RESEARCH PAPER

Fuzzy Metric Spaces: A survey On Fixed Point Results and Contraction Principles

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Abstract: This paper present extensive review of fuzzy metric spaces and fixed-point theory. This discuss several contraction principles that guarantee the existence and uniqueness of Fixed point. This approaches not only provides a framework for understanding the relationship between fixed point theory and contraction mapping within fuzzy metric space.

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Introduction and Preliminaries:

Fixed point theory is fundamental area of mathematical analysis that investigate conditions under which a function admits a point that remains invariant under the action of the function .such a point is called a fixed point . Formally ,for a mapping $T: X \to X$ a point $x \epsilon X$ is a fixed point if $T(x) = x$.The theory origininated from Classical results such as the Banach contraction Principle ,Introduced by Stefan Banach , which guarantees the existence and uniqueness of fixed points for contraction mappings in complete metric space .since then ,fixed point theory has expanded significantly to include various generalized spaces such as Normed spaces, Banach spaces, Cone spaces, Fuzzy metric spaces, Intuitionistic fuzzy spaces and Partially Ordered Sets.

Fixed point results play a crucial role in many branches of mathematics and applied sciences. It provides powerful tools for proving the existence and convergence of solutions to differential equations, integral equations, optimization problems, equilibrium problem, game theory, economics and numerical analysis.

Recent research focuses on developing new types of contractive conditions, extending classical results to generalized spaces, and designing iterative algorithms for approximating fixed points.

The fuzzy set theory and applications has established as one of the most active areas of research in many branches of mathematics and engineering. This theory was introduced by Zadeh [22] in 1965 and since then a large number of research papers have published by using the notion of fuzzy sets, fuzzy numbers and fuzzification of several classical theories has also been introduced.

Definition 1. Metric d on the set X is a function defined on the Cartesian product X x X and taking its values in the set £. of reals such that the following conditions are valid:

(1)$d(x, y) \geq 0$ for all x, y e X (positivity),
(2) $d(x, y) = 0$ if and only if x=y, $x, y \epsilon X$ (identity),
(3))$d(x, y) = d(y, x)$ for all $x, y \epsilon X$ (symmetry),
(4) $d(x, y) = d(x, z) + d(z, y)$for all $x, y, z \epsilon X$ (triangle inequality).

It is well-known fact that in practice when measuring a distance, we are not able, in general, to measure it precisely. This can be explicitly seen from the fact that measuring several times the same distance the results may differ. The fuzzy approach to the notion of distance follows from the idea that the distance between two points is not an actually existing real number which we have to find or to approximate, but that it is a fuzzy notion, i.e. the only way in which the distance in question can be described is to ascribe some values from <0, 1> to. various sentences proclaiming something concerning this distance.

Definition2.[1] Fuzzy metric R on the set X is a fuzzy set in the Cartesian product $X\ x\ X\ xE$ the characteristic function f_R of which satisfies:

(1)$f_R(x, y, \lambda) = 0$ for all $x, y \epsilon X\ for\ all\ \lambda \leq 0$
(2) $f_R(x, y, \lambda) = 1$ for A $\lambda > 0$ if and only if $x = y$,
(3)) $f_R(x, y, \lambda) = f_R(y, x, \lambda) = 0$ all $x, y\ \epsilon X$ and all λ $\lambda \epsilon\ E_1$
(4) $f_R(x, y, \lambda + \mu) \geq S(\ f_R(x, y, \lambda), f_R(y, z, \mu)$ where S is a measurable binary
real function defined on$< 0, 1 >\ x\ < 0, 1 >$ taking its values in <0, 1> and such
that $S(1, 1)\ =\ 1$,

(5) $f_R(x,y,\lambda)$is for every pair $< x,y > \epsilon X x X$ a left-continuous and non-decreasing
function of X such that $lim f_R(x,y,\lambda) = 1 \ if \ \lambda \to \infty$.

In particular, Georgeo and Veeramani [2] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [1]. and defined the Hausdorff topology of fuzzy metric spaces. Let us consider the following notations and basic definitions from [2].

Definition 3. [2] A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t − norm if it satisfies the following conditions:

1. $*$ is associative and commutative;
2. $*$ is continuous;
3. $a * 1 = a \ for \ all \ a \in [0,1]$;
4. $a * b \leq c * d \ whenever \ a \leq c \ and \ b \leq d, and \ a,b,c,d \in [0,1]$.

Example . $a * b = min\{a,b\}$ and $a * b = a \cdot b$ are two common examples of continuous t − norms.

Definition 4. [2] The 3 − tuple $(X,M,*)$ is said to be a fuzzy metric space if X is a nonempty set, $*$ is a continuous
t − norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions $for \ all \ x,y,z \in X \ and \ s,t > 0$:

1. $M(x,y,t) > 0$;
2. $M(x,y,t) = 1 \ if \ and \ only \ if \ x = y$;
3. $M(x,y,t) = M(y,x,t)$
4. $M(x,y,s+t) \geq M(x,y,s) * M(x,y,t)$
5. $M(x,y, \cdot) : (0,\infty) \to [0,1] \ is \ continuous.$

If $(X,M,*)$ is a fuzzy metric space then we will call $(M,*)$, or simply M is a fuzzy metric on X. Let (X,D) be a metric space. Denote by $a \cdot b$ the usual multiplication $for \ all \ a,b \in [0,1]$, and let M_d be the fuzzy set defined on $X_2 \times (0,\infty)$ by

$$M_d(x,y,t) = \frac{t}{t + d(x,y)}$$

Then $(M_d, \cdot)$ is a fuzzy metric on X called standard fuzzy metric space .

By definition, a sequence (x_n) in a fuzzy metric space $(X,M,*)$ converges
to a point $x \in X$, if $M(x_n,x,t) = 1$ for all t > 0.

In notions of Cauchy sequences and complete fuzzy metric spaces are
defined.

