RESEARCH PAPER

"Thermo-Viscoelastic Vibration Study of Non-Homogeneous Rectangular Plates Having Exponentially Varying Thickness Using Galerkin Approximation"

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ABSTRACT

The vibration characteristics of structural plates are of considerable importance in engineering systems such as aircraft components, nuclear reactors, communication devices, and mechanical structures. Variations in material composition, thickness distribution, and temperature field can significantly influence the dynamic behavior of such elements. In the present work, a thermo-viscoelastic analysis of a non-homogeneous rectangular plate with exponentially varying thickness is carried out. The plate is assumed to possess mixed boundary conditions, with two opposite edges clamped and the remaining two edges simply supported.

The material behavior is represented through Kelvin's viscoelastic model, which accounts for both elastic and damping effects. A mathematical model governing the transverse vibration of the plate is developed by incorporating non-homogeneity and thermal effects. Galerkin's approximation method is employed to transform the governing differential equation into a frequency equation using an admissible two-term deflection function. The influence of exponential thickness variation, thermal gradient, aspect ratio, and non-homogeneity parameter on the vibrational response is investigated in detail. Numerical results are presented for the first two vibration modes, and the corresponding values of deflection and logarithmic decrement are determined. The study indicates that changes in temperature distribution and geometric parameters have a pronounced impact on the natural frequencies and damping characteristics of the plate. The findings may contribute to the optimal design of lightweight structural components operating under thermal environments.

Keywords: Thermo-viscoelasticity, Non-homogeneous plate, Exponential thickness variation, Thermal effects, Galerkin method, Natural frequency, Logarithmic decrement, Structural vibration.

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1. INTRODUCTION

Rectangular plates constitute an essential class of structural elements and are extensively employed in aerospace engineering, nuclear reactors, marine structures, communication systems, mechanical devices, and civil engineering constructions. The dynamic behavior of these components is of considerable importance because excessive vibration may adversely affect structural integrity, service life, and operational efficiency. In many practical applications, plate structures operate under elevated temperature environments and possess variable thickness distributions, making the study of their vibration characteristics a topic of significant engineering interest.Thermal loading can considerably alter the mechanical properties of materials. As temperature increases, parameters such as Young's modulus and stiffness undergo noticeable changes, thereby influencing the natural frequencies and damping behavior of structural components. Similarly, variations in thickness and material non-homogeneity modify the mass and rigidity distribution within a plate, resulting in changes in its vibrational response. Therefore, a combined investigation of thermal effects, viscoelasticity, non-homogeneity, and thickness variation is essential for the accurate prediction of structural performance.

A considerable amount of research has been devoted to the vibration analysis of plates subjected to thermal environments. Tomar and Gupta [1] investigated the influence of temperature on the frequencies of orthotropic rectangular plates with linearly varying thickness. The effect of thermal gradients on the natural frequencies of rectangular plates was examined by Fnconneau and Marangoni [2]. Rao and Satyanarayana [3] studied frequency variations in tapered rectangular plates subjected to thermal gradients. The vibration behavior of orthotropic elliptic plates with non-uniform thickness and temperature distributions was analyzed by Tomar and Gupta [4], while the same authors [5] further examined thermal effects on axisymmetric vibrations of orthotropic circular plates with parabolically varying thickness.Additional contributions have been made in the area of variable-thickness plate vibrations. Bambill et al. [6] investigated transverse vibrations of orthotropic rectangular plates with linearly varying thickness and a free edge. Li and Zhou [7] employed a shooting method to analyze nonlinear vibration and thermal buckling of heated orthotropic circular plates. Hoff [8] highlighted the significance of temperature-dependent material properties in structural analysis, demonstrating that elastic coefficients become functions of spatial variables under thermal conditions. Pronsato et al. [9] studied the transverse vibration characteristics of rectangular membranes having discontinuously varying density distributions.

Subsequent investigations extended these studies to viscoelastic and non-homogeneous plate structures. Gupta and Kumar [10,11] examined thermal effects on non-homogeneous viscoelastic rectangular plates with parabolic and linear thickness variations, respectively. Gupta and Khanna [12] analyzed vibration characteristics of viscoelastic rectangular plates with bidirectional linear thickness variation. The free vibration of clamped viscoelastic rectangular plates possessing exponential thickness variation in two directions was reported by Gupta et al. [13]. Thermal vibration analysis of clamped viscoelastic rectangular plates with linear thickness variation was further explored by Gupta and Kaur [14]. Gupta, Kumar and Gupta [15] investigated vibration behavior of viscoelastic parallelogram plates with parabolic thickness variation, while Gupta and Singhal [16] studied the influence of non-homogeneity on thermally induced vibration of orthotropic viscoelastic rectangular plates.Although extensive literature exists on thermal vibration analysis of homogeneous and non-homogeneous plates with linear or parabolic thickness variation, comparatively less attention has been paid to thermo-viscoelastic rectangular plates possessing exponentially varying thickness. Such plate configurations are encountered in lightweight engineering structures where optimized stiffness-to-weight ratios are desired.

