RESEARCH PAPER

M-Connectedness in n-Cylindrical Neutrosophic Topologies: New Theoretical Results and a Drug Delivery Application

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Abstract: In this paper, we explore several topological concepts within the framework of n-cylindrical neutrosophic topological spaces. In particular, we examine n-cylindrical neutrosophic M-connected spaces and M-disconnected spaces. We establish fundamental properties and theorems related to connectedness under these generalized notions. As a real-life illustration of the proposed framework, we further develop an n-cylindrical neutrosophic multi-criteria decision-making model for selecting an optimal nanocarrier system in drug delivery technology, wherein five candidate nanocarriers are ranked using an n-cylindrical neutrosophic weighted averaging operator and an associated score function.

Keywords and phrases: n-cylindrical neutrosophic M-open set, n-cylindrical neutrosophic M-connected spaces, n-cylindrical neutrosophic M-disconnected spaces.

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1 Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [29], Chang [9] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy sets (ifs), was introduced by Atanassov in 1986 [4]. Building on this, Coker [10] introduced the concept of intuitionistic fuzzy topological spaces ($ifts$) based on ifs. Jeon et al. [12] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [21, 22], a new direction in uncertainty modeling emerged. Salama and Alblowi [17] introduced neutrosophic crisp sets and neutrosophic topological spaces (Nts), extending $ifts$ and incorporating degrees of membership, indeterminacy, and non-membership for each element. Neutrosophic has since formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [3] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [27] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [18] introduced the concept of n-cylindrical neutrosophic sets (abbreviated as $n\text{-}CyNS$), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic sets (ns), $n\text{-}CyNs$ represents the most extensive generalization of fuzzy sets (fs). In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \le \beta_A \le 1$ and $0 \le \alpha_A^n(x) + \gamma_A^n(x) \le 1, n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [20] introduced the notion of $n\text{-}CyN$ continuity for functions between two n-cylindrical fuzzy neutrosophic topological spaces ($n\text{-}CyNts$). They also defined the $n\text{-}CyN$ interior ($n\text{-}CyNint$) and $n\text{-}CyN$ closure ($n\text{-}CyNcl$) of subsets within $n\text{-}CyNts$.

In a related development, El-Maghrabi and Al-Juhani [11] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [16] also found M-open sets in nano topological spaces. Vadivel et al. [23, 24, 25] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [13] and Vadivel et al. [26]

introduced M-open sets in fuzzy and neutrosophic nano topological spaces. Anandhi et al. [1, 2, 28] introduced n-Cylindrical neutrosophic M open sets in n-Cylindrical neutrosophic topological spaces.

Beyond their theoretical development, neutrosophic and n-cylindrical neutrosophic structures have proved useful in modelling real-world decision-making problems involving simultaneous positive, neutral and negative evidence, such as medical diagnosis, pattern recognition and multi-criteria evaluation. Drug delivery technology, in particular, requires the selection of a nanocarrier system among several competing alternatives (liposomes, polymeric nanoparticles, solid lipid nanoparticles, dendrimers, micelles, and so on) on the basis of criteria such as encapsulation efficiency, release behaviour, biocompatibility, targeting specificity and stability, and expert judgment on these criteria naturally involves hesitation and conflicting evidence. Motivated by this, in the present paper we also demonstrate how the $n\text{-}CyNS$ framework can be applied to such a multi-criteria nanocarrier selection problem.

The subsequent sections of this paper are organized as follows: Section 2 provides a brief review of fundamental definitions related to intuitionistic fuzzy sets (ifs's), neutrosophic sets (NS's), n-cylindrical neutrosophic sets ($n\text{-}CyNS$'s), and mappings on n-cylindrical neutrosophic topological spaces ($n\text{-}CyNts$). Sections 3 introduce the concepts of $n\text{-}CyNM$-connected spaces and $n\text{-}CyNM$-disconnected spaces within $n\text{-}CyNts$, along with an exploration of their fundamental properties supported by illustrative examples. Section 4 presents a real-life application of the $n\text{-}CyNS$ framework to the selection of an optimal nanocarrier in drug delivery technology. The paper is concluded in section 5.