The sequence (x_n) in X is called $G - Cauchy$ if

$$\forall m \in N M\left(x_n, x_{\{n+m\}}, t\right) = 1 , (t > 0)$$

A fuzzy metric space in which every G-Cauchy sequence is convergent is called a $G - complete \ fuzzy \ metric \ space.$

Definition 5. (George and Veeramani, [2]). A sequence (xn) in a fuzzy metric space $(X,M,*)$ is said to be Cauchy (or M-Cauchy) if, for each $r \in (0,1)$ and each $t > 0$, there exists $N \in N$ such that

$$\forall m,n \geq N, \quad M(x_n,x_m,t) > 1 - r.$$

A fuzzy metric space in which every Cauchy sequence is convergent, is called a complete (or M-complete) fuzzy metric space.

Note that a metric space (X, d) is complete if and only if the induced fuzzy metric space $(X,M_d,*)$ is complete.

Definition 6 .([3]) A 3-tuple (X,M,T) is called a $fuzzy \ b - metric \ space$ if X is an arbitrary
(nonempty) set, T is a continuous t-norm, and M is a fuzzy set on $X_2 \times (0,\infty)$
satisfying the following conditions for all $x,y,z \in X, t,s > 0$ and a given real number $b \geq 1$:

$(b1)$ $M(x,y,t) > 0,$
$(b2)$ $M(x,y,t) = 1 \ if \ and \ only \ if \ x = y,$
$(b3)$ $M(x,y,t) = M(y,x,t),),$
$(b4)$ $T(M(x,y,tb), M(y,z,sb)) \leq M(x,z,t + s$
$(b5)$ $M(x,y, \cdot) : (0,\infty) \to [0,1] \ is \ continuous.$

The class $of \ fuzzy \ b - metric \ spaces$ is effectively larger than that of fuzzy metric spaces.
The class of fuzzy b-metric spaces is effectively larger than that of fuzzy metric spaces,
since a fuzzy b-metric is a fuzzy metric when b = 1.

Example1 : Let $M(x,y,t) = e^{\frac{-|x-y|^p}{t}}$
where $p > 1$ is a real number. Then M is a fuzzy b-metric with $b = 2p - 1$.

Example2 : Let $M(x,y,t) = e^{-d(x,y)}t$ or $M(x,y,t) = \frac{t}{t+d(x,y)}$
, where d is $a \ b - metric$ on X,
and let $T(a,c) = a \cdot c \ for \ a,c \in [0,1]$. Then it is easy to show that M is $a \ fuzzy \ b - metric$.

A sequence $\{x_n\}$:

(a) converges to x if $M(x_n,x,t) \to 1$ as $n \to \infty for \ each \ t > 0$. In this case, we write

$$x_n = x;$$

(b) is called a Cauchy sequence if for all $0 < \varepsilon < 1 \ and \ t > 0$, there exists $n0 \in N$ such that

$$M(x_n,x_m,t) > 1 - \varepsilon \ for \ all \ n,m \geq n_0$$

Definition 7. The fuzzy b-metric space (X,M,T) is said to be complete if every Cauchy sequence is convergent.

Theorem 1. [4]Let (X,M,T) be a complete fuzzy b-metric space, and let $f : X \to X$. Suppose
that there exists $\lambda \in (0,\frac{1}{b})$ such that

$M(fx,fy,t) \geq M(x,y\frac{t}{\lambda}) , x,y \in X, t > 0, \lambda$

and there exist $x_0 \in X \ and \ v \in (0,1)$ such that

$T_{i=1}^{\infty} M\left(x_0,y_0,\frac{t}{v^i}\right) = 1 \quad , \ t > 0$

Then f has a unique fixed point in X.
In 2017 [6] Mahmood, presented the extended fuzzy b metric space

Definition8. A 4-tuple $(Z, \Delta\alpha, *, \alpha)$ is called an extended fuzzy b-metric space if Z is a (nonempty) set, where $\alpha: Z \times Z \longrightarrow [1, \infty)$, $*$ is a continuous t-norm, and $\Delta\alpha$ is a fuzzy set on $Z \times Z \times (0, \infty)$, satisfying the following conditions, for all $\nu, \omega, \varkappa \in Z$ and $r, s > 0$:

$(\Delta 1)$ $\Delta\alpha(\nu, \omega, 0) = 0$
$(\Delta 2)$ $\Delta\alpha(\nu, \omega, r) = 1 \Leftrightarrow \nu = \omega$
$(\Delta 3)$ $\Delta\alpha(\nu, \omega, r) = \Delta\alpha(\omega, \nu, r)$
$(\Delta 4)$ $\Delta\alpha(\nu, \varkappa, \alpha(\nu, \varkappa)(r + s)) \geq \Delta\alpha(\nu, \omega, r) * \Delta\alpha(\omega, \varkappa, s)$
$(\Delta 5)$ $\Delta\alpha(\nu, \omega, \cdot): (0, \infty) \longrightarrow [0, 1]$ $is\ continuous$

Definition 9. [14](Poovaragavan & Jeyaraman (2019)).
A quadruple $(G, B, *, b)$ is known as Generalized b-FMS (shortly, G(b)-FMS) with $b \geq 1$ if $*$ is a continuous t-norm (shortly, CTN),
G is any nonempty set and B is a fuzzy set on $B^3 \times (0, \infty)$ fulfilling the accompanying restrictions for each $\delta, \eta, \zeta, a \in G$ and $r, s > 0$:

$(B1)$ $B(\delta, \eta, \zeta, r) > 0$;
$(B2)$ $B(\delta, \eta, \zeta, r) = 1$ $if\ and\ only\ if$ $\delta = \eta = \zeta$;
$(B3)$ $B(\delta, \eta, \zeta, r) = B(p(\delta, \eta, \zeta,), r)$, $where\ p\ is\ a\ permutation\ function$;
$(B4)$ $B(\delta, \eta, \zeta, r + s) \geq B(\delta, \eta, a, rb) * B(a, \zeta, \zeta, sb)$;
$(B5)$ $B(\delta, \eta, \zeta, \cdot) : (0, \infty) \to [0, 1]$ $is\ continuous$.

It is clear that G(b)-FMS becomes a Generalized-FMS when b takes the value 1, but the converse fails in general.

In 2025 [15]introduces the concept of generalized extended b-FMS, which guarantees the existence and uniqueness of fixed points for contractions.