The present study focuses on the vibration behavior of a non-homogeneous viscoelastic rectangular plate having exponentially varying thickness under the influence of a thermal gradient. Kelvin's viscoelastic model is adopted to represent material damping effects. The plate is assumed to be clamped along two opposite edges and simply supported along the remaining two edges. Galerkin's approximation technique is employed to obtain the governing frequency equation. The influence of thermal gradient, non-homogeneity parameter, exponential thickness variation parameter, and aspect ratio on the first two modes of vibration is investigated through the evaluation of deflection and logarithmic decrement. The results provide valuable information for the design and analysis of thermally loaded structural components used in advanced engineering applications.

2. ANALYSIS AND EQUATION OF MOTION

The governing differential equation of transverse motion of a visco-elastic non-homogeneous plate of variable thickness in Cartesian co-ordinates, as by Gupta and Kumar [11],

$$\widetilde{D}[D_1(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4})+2\frac{\partial D_1}{\partial x}(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2})+2\frac{\partial D_1}{\partial y}(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y})$$
$$+\frac{\partial^2 D_1}{\partial x^2}(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2})+\frac{\partial^2 D_1}{\partial y^2}(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2})+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}]+\rho h\frac{\partial^2 w}{\partial t^2}=0 \quad (1)$$

The solution of equation (1) can be sought in the form of product of two functions as follows:

$$w=w(x,y,t)=W(x,y)T(t) \quad (2)$$

where W(x,y) is the deflection function and T(t) is time function.

Using equation (2) in (1) after simplification and taking both sides of are equal to a constant p^2, we have

$$D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right)$$
$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}-\rho h p^2 W=0 \quad (3)$$

and

$$\frac{d^2T}{dt^2}+p^2\widetilde{D}T=0 \quad (4)$$

Equation (3) is differential equation of transverse motion and equation (4) is a differential equation of time function of free vibration of non-homogeneous visco-elastic plate of with varying exponentially thickness.

It is assumed that the non-homogeneous visco-elastic rectangular plate is subjected to a steady one-dimensional temperature distribution along x-direction.

$$\boldsymbol{\tau} = \boldsymbol{\tau_0}\left(\mathbf{1} - \frac{x}{a}\right) \quad (5)$$

where $\boldsymbol{\tau}$ denotes the temperature excess above the reference temperature at any point at distance $\frac{x}{a}$ and $\boldsymbol{\tau_0}$ denotes the temperature excess above reference temperature at the end i.e. x=a.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$\boldsymbol{E} = \boldsymbol{E_0}(\mathbf{1} - \boldsymbol{\gamma\tau}) \quad (6)$$

where E_0 is the value of the Young's modulus at some reference temperature i.e. $\boldsymbol{\tau} = \mathbf{0}$ and $\boldsymbol{\gamma}$ is the slope of the variation of E with $\boldsymbol{\tau}$. The modulus variations, in view of expression (5) and (6) become

$$\boldsymbol{E}(\boldsymbol{x}) = \boldsymbol{E_0}\left(\mathbf{1} - \boldsymbol{\alpha}\left(\mathbf{1} - \frac{x}{a}\right)\right) \quad (7)$$

where $\boldsymbol{\alpha} = \gamma\boldsymbol{\tau_0}$ $(\mathbf{0} \leq \boldsymbol{\alpha} < \mathbf{1})$, a parameter known as temperature gradient.

It is assumed that exponentially thickness variation and non-homogeneity varies in the x-direction only, consequently, the thickness, density and flexural rigidity of the plate become functions of x only. Let the two opposite edges y=0 and y=b of the plate be simply supported**.** So that the plate when undergoing free transverse vibrations with frequency p may have levy-type solution.[11]

$$\boldsymbol{W}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{W_1}(\boldsymbol{x})\boldsymbol{sin}\left(\frac{\pi y}{b}\right) \quad (8)$$