2 Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 *[29] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: A \to [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A, for all $x \in X$.*

Definition 2.2 *[9] A fuzzy topological space (briefly, fts) is a pair (X, τ), where X is any set and τ is a family of fuzzy sets in X satisfying following axioms:*

1. $\phi, X \in \tau$,
2. If $A, B \in \tau$, then $A \cap B$ in τ,
3. If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 *[4] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x): x \in X\rangle\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\alpha_A(x)$ and $\gamma_A(x)$ satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ $ifs(X)$ denotes the set of all the ifs's on X.*

Definition 2.4 *[22] An neutrosophic set A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle: x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral and $\gamma_A(x)$ is the degree of negative membership. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.*

Definition 2.5 *[22] An neutrosophic topology (Nt) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:*

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\cup G_i \in \tau_N$, for all $\{G_i: i \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called a neutrosophic topological spaces (briefly, Nts) and any neutrosophic set in τ is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_X are called neutrosophic open sets. A neutrosophic set F is closed if and only if $C(F)$ is neutrosophic open.

Definition 2.6 *[23] An n-cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle: x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, is an integer. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components and $\beta_A(x)$ is 100% independent.*

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-cylindrical neutrosophic number (briefly, n - $CyNN$) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 *[18] Let $\{A_i: i \in I\}$ be an arbitrary family of $n\text{-}CyNS$ in X. Then, $\cap A_i = \{\langle x, inf(\alpha_{A_i}(x)), inf(\beta_{A_i}(x)), sup(\gamma_{A_i}(x))\rangle: x \in X\}$, $\cup A_i = \{\langle x, sup(\alpha_{A_i}(x)), sup(\beta_{A_i}(x)), inf(\gamma_{A_i}(x))\rangle: x \in X\}$.*

Definition 2.8 *[18] $0_{CyN} = \{\langle x, 0,0,1\rangle: x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle: x \in X\}$*

Definition 2.9 *[18]* **(The Basic**

Connectives) *Let $\tau_{CyN}(X)$ denote the family of all n-$CyNS$'s on X.*

1. **Inclusion:** For every two $A, B \in \tau_{CyN}(X)$, the inclusion of two n-$CyNS$'s A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \leq \beta_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$) and ($A \subseteq B$ and $B \subseteq A$).
2. **Union:** For every two $A, B \in \tau_{CyN}(X)$, the union of two n-$CyNS$'s A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.
3. **Intersection:** For every two $A, B \in \tau_{CyN}(X)$, the intersection of two n-$CyNS$'s A and B is
$$A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}.$$
4. **Complementary:** For every $A \in \tau_{CyN}(X)$, the complement of an n-$CyNS$ A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Remark 2.1 *[18]*

1. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
2. $A \cup B = B \cup A$ & $A \cap B = B \cap A$,
3. $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$,
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,
5. $A \cap A = A$ & $A \cup A = A$,
6. De Morgan's Law for A & B ie., $(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$.

Definition 2.10 *[20] An n-cylindrical neutrosophic topology (briefly, n-$CyNt$) on a non-empty set X is a family, τ_{CyN}, of n-$CyNS$ in X which satisfies the following conditions:*

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. $A_1 \cap A_2 \in \tau_{CyN}$,
3. $\cup A_i \in \tau_{CyN}$, for any arbitrary family $A_i \in \tau_{CyN}, i \in I$.

The pair (X, τ_{CyN}) is called an n-cylindrical neutrosophic topological Spaces (briefly, n-$CyNts$) and any n-$CyNS$ belongs to τ_{CyN} is called an n-cylindrical neutrosophic open set (briefly, n-$CyNos$) and the complement of n-$CyNos$ is called n-cylindrical neutrosophic closed set (briefly, n-$CyNcs$) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-$CyNts$ and the topology containing all the n-CyN subsets is called discrete n-$CyNts$.