EXTENDED b-FUZZY METRIC SPACES

Definition 10.. A quadruple $(G, B_\alpha, *, \alpha)$ is a GNE(b)-FMS if $\alpha : G^3 \to [1, \infty)$, $*$ is a CTN, G is any nonempty set and Bα is a fuzzy set on $G^3 \times (0, \infty)$, fulfilling the following conditions for each $\delta, \eta, \zeta, a \in G$ and $r, s > 0$:

$(B_\alpha - 1)B_\alpha(\delta, \eta, \zeta, r) > 0$;
$(B_\alpha - 2)B_\alpha(\delta, \eta, \zeta, r) = 1$ $if\ and\ only\ if$ $\delta = \eta = \zeta$;
$(B_\alpha - 3)B_\alpha(\delta, \eta, \zeta, r) = B_\alpha(p(\delta, \eta, \zeta,), r)$, $where\ p\ is\ a\ permutation\ function$;
$(B_\alpha - 4)B_\alpha(\delta, \eta, \zeta, r + s)$
$$\geq B\_\alpha(\delta, \eta, a, \frac{r}{\alpha(\delta, \eta, \zeta)}) * B\_\alpha(a, \zeta, \zeta, \frac{s}{\alpha(\delta, \eta, \zeta)})$$
$(B_\alpha - 5)B_\alpha(\delta, \eta, \zeta, \cdot) : (0, \infty) \to [0,1]$ $is\ continuous$.

Note that GNE(b)-FMS is a G(b)-FMS if $b = \alpha(\delta, \eta, \zeta), for\ some\ b \geq 1$.

Example3. Let G = {1,2,3} and $\alpha : G^3 \to [1, \infty)$ be a function defined by
$\alpha(\delta, \eta, \zeta) = 1 + \delta + \eta + \zeta$. Define the fuzzy set $B\alpha : G3 \times (0, \infty) \to [0,1]$ by
$$B\alpha(\delta, \eta, \zeta, r) = \frac{inf\{\delta, \eta, \zeta\} + r}{sup\{\delta, \eta, \zeta\} + r}$$
where CTN $*$ is defined as $\beta * \gamma = \beta\gamma$ for all $\beta, \gamma \in [0,1]$. Clearly, (G,Bα,∗,α) is GNE(b)-FMS.,

Theorem2. Let $(G, B_\alpha, *, \alpha)$ be a symmetric G-complete GNE(b)-FMS with the mapping $\alpha : G^3 \to [1, \infty)$ such that
$$B_\alpha(\delta, \eta, \zeta, r) = 1$$
Let $\sigma : G \to G$ be a mapping which fulfills the condition, $B_\alpha(\sigma\delta, \sigma\eta, \sigma\zeta, \kappa r) \geq B_\alpha(\delta, \eta, \zeta, r)$ $for\ all\ \delta, \eta, \zeta \in G$, where $0 < \kappa < 1$. Further, suppose that for an arbitrary $\vartheta_0 \in G$ and $r, \gamma \in N$, we have
$$\alpha(\vartheta_r, \vartheta_r, \vartheta_{r+\gamma}) < \frac{1}{\kappa}$$
where $\vartheta_r = \sigma^r \vartheta_0$. Then, σ has a unique fixed point.

In [15] the **Definition11**. of Generalized μ-Extended b-FMS (shortly, GNE(b,μ)-FMS), which is the further extension of the concept of GNE(b)-FMS. 4 μ-EXTENDED **b**-FUZZY METRIC SPACES

Definition. A five-tuple $(G, B_\mu, *, \alpha, \mu)$ is called a GNE(b,μ)-FMS if $\alpha, \mu : G^3 \to [1, \infty)$,$*$ is a CTN, G is any non empty set and B_μ is a fuzzy set on $G^3 \times (0, \infty)$ fulfilling the accompanying restrictions for each $\delta, \eta, \zeta, a \in G$ and $r, s > 0$:

$(B_\mu - 1)$ $B\mu(\delta, \eta, \zeta, r) > 0$;
$(B_\mu - 2)$ $B\mu(\delta, \eta, \zeta, r) = 1$ $if\ and\ only\ if$ $\delta = \eta = \zeta$;
$(B_\mu - 3)B_\mu(\delta, \eta, \zeta, r) = B_\mu(p(\delta, \eta, \zeta,), r)$, $where\ p\ is\ a\ permutation\ function$;
$(B_\mu - 4)B_\mu(\delta, \eta, \zeta, r + s)$
$$\geq B_\mu\left(\delta, \eta, a, \frac{r}{\alpha(\delta, \eta, \zeta)}\right) * B_\mu\left(a, \zeta, \zeta, \frac{s}{\alpha\mu(\delta, \eta,}\right)$$
$(B_\mu - 5)B_\mu(\delta, \eta, \zeta, \cdot) : (0, \infty) \to [0, 1]$ $is\ continuous$.

Theorem3. Let $(G, B_\mu, *, \alpha, \mu)$ be a symmetric G-complete GNE(b, μ)-FMS with the

Mappings $\alpha, \mu : G^3 \to [1, \infty)$ such that

$$B_\mu(\delta, \eta, \zeta, r) = 1$$

Let $\sigma : G \to G$ be a mapping which fulfills the condition

$B_\mu(\sigma\delta, \sigma\eta, \sigma\zeta, \kappa r) \geq B_\mu(\delta, \eta, \zeta, r)$ $for\ all\ \delta, \eta, \zeta \in G$,

where $0 < \kappa < 1$. Further, suppose that for an arbitrary $\vartheta_0 \in G$ and $r, \gamma \in N$,

$$max\{(\vartheta_j, \vartheta_j, \vartheta_{j+\gamma})), \mu(\vartheta_j, \vartheta_j, \vartheta_{j+\gamma}))\} < \frac{1}{\kappa}$$

where $\vartheta_r = \sigma^r \vartheta_0$. Then, σ has a unique fixed point.

In [7] Introduce the notion of generalized fuzzy metric space.