Substitution of equation (8) in (3)

$$\boldsymbol{D_1}\left[\frac{\partial^4 \boldsymbol{W_1}}{\partial \boldsymbol{x}^4} - 2\left(\frac{\boldsymbol{\pi}}{\boldsymbol{b}}\right)^2 \frac{\partial^2 \boldsymbol{W_1}}{\partial \boldsymbol{x}^2} + \left(\frac{\boldsymbol{\pi}}{\boldsymbol{b}}\right)^4 \boldsymbol{W_1}\right] + 2\frac{\partial \boldsymbol{D_1}}{\partial \boldsymbol{x}}\left[\frac{\partial^3 \boldsymbol{W_1}}{\partial \boldsymbol{x}^3} - \left(\frac{\boldsymbol{\pi}}{\boldsymbol{b}}\right)^2 \frac{\partial \boldsymbol{W_1}}{\partial \boldsymbol{x}}\right]$$
$$+\frac{\partial^2 \boldsymbol{D_1}}{\partial \boldsymbol{x}^2}\left[\frac{\partial^2 \boldsymbol{W_1}}{\partial \boldsymbol{x}^2} - \boldsymbol{\nu}\left(\frac{\boldsymbol{\pi}}{\boldsymbol{b}}\right)^2 \boldsymbol{W_1}\right] - \boldsymbol{\rho p^2 h W_1} = \mathbf{0} \quad (9)$$

Thus (9) reduces to a form independent of y and upon introducing the non-dimensional variables

$$\overline{\boldsymbol{H}} = \frac{h}{a}, \quad \bar{\boldsymbol{\rho}} = \frac{\rho}{a}, \quad \bar{\boldsymbol{D}} = \frac{D_1}{a^3}, \quad \bar{\boldsymbol{W}} = \frac{W_1}{a}, \quad \boldsymbol{X} = \frac{x}{a} \quad (10)$$

It becomes, in non-dimensional form,

$$\bar{\boldsymbol{D}}\left(\frac{\partial^4 \bar{W}}{\partial X^4} - \mathbf{2}\boldsymbol{r}^2\frac{\partial^2 \bar{W}}{\partial X^2} + \boldsymbol{r}^4\bar{\boldsymbol{W}}\right) + \mathbf{2}\frac{\partial \bar{D}}{\partial X}\left(\frac{\partial^3 \bar{W}}{\partial X^3} - \boldsymbol{r}^2\frac{\partial \bar{W}}{\partial X}\right) + \frac{\partial^2 \bar{D}}{\partial X^2}\left(\frac{\partial^2 \bar{W}}{\partial X^2} - \boldsymbol{\nu r^2}\bar{\boldsymbol{W}}\right) - \boldsymbol{a}^3\bar{\boldsymbol{\rho}}\boldsymbol{p}^2\bar{\boldsymbol{H}}\bar{\boldsymbol{W}} = \mathbf{0} \quad (11)$$

where,

$$\boldsymbol{r} = \frac{\pi a}{b} \quad (12)$$

In view of the previous assumption, the thickness varies exponentially in the x-direction and non-homogeneity varies linearly in the x-direction only, one assumes,

$$\overline{\boldsymbol{H}}(\boldsymbol{X}) = \boldsymbol{H_0}\left(\boldsymbol{e}^{\boldsymbol{\beta X}}\right) \quad (13)$$

where $\boldsymbol{\beta}$ is the taper constant and $\boldsymbol{H_0} = \overline{\boldsymbol{H}}\big|_{x=0}$

and

$$\bar{\boldsymbol{\rho}} = \boldsymbol{\rho_0}(\mathbf{1} - \boldsymbol{\alpha_3 X}) \quad (14)$$

where is the non-homogeneity constant and $\boldsymbol{\rho_0} = \bar{\boldsymbol{\rho}}|_{x=0}$

The rigidity given by equation

$$\bar{\boldsymbol{D}} = \frac{E_0 H_0^3 \left(e^{\beta X}\right)^3 [1 - \alpha(1 - X)]}{12(1 - \nu^2)} \quad (15)$$

Using equation (13), (14) and (15) in equation (11), one obtains

$$\Phi_1\left(\frac{\partial^4 \bar{W}}{\partial X^4} - 2r^2\frac{\partial^2 \bar{W}}{\partial X^2} + r^4\bar{W}\right) + \Phi_2\left(\frac{\partial^3 \bar{W}}{\partial X^3} - r^2\frac{\partial \bar{W}}{\partial X}\right) + \Phi_3\left(\frac{\partial^2 \bar{W}}{\partial X^2} - \nu r^2\bar{W}\right)$$
$$-\boldsymbol{p}^2(\mathbf{1} - \boldsymbol{\alpha_3 X})\boldsymbol{\ell}\bar{\boldsymbol{W}} = \mathbf{0} \quad (16)$$

Here,

$$\Phi_1 = (\mathbf{1} - \boldsymbol{\alpha} + \boldsymbol{\alpha X})(\mathbf{1} - \boldsymbol{\beta X})^2, \ \Phi_2 = \mathbf{2}(\mathbf{1} - \boldsymbol{\beta X})(\mathbf{3}\boldsymbol{\alpha\beta} - \mathbf{3}\boldsymbol{\beta} + \boldsymbol{\alpha} - \mathbf{4}\boldsymbol{\alpha\beta X})$$
$$\Phi_3 = \mathbf{6}\boldsymbol{\beta}(\boldsymbol{\beta} - \boldsymbol{\alpha} - \boldsymbol{\alpha\beta} + \mathbf{2}\boldsymbol{\alpha\beta X}), \quad \boldsymbol{\ell} = \frac{12(1-\nu^2)\rho_0 a^3}{E_0 H_0^2}$$

and p^2 is a frequency parameter.