Remark 2.2 *[20] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-$CyNts$ as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.*

Definition 2.11 *[20] Let A and B be two n-cylindrical neutrosophic subsets of an n-$CyNts$. B is called neighbourhood of A if there exists an n-$CyNos$, O such that $A \subset O \subset B$.*

Proposition 2.1 *[20] $A \subset X$ is n-cylindrical neutrosophic open in (X, τ_{CyN}) if and only if it carries a neighbourhood of its subsets.*

Definition 2.12 *[20] Let (X, τ_{CyN}) be an n-$CyNts$ and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic interior (briefly, n-$CyNint$) is defined as the n-CyN union of all n-CyN open subsets of X. ie, n-$CyNint(A) = \cup \{G : G \in \tau_{CyN}$ and $G \subseteq A\}$.*

Clearly, n-$CyNint(A)$ is the biggest n-$CyNos$ that is contained by A.

Definition 2.13 *[20] Let (X, τ_{CyN}) be an n-$CyNts$ and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic closure (briefly, n-$CyNcl$) is defined as the n-CyN intersection of all n-CyN closed subsets of X. ie, n-$CyNcl(A) = \cap \{K : K \in \tau_{CyN}$ and $A \subseteq A\}$.*

Clearly, n-$CyNcl(A)$ is the smallest n-$CyNcs$ that contains A.

Definition 2.14 *[19] Let (X, τ_1) and (Y, τ_2) be two n-$CyNts$ and let $f : (X, \tau_1) \to (Y, \tau_2)$ be a n-CyN function. Then, f said to be n-CyN continuous (briefly, n-$CyNCts$) map if for any n-cylindrical neutrosophic subset A of X and for any neighbourhood $\mathfrak{V}$ of $f(A)$ there exists a neighbourhood $\mathfrak{U}$ of A such that $f(\mathfrak{U}) \subset \mathfrak{V}$.*

Definition 2.15 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and A be an n-$CyNS$. Then, A is said to be an n-CyN*

1. regular open set (briefly, n-$CyNros$), if $A = n$-$CyNint(n$-$CyNcl(A))$,
2. regular closed set (briefly, n-$CyNrcs$), if $A = n$-$CyNcl(n$-$CyNint(A))$.

Definition 2.16 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic δ-interior of A and the n-cylindrical neutrosophic δ-closure of A are denoted by n-$CyN\delta int(A)$ and n-$CyN\delta cl(A)$ are defined as follows:*

1. n-$CyN\delta int(A) = \cup \{G | G$ isan n-$CyNros$ and $G \subseteq A\}$,
2. n-$CyN\delta cl(A) = \cap \{K | K$ isan n-$CyNrcs$ and $A \subseteq K\}$.

Definition 2.17 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-$CyNS$ in X. Then, the n-cylindrical* *neutrosophic θ-interior of A and the n-cylindrical neutrosophic θ-closure of A are denoted by $n\text{-}CyN\theta int(A)$ and $n\text{-}CyN\theta cl(A)$ are defined as follows:*

1. $n\text{-}CyN\theta int(A) = \bigcup \{n-CyNint(B): B \subseteq A \,\&\, B$ is a $n-CyNcs$ in $X\}$,
2. $n\text{-}CyN\theta cl(A) = \bigcap \{n-CyNcl(B): A \subseteq B \,\&\, B$ is a $n-CyNos$ in $X\}$.