Definition12 *:* Consider a nonempty set X and a mapping $G: X \times X \times (0,1) \to [0,1]$. Define a set

$$C(G, X, \xi) = \{\{\xi_n\} subset\ XG(\xi_n, \xi, t) = 1 \qquad \forall\, t > 0\}$$

For every ξ ∈ X. Then G is said to be a generalized fuzzy metric if for all $\xi, \eta \in X$ and $t > 0$, it satisfes the following conditions

$GFM1$: $G(\xi, \eta, t) > 0$

$GFM2$: $G(\xi, \eta, t) = 1, \xi = \eta$

$GFM3$: $G(\xi, \eta, t) = G(\eta, \xi, t)$

$GFM4$: $there\ exists\ c \geq 1\ such\ that\ if\ \xi_n \in C(G, X, \xi)$

then $G(\xi, \eta, t) \geq sup\ sup\ G\left(\xi_n, \eta, \frac{t}{c}\right)$

$GFM5$: $G(\xi, \eta, .) : (0, \infty) \to [0,1] G(\cdot_{;;} : V\ (0; 1)!\ [0;\ 1] is\ continuous\ and G(\xi, \eta, t) = 1$

$Then\ (G;\ X; *)\ is\ called\ a\ generalized\ fuzzy\ metric\ space.$

Example4 *:* Consider a generalized metric space $(X; D)$.

Define a mapping $G: X \times X \times (0,1) \to [0,1]$. By

$$G(\xi, \eta, t) = \frac{e^{-d(\xi, \eta)}}{t}$$

And $C(G, X, \xi) = \{\xi_n\} subset\ XG(\xi_n, \xi, t) = 1 \quad \forall\, t > 0\}$ $G(\xi, \eta, t)$

For every $\xi \in X. and\ t > 0$ Then Ge is said to be a generalized fuzzy metric space, where the t-norm ``*" is taken as product norm i.e.

$$\xi * \eta = \xi\eta$$

Theorem4: Let $(G;\ X; *)$ be a G-complete generalized fuzzy metric space and $g: X \to X$ be a fuzzy k-quasi contraction. If there exists $a_0 \in X$ such that $\delta(G, g, a_0, t) > 0$ then $\{g^n(a_0)\}$ converges to a fixed point $u\ of\ g$. Further, if v is another fixed point of g such the

$$G(u;\ v;\ t) > 0\ then\ u = v.$$

In 2020 M. S. Sezen, 8] Controlled Fuzzy Metric Spaces

Definition13. ([8]). A 4-tuple $(Z, \Delta_\gamma, *)$ is called a control fuzzy metric space if Z is a (nonempty) set, $\gamma: Z \times Z \longrightarrow [1, \infty)$, where $*$ is a continuous $t-norm\ and\ \Delta_\gamma$ is a fuzzy set on $Z \times Z \times (0, \infty)$, satisfying the following conditions,

for all $\nu, \omega, \varkappa \in Z\ and\ r, s > 0$:

$(\Delta 1) \Delta_\gamma(\nu, \omega, 0) = 0$

$(\Delta 2) \Delta_\gamma(\nu, \omega, r) = 1 \Leftrightarrow \nu = \omega$

$(\Delta 3) \Delta_\gamma(\nu, \omega, r) = \Delta_\gamma(\omega, \nu, r)$

$(\Delta 4) \Delta_\gamma(\nu, \varkappa, r + s)$

$$\geq \Delta_\gamma\left(\nu, \omega, \left(\frac{r}{\gamma(\nu, \omega)}\right)\right) * \Delta_\gamma\left(\omega, \varkappa, \left(\frac{s}{\gamma(\omega, \varkappa)}\right)\right)$$

$(\Delta 5) \Delta_\gamma(\nu, \omega, \cdot) : (0, \infty) \longrightarrow [0, 1] is\ continuous.$

In [9] introduce orthogonal control fuzzy metric spaces and prove some fixed point results.

Definition 14.. A 4-tuple $(Z, \theta_\gamma, *, \perp)$ is called an orthogonal control fuzzy metric space if Z is an (nonempty) orthogonal set, $\gamma: Z \times Z \longrightarrow [1, \infty)$, where $*$ is a continuous *t*-norm

and θ_γ is a fuzzy set on Z × Z × (0,∞), satisfying the followingconditions:

$(\theta_\gamma 1) \theta_\gamma(\nu, \omega, r) > 0, \forall \nu, \omega \in Z, r > 0 such\ that\ \nu \perp \omega\ and\ \omega \perp \nu\ \omega \perp]$

$(\theta_\gamma 2) \theta_\gamma(\nu, \omega, r) = 1 \Leftrightarrow \nu = \omega$

$(\theta_\gamma 3) \theta_\gamma(\nu, \omega, r) = \theta_\gamma(\omega, \nu, r), \forall \nu, \omega \in Z, r > 0, \nu \perp \omega and \omega \perp \nu$

$(\theta_\gamma 4)\ \theta_\gamma((\nu, \varkappa, r + s))$

$$\geq \theta_\gamma\left(\nu, \omega, \left(\frac{r}{\gamma(\nu, \omega)}\right)\right) * \theta_\gamma\left(\omega, \varkappa, \left(\frac{s}{\gamma(\omega, \varkappa)}\right)\right), or$$

$$\theta_\gamma(\nu, \varkappa, c(\nu, \omega)\gamma(\omega, \varkappa)(r + s)) \geq \theta_\gamma(\nu, \omega, r) * \theta_\gamma(\omega, \varkappa, s), \forall \nu, \omega, \varkappa \in Z, r, s > 0$$

such that

$$\nu \perp \omega, \omega \perp \varkappa, and\ \nu \perp \varkappa$$

$(\theta_\gamma 5)\ \theta_\gamma(\nu, \omega, \cdot) : (0, \infty) \longrightarrow [0, 1] is\ continuous, \forall \nu, \omega \in Z\ such\ that\ \nu \perp \omega\ and\ \omega \perp \nu.$

Example5 . Let $Z = \{-1, 1, 2, 3, 4, \ldots\} = A \cup B$, where $A = \{-1, 1\}$ and $B = N \backslash \{1\}$. Define a binary relation $\perp$ by $\nu \perp \omega \Leftrightarrow], \omega \in \{|\nu|, |\omega|\}$. Given $\theta_\gamma: Z \times Z \times [0, \infty) \longrightarrow [0, 1]$ as

$\theta_\gamma(\nu,\omega,r) = \{1, if\ \nu = \omega, \dfrac{r + (1/\nu)}{r + (1/\omega)}\quad if\ \nu \in B\ and\ \omega \in A, \dfrac{r + (1/\omega)}{r + (1/\nu)}\quad if\ \nu \in A\ and\ \omega \in B, \dfrac{r + (1/max\{\nu,\omega\})}{r + (1/min\{\nu,\omega\})}\quad if\ otherwise,$

$with\ a\ continuous\ t - norm\ *\ defined\ by\ \ r1 * r2 = r1 \cdot r2.$

$Given\ \gamma: Z \times Z \longrightarrow [1,\infty)$ as

$\gamma(\nu,\omega) = \{1 \quad if\ \nu,\omega \in A, max\{\nu,\omega\} \quad , otherwise.$

Then $(Z,\theta_\gamma,*,\perp)$ is an orthogonal control fuzzy metric space, but it is not a control fuzzy metric space.