The deflection function $\bar{\boldsymbol{W}}(\boldsymbol{X})$, of plate is assumed to be a finite sum of characteristic functions $\bar{\boldsymbol{W}}_{\boldsymbol{k}}(\boldsymbol{X})$

$$\bar{\boldsymbol{W}}(\boldsymbol{X}) = \sum_{k=1}^{n} \boldsymbol{C_k}\bar{\boldsymbol{W}}_{\boldsymbol{k}}(\boldsymbol{X}) \quad (17)$$

where C_k's are undetermined coefficients and $\bar{\boldsymbol{W}}_{\boldsymbol{k}}$are characteristic function chosen to satisfy the boundary conditions of plate. For a rectangular plate clamped at both the edges X=0 and X=1 (and simply supported at the remaining two edges)

$$\bar{\boldsymbol{W}}|_{X=0} = \frac{\partial \bar{W}}{\partial X}\bigg|_{X=0} = \mathbf{0}, \qquad \bar{\boldsymbol{W}}|_{X=1} = \frac{\partial \bar{W}}{\partial X}\bigg|_{X=1} = \mathbf{0} \quad (18)$$

Using Galerkin's technique, one has

$$\int \boldsymbol{L}[\bar{\boldsymbol{W}}(\boldsymbol{X})]\bar{\boldsymbol{W}}(\boldsymbol{X})\boldsymbol{dX} = \mathbf{0} \quad (19)$$

where $\boldsymbol{L}[\bar{\boldsymbol{W}}(\boldsymbol{X})]$ is left hand side of equation (16). Taking the first two terms of the sum (17) for the function $\bar{\boldsymbol{W}}(\boldsymbol{X})$ as a solution of equation (16),

$$\bar{W}(X) = C_1X^2(1-X)^2 + C_2X^3(1-X)^3 \quad (20)$$

Substituting equation (20) into equation (19) and then eliminating C_1 and C_2 given the frequency equation as,

$$\begin{vmatrix} 2(A_1+B_1p^2) & (A_2+B_2p^2) \\ (A_2+B_2p^2) & 2(A_3+B_3p^2) \end{vmatrix} = 0 \quad (21)$$

The frequency equation (21) is a quadratic equation in p^2 from which the two values of p^2 can be found.
Choosing C_1=1, than $C_2=-\frac{A_4}{A_5}$
where $A_4=2(A_1+p^2B_1)$, $A_5=2(A_2+p^2B_2)$
Therefore

$$\bar{W} = X^2(1-X)^2 - \frac{A_4}{A_5}X^3(1-X)^3 \quad (22)$$

3. TIME FUNCTIONS OF VIBRATIONS OF NON-HOMOGENEOUS VISCO-ELASTIC PLATES

Time function of free vibrations of visco-elastic plates is defined by the general ordinary differential equation (4). Their form depends on the visco-elastic operator$\widetilde{D}$. For Kelvin's model, one has

$$\widetilde{D} \equiv \left(1+\frac{\eta}{G}\frac{d}{dt}\right) \quad (24)$$

Taking temperature dependence of shear modulus G and visco-elastic constants ηin the same form as that of Young's modulus,

$$G(\tau) = G_0(1-\gamma_1\tau), \qquad \eta(\tau) = \eta_0(1-\gamma_2\tau) \quad (25)$$

where G_0 is shear modulus and η_0 is visco-elastic constant at some reference temperature i.e. at $\tau = 0, \gamma_1$ and $\gamma_2$are slope variation of τ with G and η respectively. Using equation (5) in (25),

$$G(X) = G_0[1-\alpha_1(1-X)], \qquad \eta(X) = \eta_0[1-\alpha_2(1-X)] \quad (26)$$

where

$$\alpha_1 = \gamma_1\tau_0, \quad 0 \le \alpha_1 < 1, \quad \text{and} \quad \alpha_2 = \gamma_2\tau_0, \quad 0 \le \alpha_2 < 1$$

Using equation (26) in (24), we get

$$\widetilde{D} \equiv \left(1+q\frac{d}{dt}\right) \quad (27)$$

where $q = \frac{\eta_0[1-\alpha_2(1-X)]}{G_0[1-\alpha_1(1-X)]}$ (28)

Using equation (27) in equation (4), one obtains

$$\frac{d^2T}{dt^2} + p^2q\frac{dT}{dt} + p^2T = 0 \quad (29)$$

and its solution comes out as

$$T(t)=e^{-\frac{p^2qt}{2}}(e_1 \cos st + e_2 \sin st) \quad (30)$$

where $s^2 = p^2 - \frac{1}{4}p^4q^2$, and e_1 and e_2 are constants of integration.