Definition 2.18 *[1] Let (X,τ_{CyN}) be an $n\text{-}CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an $n\text{-}CyNS$ in X. A set A is said to be $n\text{-}CyN$*

1. δ-open set (briefly, $n\text{-}CyN\delta os$), if $A = n\text{-}CyN\delta int(A)$,
2. δ-pre open set (briefly, $n\text{-}CyN\delta\mathcal{P}os$), if $A \subseteq n\text{-}CyNint(n\text{-}CyN\delta cl(A))$,
3. δ-semi open set (briefly, $n\text{-}CyN\delta\mathcal{S}os$), if $A \subseteq n\text{-}CyNcl(n\text{-}CyN\delta int(A))$,
4. θ-open set (briefly, $n\text{-}CyN\theta os$), if $A = n\text{-}CyN\theta int(A)$,
5. θ-semi open set (briefly, $n\text{-}CyN\theta\mathcal{S}os$), if $A \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(A))$,
6. e-open set (briefly, $n\text{-}CyNeos$), if $A = n\text{-}CyNcl(n\text{-}CyN\delta int(A)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(A))$,
7. M-open set (briefly, $n\text{-}CyNMos$), if $A \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(A)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(A))$.

The complement of a $n\text{-}CyNMos$ (resp. $n\text{-}CyN\delta os$, $n\text{-}CyN\delta\mathcal{P}os$, $n\text{-}CyN\delta\mathcal{S}os$, $n\text{-}CyN\theta os$, $n\text{-}CyN\theta\mathcal{S}os$ & $n\text{-}CyNeos$) is called a $n\text{-}CyNM$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$ & $n\text{-}CyNe$) closed set (briefly, $n\text{-}CyNMcs$ (resp. $n\text{-}CyN\delta cs$, $n\text{-}CyN\delta\mathcal{P}cs$, $n\text{-}CyN\delta\mathcal{S}cs$, $n\text{-}CyN\theta cs$, $n\text{-}CyN\theta\mathcal{S}cs$ & $n\text{-}CyNecs$)) in X.

The family of all $n\text{-}CyNMos$ (resp. $n\text{-}CyN\delta os$, $n\text{-}CyN\delta\mathcal{P}os$, $n\text{-}CyN\delta\mathcal{S}os$, $n\text{-}CyN\theta os$, $n\text{-}CyN\theta\mathcal{S}os$ & $n\text{-}CyNeos$) of X is denoted by $n\text{-}CyNMOS(X)$, (resp. $n\text{-}CyNMCS(X)$, $n\text{-}CyN\delta OS(X)$, $n\text{-}CyN\delta CS(X)$, $n\text{-}CyN\delta\mathcal{P}OS(X)$, $n\text{-}CyN\delta\mathcal{P}CS(X)$, $n\text{-}CyN\delta\mathcal{S}OS(X)$, $n\text{-}CyN\delta\mathcal{S}CS(X)$, $n\text{-}CyN\theta OS(X)$, $n\text{-}CyN\theta CS(X)$, $n\text{-}CyN\theta\mathcal{S}OS(X)$, $n\text{-}CyN\theta\mathcal{S}CS(X)$, $n\text{-}CyNeOS(X)$ & $n\text{-}CyNeCS(X)$).

Definition 2.19 *[1] Let (X,τ_{CyN}) be an $n\text{-}CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an $n\text{-}CyNS$ in X. Then, the $n\text{-}CyN$*