Definition15. Let $(Z,\theta_\gamma,*,\perp)$ be an orthogonal control fuzzy metric space., then a sequence $\{\nu_n\}$ is said to be G-convergent to ν, where $\nu,\nu_n \in Z$ if and only if $\theta_\gamma(\nu_n,\nu,r) = 1\ for\ any\ n > 0\ and\ for\ all\ r > 0$

Definition16. Let $(Z,\theta_\gamma,*,\perp)$ be an orthogonal control fuzzy metric space. then, a sequence $\{\nu_n\}$ is said to be a G-Cauchy sequence with$\{\nu_n\} \in Z$ if and only if
$\theta_\gamma(\nu_n,\nu_{n+m},r) = 1\ for\ all\ m > 0\ and\ r > 0.$

Definition17. Let $(Z,\theta_\gamma,*,\perp)$ be an orthogonal control fuzzy metric space; then, it is G-complete if and only if every G-Cauchy sequence is convergent.

Theorem5. Let $(Z,\theta_\gamma,*,\perp)$ be an orthogonal G-complete control fuzzy metric space and $\zeta: Z \longrightarrow Z$ be an $\perp$--continuous, $\perp$-contraction, and $\perp$-preserving mapping so that

$$\theta_\gamma(\nu,\omega,r) > 0 \Rightarrow \theta_\gamma(\zeta\nu,\zeta\omega,r) \geq \Psi\, \theta_\gamma(\nu,\omega,r))$$

for all $\nu,\omega \in Z$ and r > 0. then, ζ has a unique fixed point in Z.

In **2018**, Shukla et al. [10] generalized and extended the notion of fuzzy metric
spaces due to George and Veeramani [3] and introduced the notion of complex-valued
fuzzy metric spaces.

Definition 18. (Shukla et al. [10]**).** *A binary operation* $*: I \times I \to I$ *is called a complex-valued t-norm if:*

1. $\zeta_1 * \zeta_2 = \zeta_2 * \zeta_1;$
2. $\zeta_1 * \zeta_2 \leq \zeta_3 * \zeta_4\ whenever\ \zeta_1 \leq \zeta_3, \zeta_2 \leq \zeta_4;$
3. $\zeta_1 * (\zeta_2 * \zeta_3) = (\zeta_1 * \zeta_2) * \zeta_3;$
4. $\zeta * q = q, \zeta * l = \zeta.$

$for\ all\ \zeta,\zeta_1,\zeta_2,\zeta_3,\zeta_4 \in I.$

Definition 19. (Shukla et al. [10] *Let F be a nonempty set, * a continuous complex-valued t-norm and M be a complex fuzzy set on* $F \times F \times P_\theta$ *satisfying the following conditions*

$(CFMS1)\ q \prec M(b1,b2,V);$
$(CFMS2)\ M(b1,b2,V) = l\ if\ and\ only\ if\ b1 = b2;$
$(CFMS3)\ M(b1,b2,\zeta) = M(b2,b1,\zeta);$
$(CFMS4)\ M(b1,b2,\zeta) * M(b2,b3,\zeta')\ \_\ M(b1,b3,\zeta + \zeta');$
$(CFMS5)\ M(b1,b2,\cdot): P_\theta \to I\ is\ continuous;$

for all $b1,b2,b3 \in F$ and $\zeta,\zeta' \in Pq$. Then, the triplet $(F,M,*)$ is called a complex-valued fuzzy metric space and M is called a complex-valued fuzzy metric on F. A complex-valued fuzzy metric can be thought of as the degree of nearness between two points of F with respect to a complex parameter $V \in P_\theta$. fixed point theorem proved by Humaira et al. [11]is as follows:

Theorem 6.[11]Let $(F,M,*)$ be a complete complex valued fuzzy metric space such that for any sequence $\{\zeta_q\}$ in P_θ with $\zeta_q = \infty$, we have $\zeta_q\ M(b1,b2,\zeta_q) = l$ for all $x \in F. If\ \top: F \to F$ satisfies

$$\psi(M(\top b1,\top b2,\zeta)) \geq \psi(M(b1,b2,\zeta)) - \phi(M(b1,b2,\zeta))$$

for all $b1,b2 \in F, \zeta \in P_\theta$, where $\psi,\phi: P \to P$ both are continuous, monotonic nondecreasing functions with $\psi(\zeta),\phi(\zeta) \succ \theta$ for $\zeta \in P_q\ and\ \phi(q) = \psi(q) = q$, then $\top$ has a unique fixed point.