Let us assume that the initial conditions are $T = 1$ and $\frac{dT}{dt} = 0$ at $t = 0$, equation (30) become

$$T(t)=e^{-\frac{p^2qt}{2}}\left(\cos st + \frac{p^2q}{2s}\sin st\right) \quad (31)$$

Thus deflection w(x,y,t) may be expressed from equation (2), (8), (22) and (31),

$$w(x,y,t)= \bar{W}(X)e^{-\frac{p^2qt}{2}}\left(\cos st + \frac{p^2q}{2s}\sin st\right)\sin\frac{\pi y}{b} \quad (32)$$

where p is frequency given by equation (21).
Logarithmic decrement of the vibration is given by

$$\Lambda = \log_e\frac{w_2}{w_1} \quad (33)$$

where w_1 is the deflection at any point of the plate at a time period $K=K_1$ and w_2 is the deflection at the same point at the time period succeeding K_1.

4. RESULT AND DISCUSSION

Deflection, logarithmic decrement corresponding to the first two modes of vibration at different points of C-S-C-S non-homogeneous visco-elastic rectangular plate with exponentially varying thickness have been computed for different combinations of non-homogeneity parameter, taper constant, aspect ratio and thermal constants. Results are presented in Tabular form (1-17). For numerical computation, following materials parameters are used [10]:
$E_0 = 7.08 \times 10^{10}\, N/M^2$, $G_0 = 2.682 \times 10^{10}\, N/M^2$, $\eta_0 = 1.4612 \times 10^6\, N.S/M^2$

$\boldsymbol{\rho_0 = 2.80 \times 10^3\ Kg/M^3, \nu = 0.345}$, **$H_0$=0.01 M**

Table (1-6) show that the deflection (w) for fixed aspect ratio ($\frac{a}{b}$=1.5) starts from zero to increase then decrease to zero for first mode of vibration but for the second mode of vibration value starts zero to increase then decease then increase and finally become to zero for fixed Y and increasing value of X for time 0.K and 5.K respectively. Table (7) and (8) shows that the deflection (w) of first two modes of vibration decease with an increase in aspect ratio ($\frac{a}{b}$). Table (9) shows that the deflection (w) of first two modes of vibration decease with an increase in non-homogeneity parameter ($\boldsymbol{\alpha_3}$).

Table (10 and 12) show that logarithmic decrement ($\boldsymbol{\Lambda}$) of first two modes of vibration increases with an increase in the thermal constant (α). Table (11 and 13) show that logarithmic decrement ($\boldsymbol{\Lambda}$) of first two modes of vibration decreases with an increase in the taper constant(β). Table (14-17) show that logarithmic decrement ($\boldsymbol{\Lambda}$) of first two modes of vibration deceases with an increase in X and in the aspect ratio ($\frac{a}{b}$) respectively.

TABLE 1

Deflection w for different X, Y and β=0.0 and a/b=1.5 for all α, α_1, α_2, α_3 =0.0 at Initial time 0.K

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.014244	0.004441	0.023048	0.007185	0.023048	0.007185	0.014244	0.004441
0.4	0.031146	-0.00194	0.050395	-0.00314	0.050395	-0.00314	0.031146	-0.00194
0.6	0.031146	-0.00194	0.050395	-0.00314	0.050395	-0.00314	0.031146	-0.00194
0.8	0.014244	0.004441	0.023048	0.007185	0.023048	0.007185	0.014244	0.004441
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 2

Deflection w for different X, Y and α =0.8, β=0.6 and a/b=1.5 for all α_1, α_2, α_3 =0.0 at Initial time 0.K

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.016533	0.004474	0.026750	0.007239	0.026750	0.007239	0.016533	0.004474
0.4	0.038869	-0.00183	0.062892	-0.00296	0.062892	-0.00296	0.038869	-0.00183
0.6	0.038869	-0.00183	0.062892	-0.00296	0.062892	-0.00296	0.038869	-0.00183
0.8	0.016533	0.004474	0.026750	0.007239	0.026750	0.007239	0.016533	0.004474
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 3

Deflection w for different X, Y and α =0.8, β=0.0, α_1=0.2, α_2 =0.3, α_3 =0.0 and a/b=1.5 at time 5.K

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.014244	0.004441	0.023048	0.007185	0.023048	0.007185	0.014244	0.004441
0.4	0.031146	-0.00194	0.050395	-0.00314	0.050395	-0.00314	0.031146	-0.00194
0.6	0.031146	-0.00194	0.050395	-0.00314	0.050395	-0.00314	0.031146	-0.00194
0.8	0.014244	0.004441	0.023048	0.007185	0.023048	0.007185	0.014244	0.004441