1. M-interior (resp. $n\text{-}CyN\delta$-interior, $n\text{-}CyN\delta\mathcal{P}$-interior, $n\text{-}CyN\delta\mathcal{S}$-interior, $n\text{-}CyN\theta$-interior, $n\text{-}CyN\theta\mathcal{S}$-interior & $n\text{-}CyNe$-interior) of A (briefly, $n\text{-}CyNMint(A)$ (resp. $n\text{-}CyN\delta int(A)$, $n\text{-}CyN\delta\mathcal{P}int(A)$, $n\text{-}CyN\delta\mathcal{S}int(A)$, $n\text{-}CyN\theta\ int(A)$, $n\text{-}CyN\theta\mathcal{S}int(A)$ & $n\text{-}CyNeint(A)$) is defined by $n\text{-}CyNMint(A)$ (resp. $n\text{-}CyN\delta int(A)$, $n\text{-}CyN\delta\mathcal{P}int(A)$, $n\text{-}CyN\delta\mathcal{S}int(A)$, $n\text{-}CyN\theta int(A)$, $n\text{-}CyN\theta\mathcal{S}int(A)$ & $n\text{-}CyNeint(A)$) $= \bigcup \{G: G \subseteq A$ and G is a $n\text{-}CyNMos$ (resp. $n\text{-}CyN\delta os$, $n\text{-}CyN\delta\mathcal{P}os$, $n\text{-}CyN\delta\mathcal{S}os$, $n\text{-}CyN\theta os$, $n\text{-}CyN\theta\mathcal{S}os$ & $n\text{-}CyNeos$) in $X\}$,
2. M-closure (resp. $n\text{-}CyN\delta$-closure, $n\text{-}CyN\delta\mathcal{P}$-closure, $n\text{-}CyN\delta\mathcal{S}$-closure, $n\text{-}CyN\theta$-closure, $n\text{-}CyN\theta\mathcal{S}$-closure & $n\text{-}CyNe$-closure) of A (briefly, $n\text{-}CyNMcl(A)$ (resp. $n\text{-}CyN\delta cl(A)$, $n\text{-}CyN\delta\mathcal{P}cl(A)$, $n\text{-}CyN\delta\mathcal{S}cl(A)$, $n\text{-}CyN\theta cl(A)$, $n\text{-}CyN\theta\mathcal{S}cl(A)$ & $n\text{-}CyNecl(A)$) is defined by $n\text{-}CyNMcl(A)$ (resp. $n\text{-}CyN\delta cl(A)$, $n\text{-}CyN\delta\mathcal{P}cl(A)$, $n\text{-}CyN\delta\mathcal{S}cl(A)$, $n\text{-}CyN\theta cl(A)$, $n\text{-}CyN\theta\mathcal{S}cl(A)$ & $n\text{-}CyNecl(A)$) $= \bigcap \{K: K \subseteq A$ and K is a $n\text{-}CyNMcs$ (resp. $n\text{-}CyN\delta cs$, $n\text{-}CyN\delta\mathcal{P}cs$, $n\text{-}CyN\delta\mathcal{S}cs$, $n\text{-}CyN\theta cs$, $n\text{-}CyN\theta\mathcal{S}cs$ & $n\text{-}CyNecs$) in $X\}$.

Definition 2.20 *[2] Let (X,τ_1) and (Y,τ_2) be any two $n\text{-}CyNts$'s. A map $f:(X,\tau_1) \to (Y,\tau_2)$ is said to be a $n\text{-}CyN$*

1. δ-continuous map (briefly, $n\text{-}CyN\delta Cts$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyN\delta os$ in (X,τ_1),
2. δ-pre continuous map (briefly, $n\text{-}CyN\delta\mathcal{P}Cts$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1),
3. δ-semi continuous map (briefly, $n\text{-}CyN\delta\mathcal{S}Cts$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1),
4. θ-continuous map (briefly, $n\text{-}CyN\theta Cts$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyN\theta os$ in (X,τ_1),
5. θ-semi continuous map (briefly, $n\text{-}CyN\theta\mathcal{S}os$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyN\theta\mathcal{S}Cts$ in (X,τ_1),
6. e-continuous map (briefly, $n\text{-}CyNeCts$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyNeos$ in (X,τ_1),
7. M-continuous map (briefly, $n\text{-}CyNMCts$ map), if the inverse image of every $n\text{-}CyNos$ in (Y,τ_2) is a $n\text{-}CyNMos$ in (X,τ_1).

Definition 2.21 *[2] Let (X,τ_1) and (Y,τ_2) be any two $n\text{-}CyNts$'s. A map $f:(X,\tau_1) \to (Y,\tau_2)$ is said to be a $n\text{-}CyN$*

1. δ-irresolute map (briefly, $n\text{-}CyN\delta Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta os$ in (X,τ_1) for every $n\text{-}CyN\delta os$ B of (Y,τ_2),
2. δ-pre irresolute map (briefly, $n\text{-}CyN\delta\mathcal{P}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1) for every $n\text{-}CyN\delta\mathcal{P}os$ B of (Y,τ_2),
3. δ-semi irresolute map (briefly, $n\text{-}CyN\delta\mathcal{S}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1) for every $n\text{-}CyN\delta\mathcal{S}os$ B of (Y,τ_2),
4. θ-irresolute map (briefly,