In 2015 [12] the framework of complete fuzzy cone metric spaces

Definition 20.. If P is a cone of E, S is an arbitrary set, $*$ is a continuous t-norm and ζ is a fuzzy on $X^2 \times int(P)$, then a 3-tuple $(S,\zeta,*)$ is considered fuzzy cone metric space satisfying the following conditions, for all $t,s \in int(P)$,

$a,b,z \in S\ and\ t > 0, s > 0.$

1. $(FCM1)\ \zeta(p,q,t) > 0,$
2. $(FCM2)\ \zeta(p,q,t) = 1\ if\ and\ only\ if\ when\ p\ equals\ q,$
3. $(FCM3)\ \zeta(p,q,t) = \zeta(q,p,t),$
4. $(FCM4)\ \zeta(p,q,t) * \zeta(q,z,s) \leq \zeta(p,z,t + s),$
5. $(FCM5)\ \zeta(p,q,.): int(P) \to [0,1]\ is\ continuous.$

In2025 [13]Describe Efuzzy cone metric space (or E − FCMS)

Definition21. Let ζ be a fuzzy set $on\ S \times S \times (0,\infty]$, let $S \neq \phi$, and let $*$ be a
continuous t-norm, and ζ such that

1. $(EFCM1)\ \zeta(p,q,t) > 0,$
2. $(EFCM2)\ \zeta(p,q,t) = 1\ if\ and\ only\ if\ when\ p\ equals\ q,$
3. $(EFCM3)\ \zeta(p,q,t) = \zeta(q,p,t),$
4. $(EFCM4)\ \zeta(p,q,t) * \zeta(q,z,s) \leq \zeta(p,z,t + s),$

5. $(EFCM5)\ \zeta(p,q,.) : int(P) \to (0,1]$ *is left continuous.*
6. $(EFCM6)$ *The collection* $\{\zeta(p,q,.) : (0,r) \to (0,1];\ (p,q) \in S^2\}$ *is uniformly equicontinuous, for some r positive,*

for all t, s > 0 and p, q, z ∈ S. Then, the ordered triple (S, ζ, ∗) is called a E fuzzy cone metric space (or E − FCMS).

Theorem7. Consider the E- fuzzy cone metric space $(S,\zeta,*)$ to be complete. Consider a contractive mapping $\chi : S \to S$ with a contractive constant k, which means there exists $k \in]0,1[$ such that

$$\frac{1}{\zeta(\chi^a,\chi^b,t)} - 1 \le k((\frac{1}{\zeta(a,b,t)} - 1)$$

$\forall\, a,b \in S$ and $\forall\, t > 0$. Consequently, the sequence $\{\chi^n a\}$ converges to a^* and χ has a unique fixed point a^* for all $a \in S$.

In [17] **Bipolar fuzzy metric spaces**

Definition22: (Menger 1942). Continuous *t*-norms are binary operations on the interval [0, 1], it means that$\Delta : [0,1]\times[0,1] \to [0,1]$ satisfies the following situations with neutral element

1. Δ *is commutative and associative*
2. Δ *is continuous*
3. $a\Delta 1 = a$ *for all* $a \in [0,1]$
4. $a\Delta b \le c\Delta d$ *where* $a \le c, b \le d$ *for all* $a,b,c,d \in [0,1]$.

Definition23: A doubled arithmetic operation $\nabla : [-1,0] \times [-1,0] \to [-1,1]$ is referred to as bipolar continuous symmetry *ts*-conorm if ∇ applies the required specifications:

1. ∇ *is commutative and associative*
2. ∇ *is continuous*
3. $a\nabla 0 = a$ *for all* $a \in [-1,0]$
4. $a\nabla b \le c\nabla d$ *where* $a \le c, b \le d$ *for all* $a,b,c,d \in [-1,0]$.

Definition 24: The tetra set (A,B,Δ,∇) is called bipolar fuzzy metric space (BFMS) when the following conditions are satisfied for all $u,v,z \in A$, where A be arbitrary set, $\{< u, \mu\_A\^r\ (u), \mu\_A\^l\ (u)\ > : u \in A\}$ be BFS such that $\mu\_A\^r\ (u), \mu\_A\^l$ are defined as fuzzy sets on $A \times A \times (0,\infty), B : A \times A \times (0,\infty) \to [-1,1], \alpha,\gamma > 0$ *and* Δ,∇ show the continuous *t*-norm and continuous symmetry t_s −conorm, respectively:

1. $0 \le \mu_A^r(u)(u,v,\gamma) \le 1, -1 \le \mu_A^l(u,v,\gamma) \le 0$
2. $\mu_A^r(u,v,\gamma) + \mu_A^l(u,v,\gamma) \le 1$
3. $\mu_A^r(u,v,\gamma) = 1 \Leftrightarrow u = v$
4. $\mu_A^r(u,v,\gamma) = \mu_A^r(v,u,\gamma)$
5. $\mu_A^r(u,v,\gamma)\Delta\mu_A^r(v,z,\alpha) \le \mu_A^r(u,z,\gamma+\alpha)$
6. $\mu_A^r(u,v,.) : [0,\infty) \to [-1,1]$ *is continuous*
7. $\mu_A^r(u,v,\gamma) = 1 (\forall\gamma > 0)$
8. $\mu_A^l(u,v,\gamma) = -1 \Leftrightarrow u = v$
9. $\mu_A^l(u,v,\gamma) = \mu_A^l(v,u,\gamma)$
10. $\mu_A^l(u,v,\gamma)\nabla\mu_A^l(v,z,\alpha) \ge \mu_A^l(u,z,\gamma+\alpha)$
11. $\mu_A^l(u,v,.) : [0,\infty) \to [-1,1]$ *is continuous.*
12. $\mu_A^l(u,v,\gamma) = -1, (\forall\gamma > 0)$.

The functions $.\mu_A^r(u,v,\gamma)$ and $\mu_A^l(u,v,\gamma)$ represent the value of closeness and the value of distance between *u* and *v* with respect to *γ* , respectively.

Example6. Let *(A, d)* be a crisp metric space. Give the functions *Δ* and ∇ as default

$$u\Delta v = min\{u,v\}, u\nabla v = max\{u,v\}.$$

$$\mu_A^r(u) = \frac{\gamma}{\gamma + d(u,v)}$$

$$\mu_A^l(u) = \frac{-\gamma}{\gamma + d(u,v),}$$

for all $\gamma > 0, u,v \in A$

Then, (A,B,Δ,∇) *be* bipolar fuzzy metric space by considering

$$B : A \times A \times (0,\infty) \to [-1,1].$$

In [16] new way to introduce the notion of fuzzy bipolar metric space

Fuzzy bipolar metric spaces

Definition25. Let X and Y be two non-empty sets. A quadruple $(X,Y,Mb,*)$ is said to be fuzzy bipolar metric space (FB-space), where _ is continuous t-norm and Mb is a fuzzy set on $X \times Y \times (0,\infty)$, satisfying the following conditions for all $t,s,r > 0$:

$(FB-1)\ Mb(x,y,t) > 0$ *for all* $(x,y) \epsilon X\_Y;$

$(FB-2)\ Mb(x,y,t) = 1$ *if and only if* $x = y$ *for* $x \epsilon X$ *and* $y \epsilon Y;$

$(FB-3)\ Mb(x,y,t) = Mb(y,x,t)$ *for all* $x,y \epsilon X \cap Y;$

$(FB-4)\ Mb(x1,y2,t+s+r) > Mb(x1,y1,t)\_Mb(x2,y1,s)\_Mb(x2,y2,r)$ *for all* $x1,x2 \epsilon X$ *an*

$(FB-5)\ Mb(x,y,.) : [0,1) \to [0,1]$ *is left continuous;*

$(FB-6)\ Mb(x,y,.)$ *is non − decreasing for all* $x \epsilon X$ *and* $y \epsilon Y$.

Example7. Let (X,Y,d) be an bipolar metric space. For all $x \epsilon X, y \epsilon Y,$ *and* $t > 0$, denote

$$Mb(x,y,t) = \frac{t}{t + d(x,y)}$$

.Then $(X,Y,Mb,*)$ is fuzzy bipolar metric space, where is a t-norm defined as

$a * b = ab$ or $a \_ * b = min\{a, b\}$.

Theorem8. Let $(X, Y, Mb, *)$ be a complete fuzzy bipolar metric space such that

$$Mb(x, y, t) = 1 \; for\; all\; x \in X, y \in Y.$$

Let $T : X \cup Y \to X \cup Y$ be mapping satisfying

$(i)\; T(X) \subseteq X\; and\; T(Y) \subseteq Y;$

$(ii)\; Mb(T(x), T(y), kt) > Mb(x, y, t)\; for\; all\; x \in X, y \in Y\; and\; t > 0, where\; k \in (0, 1).$

Then T has a unique fixed point.

In 19] uses Intuitionistic fuzzy metric space

Definition26. A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions:

$(i)\; *\; is\; commutative\; and\; associative;$

$(ii)\; *\; is\; continuous;$

$(iii) a * 1 = a\; for\; all\; a \in [0, 1];$

$(iv) a * b \leq c * d\; whenever\; a \leq c, b \leq d\; and\; a, b, c, d \in [0, 1].$

Definition27. A binary operation $\diamond: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:

$(i)\; \diamond\; is\; commutative\; and\; associative;$

$(ii)\; \diamond\; is\; continuous;$

$(iii)\; a \diamond 1 = a\; for\; all\; a \in [0, 1];$

$(iv)\; a \diamond b \leq c \diamond d\; whenever\; a \leq c, b \leq d\; and\; a, b, c, d \in [0, 1].$

Definition28. [19] .Let $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$is a continuous t-conorm and M,N are fuzzy sets on $X \times X \times [0, \infty)$ satisfying the following conditions:

$(i)\; M(x, y, t) + N(x, y, t) \leq 1\; for\; all\; x, y \in X\; and\; t > 0;$

$(ii)\; M(x, y, 0) = 0\; for\; all\; x, y \in X;$

$(iii)\; M(x, y, t) = 1\; for\; all\; x, y \in X\; and\; t > 0\; if\; and\; only\; if\; x = y;$

$(iv)\; M(x, y, t) = M(y, x, t)\; for\; allx, y \in X\; and\; t > 0;$

$(v)\; M(x, y, t) \_ M(y, z, s) \leq M(x, z, t + s)\; for\; all\; x, y, z \in X\; and\; t, s > 0;$

$(vi)\; M(x, y, .) : [0,1) \to [0, 1]\; is\; left\; continuous\; for\; all\; x, y, \in X;$

$(vii) M(x, y, t) = 1\; for\; all\; x, y \in X\; and\; t > 0;$

$(vii)\; N(x, y, 0) = 1\; for\; all\; x, y \in X;$

$(viii)\; N(x, y, t) = 0\; for\; all\; x, y \in X\; and\; t > 0\; if\; and\; only\; if\; x = y;$

$(ix)\; N(x, y, t) = N(y, x, t)\; for\; all\; x, y \in X\; and\; t > 0;$

$(x)\; N(x, y, t) \diamond N(y, z, s) > N(x, z, t + s)\; for\; all\; x, y, z \in X\; and\; t, s > 0;$

$(xi)\; N(x, y, .) : [0,1) \to [0, 1]\; is\; right\; continuous\; for\; all\; x, y, z \in X;$

$(xii) N(x, y, t) = 0 for\; all\; x, y\; X\; and\; t > 0.$

Then (M,N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Example8 . Let $X = \left\{\frac{1}{n}; n = 1, 2, 3, \dots\right\} \cup \{0\}$and let $*$be the continuous t-norm and $\diamond$be the continuoust-conorm defined by $a * b = ab$ and $a \diamond b = min\{1, a + b\}$ for all $a, b \in [0, 1]$ for each $x, y \in X\; and\; t > 0$,

define (M,N) by

$$M(x, y, t) = \{\; 0,\; t = 0, \frac{t}{t + d(x, y)} \quad , t > 0,$$

$$N(x, y, t) = \{\; 1\; , t = 0, \frac{d(x, y)}{t + d(x, y)} \quad , t > 0,$$

Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space.

Theorem9. [19] Let A and B be self mappings of a complete intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ such that $a * b = min\{ab\}$ and $a \diamond b = min\{a, b\}$ for all $a, b \in X$. Assume $M(xo, A(xo), t) > 0$ and $N(xo, A(xo), t) < 1$ for all $xo \in X, \forall t > 0$, let $A, B\; be\; self\; mappings\; of\; X$ satisfying the following conditions:

(1) A, B are two continuous and intuitionistic ψ -contractive mappings;

$(2) M(A(x), B(y), t) \geq \_(min\{M(x, y, t), M(A(x), x, t), M(y, B(y), t)\}),$

$N(A(x), B(y), t) \leq (max\{N(x, y, t), N(A(x), x, t), N(y, B(y), t)\})$

for all $x, y \in X\; and\; t > 0;$

(3) $\{x_n\} is\; (A, B) - sequence\; of\; initial\; point\; x_0$

Then A and B have a unique common fixed point in X.