1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 4

Deflection w for different X, Y and α =0.8, β=0.6, α_1=0.2, α_2=0.3, α_3=0.0 and a/b=1.5 at time 5.K

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.016533	0.004474	0.026750	0.007239	0.026750	0.007239	0.016533	0.004474
0.4	0.038869	-0.00183	0.062892	-0.00296	0.062892	-0.00296	0.038869	-0.00183
0.6	0.038869	-0.00183	0.062892	-0.00296	0.062892	-0.00296	0.038869	-0.00183
0.8	0.016533	0.004474	0.026750	0.007239	0.026750	0.007239	0.016533	0.004474
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 5

Deflection w for different X, Y and α =0.8, β=0.0, α_1=0.2, α_2=0.3, α_3=0.4 and a/b=1.5 at time 5.K

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.014244	0.004441	0.023048	0.007185	0.023048	0.007185	0.014244	0.004441
0.4	0.031146	-0.00194	0.050395	-0.00314	0.050395	-0.00314	0.031146	-0.00194
0.6	0.031146	-0.00194	0.050395	-0.00314	0.050395	-0.00314	0.031146	-0.00194
0.8	0.014244	0.004441	0.023048	0.007185	0.023048	0.007185	0.014244	0.004441
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 6

Deflection w for different X, Y and α =0.8, β=0.6, α_1=0.2, α_2=0.3, α_3=0.4 and a/b=1.5 at time 5.K

X	Y=0.2		Y=0.4		Y=0.6		Y=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.016533	0.004474	0.02675	0.007239	0.02675	0.007239	0.016533	0.004474
0.4	0.038869	-0.00183	0.062892	-0.00296	0.062892	-0.00296	0.038869	-0.00183
0.6	0.038869	-0.00183	0.062892	-0.00296	0.062892	-0.00296	0.038869	-0.00183
0.8	0.016533	0.004474	0.02675	0.007239	0.02675	0.007239	0.016533	0.004474
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 7

Deflection w for different a/b for all α_3 at time 0.K

a/b	α =0.8, β=0.6, α_1=0.2, α_2=0.3				α =0.8, β=0.0, α_1=0.2, α_2=0.3			
	X=0.2, Y=0.4		X=0.4, Y=0.2		X=0.2, Y=0.4		X=0.4, Y=0.2	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	0.030096	0.007273	0.045848	-0.00176	0.026081	0.007231	0.037473	-0.00185
1.0	0.028528	0.007258	0.042576	-0.00179	0.024681	0.007212	0.034553	-0.00189
1.5	0.026750	0.007239	0.038869	-0.00183	0.023048	0.007185	0.031146	-0.00194
2.0	0.025257	0.007220	0.035754	-0.00187	0.021577	0.007156	0.028078	-0.00200
2.5	0.024223	0.007205	0.033598	-0.00190	0.020397	0.007127	0.025618	-0.00206

TABLE 8

Deflection w for different a/b at time 5.K

a/b	$\alpha=0.8, \beta=0.6, \alpha_1=0.2, \alpha_2=0.3, \alpha_3=0.4$				$\alpha=0.8, \beta=0.0, \alpha_1=0.2, \alpha_2=0.2, \alpha_3=0.4$			
	X=0.2, Y=0.4		**X=0.4, Y=0.2**		**X=0.2, Y=0.4**		**X=0.4, Y=0.2**	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.5	0.020651	0.000909	0.031108	-0.00021	0.020372	0.001886	0.029055	-0.00046
1.0	0.018209	0.000837	0.026814	-0.00019	0.018278	0.001775	0.025361	-0.00045
1.5	0.014802	0.000725	0.021131	-0.00017	0.015365	0.001599	0.020513	-0.00041
2.0	0.011193	0.000587	0.015464	-0.00014	0.012228	0.001373	0.015645	-0.00037
2.5	0.007944	0.000441	0.010658	-0.00011	0.009286	0.001121	0.011391	-0.00031

TABLE 9

Deflection w for different non-homogeneity (α_3) for all a/b=1.5 at time 5.K

α_3	$\alpha=0.8, \alpha_1=0.2, \alpha_2=0.3, \beta=0.0$				$\alpha=0.8, \alpha_1=0.2, \alpha_2=0.3, \beta=0.6$			
	X=0.2, Y=0.4		**X=0.4, Y=0.2**		**X=0.2, Y=0.4**		**X=0.4, Y=0.2**	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	0.030096	0.007273	0.045848	-0.00176	0.026081	0.007231	0.037473	-0.00185
0.2	0.028528	0.007258	0.042576	-0.00179	0.024681	0.007212	0.034553	-0.00189
0.4	0.02675	0.007239	0.038869	-0.00183	0.023048	0.007185	0.031146	-0.00194
0.6	0.025257	0.007220	0.035754	-0.00187	0.021577	0.007156	0.028078	-0.00200
0.8	0.024223	0.007205	0.033598	-0.0019	0.020397	0.007127	0.025618	-0.00206