$n\text{-}CyN\theta Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\theta os$ in (X,τ_1) for every $n\text{-}CyN\theta os$ B of (Y,τ_2),

5. θ-semi irresolute map (briefly, $n\text{-}CyN\theta\mathcal{S}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\theta\mathcal{S}os$ in (X,τ_1) for every $n\text{-}CyN\theta\mathcal{S}os$ B of (Y,τ_2),

6. e-irresolute map (briefly, $n\text{-}CyNeIrr$ map), if $f^{-1}(B)$ is a $n\text{-}CyNeos$ in (X,τ_1) for every $n\text{-}CyNeos$ B of (Y,τ_2),

7. M-irresolute map (briefly, $n\text{-}CyNMIrr$ map), if $f^{-1}(B)$ is a $n\text{-}CyNMos$ in (X,τ_1) for every $n\text{-}CyNMos$ B of (Y,τ_2).

3 $\boldsymbol{n}$-Cylindrical neutrosophic $\boldsymbol{M}$-connected spaces

We will introduce n-cylindrical neutrosophic M-connected and look at some of its feature in this section.

Definition 3.1 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$. Let $A=\{\langle\alpha_A(x),\beta_A(x),\gamma_A(x)\rangle : x\in X\}\}$ and $B=\{\langle\alpha_B(x),\beta_B(x),\gamma_B(x)\rangle : x\in X\}$ are $n\text{-}CyNS$'s in X. Then, (X,τ_{CyN}) is $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$ and $n\text{-}CyNM$) disconnected (briefly, $n\text{-}CyN\mathcal{D}Con$ (resp. $n\text{-}CyN\delta\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyNe\mathcal{D}Con$ and $n\text{-}CyNM\mathcal{D}Con$)) if there exists $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$ and $n\text{-}CyNMo$) sets A, B in X, $A\neq 0_{CyN}$, $B\neq 0_{CyN}$, such that $A\cup B=1_{CyN}$ and $A\cap B=0_{CyN}$. That is, $\alpha_A\vee\alpha_B=1_{CyN}$, $\beta_A\vee\beta_B=1_{CyN}$, and $\gamma_A\wedge\gamma_B=0_{CyN}$; $\alpha_A\wedge\alpha_B=0_{CyN}$, $\beta_A\wedge\beta_B=0_{CyN}$, and $\gamma_A\vee\gamma_B=1_{CyN}$.*

If X is not $n\text{-}CyN\mathcal{D}Con$ (resp. $n\text{-}CyN\delta\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyNe\mathcal{D}Con$ and $n\text{-}CyNM\mathcal{D}Con$) then it is said to be $n\text{-}CyN$ (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, θ, $\theta\mathcal{S}$, e and M) connected (briefly, $n\text{-}CyN\mathcal{C}on$ (resp. $n\text{-}CyN\delta\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}\mathcal{C}on$, $n\text{-}CyNe\mathcal{C}on$ and $n\text{-}CyNM\mathcal{C}on$)).

Example 3.1 Let $X=\{x_1,x_2\}$ and the n-cylindrical neutrosophic sets A_1,A_2,A_3 and A_4 in X are defined as

$A_1=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\}$,

$A_2=\{\langle x_1,0.1150731,0.50,0.1950739\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\}$,

$A_3=\{\langle x_1,0.1950739,0.50,0.1150731\rangle,\langle x_2,0.1750737,0.50,0.1350733\rangle\}$,

$A_4=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1350733,0.50,0.1750737\rangle\}$.

