma_1\text{-Int}(B)) = \bigcup_{i \in \Delta} F_i$, where $F_i \in (1,2)\text{P-CL}(X)$. Thus $f^{-1}(\sigma_1\text{-Int}(B)) \subseteq \tau_1\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(B)))$ and

$f^{-1}(\sigma_1\text{-Int}(B)) = \bigcup_{i \in \Delta} F_i$, where $F_i \in (1,2)\text{P-CL}(X)$.

(iii) \Rightarrow (i) Let B be subset of Y , then $\sigma_1\text{-Int}(B) = B$ and by (iii), $f^{-1}(B) \subseteq \tau_1\text{-Cl}(\tau_1\tau_2\text{-Int}(f^{-1}(B)))$ and $f^{-1}(B) = \bigcup_{i \in \Delta} F_i$, where $F_i \in (1,2)\text{P-CL}(X)$ implies $f^{-1}(B) \in (1,2)\text{S}_p\text{-O}(X)$. Hence, f is (1,2) S_p -continuous.

(ii) \Rightarrow (iv) Let $A \subseteq X$, then $f(A) \subseteq Y$ and then by (ii), $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(f(A)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A)))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A))) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$. Therefore $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(f(A)))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A)))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A))) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$. Hence, $f(\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(f(A)))) \subseteq \sigma_1\sigma_2\text{-Cl}(f(A))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(A))) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$.

(iv) \Rightarrow (ii) Let $B \subseteq Y$, then $f^{-1}(B) \subseteq X$. Therefore by (iv), $f^{-1}(\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(B)))) \subseteq (\sigma_1\sigma_2\text{-Cl}(f(f^{-1}(B)))) \subseteq \sigma_1\sigma_2\text{-Cl}(B)$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(f(f^{-1}(B)))) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$ which implies that $\tau_1\text{-Int}(\tau_1\tau_2\text{-Cl}(f^{-1}(B))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B))$ and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)) = \bigcap_{i \in \Delta} V_i$ where $V_i \in (1,2)\text{P-O}(X)$.

ACKNOWLEDGEMENT

I acknowledge the editor in chief for providing the timing help to publish my paper.

CONCLUSION

In this paper, the new notion of (1,2) S_p -continuous function was introduced and some of its characterizations are discussed. Later Research be reached out with certain applications.

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