TABLE 10

Logarithmic decrement Λ for different thermal constant (α) for all X, Y (from 0.2 to 0.4) $\alpha_1=0.0$, $\alpha_2=0.0$ and a/b=1.5

α	$\beta=0.0, \alpha_3=0.0$		$\beta=0.6, \alpha_3=0.0$		$\beta=0.0, \alpha_3=0.4$		$\beta=0.6, \alpha_3=0.4$	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	-0.10349	-0.38379	-0.14313	-0.53888	-0.11571	-0.42932	-0.16003	-0.60315
0.2	-0.09818	-0.36402	-0.1371	-0.51904	-0.10977	-0.40718	-0.15329	-0.5809
0.4	-0.09256	-0.34313	-0.13078	-0.49846	-0.10349	-0.38379	-0.14622	-0.55781
0.6	-0.08658	-0.32091	-0.1241	-0.47702	-0.09681	-0.35891	-0.13876	-0.53377
0.8	-0.08016	-0.29704	-0.117	-0.45461	-0.08962	-0.3322	-0.13082	-0.50865

TABLE 11

Logarithmic decrement Λ for different taper constant (β) for all X, Y (from 0.2 to 0.4) $\alpha_1=0.0$, $\alpha_2=0.0$ and a/b=1.5

β	$\alpha=0.0, \alpha_3=0.0$		$\alpha=0.8, \alpha_3=0.0$		$\alpha=0.0, \alpha_3=0.4$		$\alpha=0.8, \alpha_3=0.4$	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	-0.10349	-0.38379	-0.08016	-0.29704	-0.11571	-0.42932	-0.08962	-0.3322
0.2	-0.11473	-0.42645	-0.09068	-0.34085	-0.12828	-0.4771	-0.10139	-0.38123
0.4	-0.1279	-0.47799	-0.10293	-0.3933	-0.14301	-0.53486	-0.11509	-0.43996
0.6	-0.14313	-0.53888	-0.117	-0.45461	-0.16003	-0.60315	-0.13082	-0.50865
0.8	-0.16042	-0.60907	-0.13291	-0.52456	-0.17937	-0.68194	-0.1486	-0.58708

TABLE 12

Logarithmic decrement Λ for different thermal constant (α) for all X=0.4, Y=0.2, α_1=0.2, α_2=0.3 and a/b=1.5

α	β =0.0, α_3=0.0		β =0.6, α_3=0.0		β =0.0, α_3=0.4		β =0.6, α_3=0.4	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	-0.09643	-0.35753	-0.13336	-0.50185	-0.10782	-0.39991	-0.14911	-0.56161
0.2	-0.09148	-0.33912	-0.12775	-0.4834	-0.10229	-0.3793	-0.14284	-0.54093
0.4	-0.08625	-0.31967	-0.12186	-0.46425	-0.09643	-0.35753	-0.13625	-0.51946
0.6	-0.08068	-0.29897	-0.11564	-0.4443	-0.0902	-0.33436	-0.12929	-0.4971
0.8	-0.0747	-0.27674	-0.10902	-0.42345	-0.08351	-0.30949	-0.12189	-0.47373

TABLE 13

Logarithmic decrement Λ for different taper constant (β) for all X=0.4, Y=0.2, α_1=0.2, α_2=0.3 and a/b=1.5

β	α=0.0, α_3=0.0		α=0.8, α_3=0.0		α=0.0, α_3=0.4		α=0.8, α_3=0.4	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	-0.09643	-0.35753	-0.0747	-0.27674	-0.10782	-0.39991	-0.08351	-0.30949
0.2	-0.10691	-0.39723	-0.0845	-0.31754	-0.11953	-0.44437	-0.09448	-0.35515
0.4	-0.11918	-0.4452	-0.09591	-0.36638	-0.13325	-0.49811	-0.10724	-0.40981
0.6	-0.13336	-0.50185	-0.10902	-0.42345	-0.14911	-0.56161	-0.12189	-0.47373
0.8	-0.14948	-0.56712	-0.12384	-0.48853	-0.16713	-0.63484	-0.13847	-0.54668

TABLE 14

Logarithmic decrement Λ for different X for all Y (from 0.2 to 0.8) and α=0.8, α_1=0.2, α_2=0.3 and a/b=1.5