Then, we have $\tau_{CyN}=\{0_{CyN},1_{CyN},A_1,A_2,A_3,A_4\}$ be a n-cylindrical neutrosophic topology on X. Then,

1. A_1 and A_2 are $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets. Then, X is a $n\text{-}CyN\mathcal{C}on$ (resp. $n\text{-}CyN\delta\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}\mathcal{C}on$, $n\text{-}CyNe\mathcal{C}on$, and $n\text{-}CyNM\mathcal{C}on$). 2. A_2 and A_3 are $n\text{-}CyN\theta os$'s. Then, X is a $n\text{-}CyN\theta\mathcal{C}on$

Example 3.2 *In Example 3.1, A_3 and A_4 are $n\text{-}CyNMos$'s. Then X is $n\text{-}CyNM\mathcal{D}Con$.*

Definition 3.2 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$ and $\mathcal{S}=\{\langle\alpha_{\mathcal{S}}(x),\beta_{\mathcal{S}}(x),\beta_{\mathcal{S}}(x)\rangle : x\in X\}$ be an $n\text{-}CyNS$ in X.*

1. If there exists $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets $\mathcal{L}$ and $\mathcal{M}$ in X satisfying the following properties, then $\mathcal{S}$ is called $n\text{-}CyN$ (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, θ, $\theta\mathcal{S}$, e, and M) C_i-disconnected (briefly, $n\text{-}CyNC_i\mathcal{D}Con$ (resp. $n\text{-}CyN\delta C_i\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}C_i\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}C_i\mathcal{D}Con$, $n\text{-}CyN\theta C_i\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}C_i\mathcal{D}Con$, $n\text{-}CyNeC_i\mathcal{D}Con$, and $n\text{-}CyNMC_i\mathcal{D}Con$)), where $i=1,2,3,4$. (a) $\mathcal{S}\subseteq\mathcal{L}\cup\mathcal{M},\mathcal{L}\cap\mathcal{M}\subseteq\mathcal{S}^c,\mathcal{S}\cap\mathcal{L}\neq 0_{CyN},\mathcal{S}\cap\mathcal{M}\neq 0_{CyN}$.

(b) $\mathcal{S}\subseteq\mathcal{L}\cup\mathcal{M},\mathcal{S}\cap\mathcal{L}\cap\mathcal{M}=0_{CyN},\mathcal{S}\cap\mathcal{L}\neq 0_{CyN},\mathcal{S}\cap\mathcal{M}\neq 0_{CyN}$.

(c) $\mathcal{S}\subseteq\mathcal{L}\cup\mathcal{M},\mathcal{L}\cap\mathcal{M}\subseteq\mathcal{S}^c,\mathcal{L}\nsubseteq\mathcal{S}^c,\mathcal{M}\nsubseteq\mathcal{S}^c$.

(d) $\mathcal{S}\subseteq\mathcal{L}\cup\mathcal{M},\mathcal{S}\cap\mathcal{L}\cap\mathcal{M}=0_{CyN},\mathcal{L}\nsubseteq\mathcal{S}^c,\mathcal{M}\nsubseteq\mathcal{S}^c$.

2. S is said to be a $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, and $n\text{-}CyNM$) C_i-connected (briefly, $n\text{-}CyNC_i\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_i\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_i\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_i\mathcal{C}on$, $n\text{-}CyN\theta C_i\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_i\mathcal{C}on$, $n\text{-}CyNeC_i\mathcal{C}on$, and $n\text{-}CyNMC_i\mathcal{C}on$)), where $i=1,2,3,4$. If S is not (briefly, $n\text{-}CyNC_i\mathcal{D}Con$ (resp. $n\text{-}CyN\delta C_i\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}C_i\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}C_i\mathcal{D}Con$, $n\text{-}CyN\theta C_i\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}C_i\mathcal{D}Con$, $n\text{-}CyNeC_i\mathcal{D}Con$, and $n\text{-}CyNMC_i\mathcal{D}Con$)), where $i=1,2,3,4$. Obviously, we can obtain the following implications between several types of $n\text{-}CyNC_i\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_i\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_i\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_i\mathcal{C}on$, $n\text{-}CyN\theta C_i\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_i\mathcal{C}on$, $n\text{-}CyNeC_i\mathcal{C}on$, and $n\text{-}CyNMC_i\mathcal{C}on$), where $i=1,2,3,4$),