In [21] Define the Let IFbMSIntuitionistic fuzzy b-metric space.

Intuitionistic fuzzy b- metric space..

Definition29.Let E is an arbitrary set, $\diamond$ and o are continuous t-norm and continuous t-conorm respectively. Let H and O be fuzzy sets on $E \times E \times (0; \infty)$ satisfying the following state, for all $\rho1; \rho2; \rho3 \in E, b \geq 1, y;\; s > 0.$

$(a)\; H(\rho1; \rho2; y) + O(\rho1; \rho2; y) \leq 1;$

$(b)\; H(\rho1; \rho2; y) > 0;$

$(c)\ H(\rho1;\rho2;y) = 1;\ for\ all\ y > 0\ iff\ \rho1 = \rho2;$
$(d)\ H(\rho1;\rho2;y) = H(\rho2;\rho1;y), for\ all\ y > 0;$
$(e)\ H(\rho1;\rho3;\ (y+s))$
$$\geq H(\rho1;\rho2;\frac{y}{b})$$
$$\diamond\ H(\rho2;\rho3;\frac{s}{b}), for\ all\ y;\ s > 0;$$
$(f) H(\rho1;\rho2;$
$:) is\ non\ decreasing\ function\ of\ R^+\ and H(\rho1;\rho2;y) = 1;$
$(g)\ O(\rho1;\rho2;y) < 1;$
$(h)\ O(\rho1;\rho2;y) = 0, for\ all\ y > 0\ iff\ \rho1 = \rho2;$
$(i)\ O(\rho1;\rho2;y) = O(\rho2;\rho1;y), for\ all\ y > 0;$
$(j)\ O(\rho1;\rho3;\ (y+s))$
$\leq\ O(\rho1;\rho2;\frac{y}{b})\ o\ O(\rho2;\rho3;\frac{s}{b}), for\ all\ y;\ s > 0;$
$(k) O(\rho1;\rho2;$
$:) is\ non\ increasing\ function\ of\ R^+\ and O(\rho1;\rho2;y) = 0:$
Also, $H(\rho1;\rho2;y) = 1$ and $O(\rho1;\rho2;y) = 0$:
Then a 5-tuple $(E;H;O;\diamond;o)$ is called an IFbMS.

In 2025 [20] extended intuitionistic fuzzy b-metric space is defined and showing the properties crucial to its structure.

Extended Intuitionistic fuzzy b- metric space.

Definition 30. A 5-tuple set $(E;H_\zeta;O_\zeta;\diamond;o)$ is said to be an **EIFbMS(**Extended Intuitionistic fuzzy b- metric space.**),** if E is a non-empty set,
$\diamond$ and o is continuous t-norm and continuous t-conorm respectively and $\zeta: E\times E \to [1,\infty)$be a function, H_ζ and O_ζ are fuzzy sets on $E\ \times E\ \times (0;\infty)$ satisfying the following conditions with
$b\ \geq 1, s; y > 0\ and\ for\ all\ \rho1;\rho2;\rho3\ \epsilon\ E.$
$(a) H_\zeta(\rho1;\rho2;y) + O_\zeta(\rho1;\rho2;y)\ \leq\ 1;$
$(b) H_\zeta(\rho1;\rho2;y) > 0$
$(c) H_\zeta(\rho1;\rho2;y) = 1\ , for\ all\ y > 0, iff \rho1 = \rho2$
$(d) H_\zeta(\rho1;\rho2;y) = H_\zeta(\rho2;\rho1;y)$ for all $y > 0$
$(e) H_\zeta\ \left(\rho1;\rho3;\zeta(\rho1;\rho3)(y+s)\right)$
$$\geq H_\zeta\ \left(\rho1;\rho2;\frac{y}{b}\right) H_\zeta$$
$$\diamond\ H_\zeta\ (\rho2;\rho3;\frac{s}{b}), for\ all\ y;\ s > 0;$$
$(f) H_\zeta\ (\rho1;\rho2;$
$:)\ is\ non\ decreasing\ function\ of\ R^+\ and H(\rho1;\rho2;y) = 1;$
$(g) O_\zeta(\rho1;\rho2;y) < 1;$
$(h) O_\zeta(\rho1;\rho2;y) = 0, for\ all\ y > 0\ iff\ \rho1 = \rho2;$
$(i)\ O_\zeta(\rho1;\rho2;y) = O_\zeta(\rho2;\rho1;y), for\ all\ y > 0;$
$(j)\ O_\zeta(\rho1;\rho3,;\zeta(\rho1;\rho3)\ (y+s))$
$\leq\ O_\zeta(\rho1;\rho2;\frac{y}{b})\ o\ O_\zeta(\rho2;\rho3;\frac{s}{b}), for\ all\ y;\ s > 0;$
$(k) O_\zeta(\rho1;\rho2;$
$:) is\ non\ increasing\ function\ of\ R^+\ and O_\zeta(\rho1;\rho2;y) = 0:$
Also $H_\zeta(\rho1;\rho2;y) = 1$ and $O_\zeta(\rho1;\rho2;y) = 0$

Theorem10.[20]. Let$(E;H_\zeta;O_\zeta;\diamond;o)$ is a complete EIFBMS with the mapping $\zeta: E\times E \to [1,\infty)$ and $for\ all\ \rho1;\rho2;\rho3\ \epsilon\ E$.such that

$$H_\zeta(\rho1;\rho2;y) = 1$$
$$O_\zeta(\rho1;\rho2;y) = 0$$

and a mapping $g: E \to E$ which satisfies the following Equation

$$H_\zeta(g\rho1, g\rho2, ky) \geq H_\zeta(\rho1,\rho2,y)$$

$$O_\zeta(g\rho1, g\rho2, y) \leq\ O_\zeta(\rho1,\rho2;y)$$

where $k\ \epsilon(0;1)$ Let us take an arbitrary $a_0\epsilon\ E$;$n;q\ 2\epsilon$ N we have $\zeta(a_n, a_{n+q}) < \frac{1}{k}$
where $a_n = g^n a_0$, then a mapping g has a unique FP.

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