X	β =0.0, α_3=0.0		β =0.6, α_3=0.0		β =0.0, α_3=0.4		β =0.6, α_3=0.4	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	-0.07253	-0.26869	-0.10585	-0.41109	-0.08109	-0.30048	-0.11835	-0.45989
0.2	-0.0747	-0.27674	-0.10902	-0.42345	-0.08351	-0.30949	-0.12189	-0.47373
0.4	-0.07668	-0.2841	-0.11191	-0.43473	-0.08573	-0.31772	-0.12513	-0.48638
0.6	-0.07849	-0.29084	-0.11456	-0.44508	-0.08776	-0.32526	-0.12809	-0.49798
0.8	-0.07253	-0.26869	-0.10585	-0.41109	-0.08109	-0.30048	-0.11835	-0.45989

TABLE 15

Logarithmic decrement Λ for different X for all Y (from 0.2 to 0.8) and α=0.0, α_1=0.2, α_2=0.3 and a/b=1.5

X	β =0.0, α_3=0.0		β =0.6, α_3=0.0		β =0.0, α_3=0.4		β =0.6, α_3=0.4	
	First Mode	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**	**First Mode**	**Second Mode**
0.0	-0.09363	-0.34711	-0.12949	-0.48717	-0.10469	-0.38824	-0.14478	-0.54516
0.2	-0.09643	-0.35753	-0.13336	-0.50185	-0.10782	-0.39991	-0.14911	-0.56161
0.4	-0.09899	-0.36704	-0.1369	-0.51526	-0.11068	-0.41056	-0.15307	-0.57665
0.6	-0.10134	-0.37576	-0.14014	-0.52756	-0.1133	-0.42033	-0.15669	-0.59045
0.8	-0.09363	-0.34711	-0.12949	-0.48717	-0.10469	-0.38824	-0.14478	-0.54516

TABLE 16

Logarithmic decrement Λ for different aspect ratio (a/b) and different X for all Y (from 0.2 to 0.8) and α=0.8, α_1=0.2, α_2=0.3, α_3=0.4, β=0.0

	X=0.2	X=0.4	X=0.6	X=0.8

TABLE 17

a/b	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	-0.0494	-0.26874	-0.05088	-0.27679	-0.05223	-0.28414	-0.05346	-0.29089
1.0	-0.06005	-0.28034	-0.06185	-0.28874	-0.06349	-0.29641	-0.06499	-0.30345
1.5	-0.08109	-0.30048	-0.08351	-0.30949	-0.08573	-0.31772	-0.08776	-0.32526
2.0	-0.11357	-0.33004	-0.11697	-0.33994	-0.12007	-0.34898	-0.12292	-0.35727
2.5	-0.15738	-0.36981	-0.16209	-0.38091	-0.16639	-0.39105	-0.17033	-0.40035

Logarithmic decrement Λ for different aspect ratio (a/b) and different X for all Y (from 0.2 to 0.8) and α=0.8, α_1=0.2, α_2 =0.3, α_3 =0.4, β =0.6

a/b	X =0.2		X=0.4		X=0.6		X=0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	-0.07531	-0.41569	-0.07757	-0.42819	-0.07962	-0.43961	-0.08151	-0.45007
1.0	-0.08978	-0.43179	-0.09247	-0.44478	-0.09492	-0.45664	-0.09717	-0.46752
1.5	-0.11835	-0.45989	-0.12189	-0.47373	-0.12513	-0.48638	-0.12809	-0.49798
2.0	-0.16276	-0.50144	-0.16763	-0.51655	-0.17208	-0.53036	-0.17615	-0.54302
2.5	-0.22299	-0.5578	-0.22967	-0.57465	-0.23577	-0.59004	-0.24136	-0.60417

5

. CONCLUSION

In this study, the dynamic analysis on thermal effect on deflection of non-homogeneous rectangular plates with exponentially varying thickness is analyzed. The differential equations have been solved using Galerkin method of separation and variable separation method. The obtained analytical solutions were used to examine the effects of taper, thermal, non-homogeneity parameter and aspect ratio.

From the parametric studies, the following observations were established:

1. Increase in aspect ratio decreases deflection of plate. 2. Increase in non-homogeneity decreases deflection of plate. 3. Increasing the taper increases deflection. 4. Increase in aspect ratio decreases logarithmic decrement. 5. Increase in non-homogeneity decreases logarithmic decrement.6. Increasing the thermal increases logarithmic decrement. 7.Increase in taper decreases logarithmic decrement. Therefore, engineers can see and made the plates in that manner so these results can fulfill those requirements

6. ABBREVIATIONS

x,y-coordinate in the plane of plate,
E-Young's modulus,

G-shear modulus,
ν-Poisson's ratio,
ρ-mass density per unit length of plate material,
D_1-flexural rigidity,

h-thickness of plate,
Ď-visco elastic operator,

t-time,
η-visco elastic constant,
w(x,y,t) - transverse deflection of plate at point,
a,b - length and breadth of the plate,

α, α_1, α_2-temperature constants,
β-taper constant,
α_3-non-homogeneity constant,
K- time period

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