[(i)] (a) $n\text{-}CyNC_1\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_1\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_1\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_1\mathcal{C}on$, $n\text{-}CyN\theta C_1\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_1\mathcal{C}on$, $n\text{-}CyNeC_1\mathcal{C}on$, and $n\text{-}CyNMC_1\mathcal{C}on$) $\Rightarrow$ $n\text{-}CyNC_2\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_2\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_2\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\ C_2\mathcal{C}on$, $n\text{-}CyN\theta C_2\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_2\mathcal{C}on$, $n\text{-}CyNeC_2\mathcal{C}on$, and $n\text{-}CyNMC_2\mathcal{C}on$).

(b) $n\text{-}CyNC_1\mathcal{C}on$ (resp.

$n\text{-}CyN\delta C_1\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_1\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_1\mathcal{C}on$, $n\text{-}CyN\theta C_1\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_1\mathcal{C}on$, $n\text{-}CyNeC_1\mathcal{C}on$, and $n\text{-}CyNMC_1\mathcal{C}on$) $\Rightarrow$ $n\text{-}CyNC_3\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_3\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_3\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\ C_3\mathcal{C}on$, $n\text{-}CyN\theta C_3\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_3\mathcal{C}on$, $n\text{-}CyNeC_3\mathcal{C}on$, and $n\text{-}CyNMC_3\mathcal{C}on$).

(c) $n\text{-}CyNC_3\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_3\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_3\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_3\mathcal{C}on$, $n\text{-}CyN\theta C_3\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_3\mathcal{C}on$, $n\text{-}CyNeC_3\mathcal{C}on$, and $n\text{-}CyNMC_3\mathcal{C}on$) $\Rightarrow$ $n\text{-}CyNC_4\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_4\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_4\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\ C_4\mathcal{C}on$, $n\text{-}CyN\theta C_4\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_4\mathcal{C}on$, $n\text{-}CyNeC_4\mathcal{C}on$, and $n\text{-}CyNMC_4\mathcal{C}on$).

(d) $n\text{-}CyNC_2\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_2\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_2\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_2\mathcal{C}on$, $n\text{-}CyN\theta C_2\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_2\mathcal{C}on$, $n\text{-}CyNeC_2\mathcal{C}on$, and $n\text{-}CyNMC_2\mathcal{C}on$) $\Rightarrow$ $n\text{-}CyNC_4\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_4\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_4\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\ C_4\mathcal{C}on$, $n\text{-}CyN\theta C_4\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_4\mathcal{C}on$, $n\text{-}CyNeC_4\mathcal{C}on$, and $n\text{-}CyNMC_4\mathcal{C}on$).

(e) $n\text{-}CyNC_1\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_1\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_1\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_1\mathcal{C}on$, $n\text{-}CyN\theta C_1\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_1\mathcal{C}on$, $n\text{-}CyNeC_1\mathcal{C}on$, and $n\text{-}CyNMC_1\mathcal{C}on$) $\Rightarrow$ $n\text{-}CyNC_4\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_4\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_4\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\ C_4\mathcal{C}on$, $n\text{-}CyN\theta C_4\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_4\mathcal{C}on$, $n\text{-}CyNeC_4\mathcal{C}on$, and $n\text{-}CyNMC_4\mathcal{C}on$).

Remark 3.1 *From the above mentioned results. We get the diagram below.*

$$\begin{tikzcd}
n\text{-}CyNC_1\mathscr{C}on \arrow[r] \arrow[d] \arrow[dr] & n\text{-}CyNC_2\mathscr{C}on \arrow[d] \\
n\text{-}CyNC_3\mathscr{C}on \arrow[r] & n\text{-}CyNC_4\mathscr{C}on
\end{tikzcd}$$

Note: $A \rightarrow B$ denotes A implies B, but not convers

Example 3.3 *Let* $X = \{x_1, x_2\}$ *and the* n*-cylindrical neutrosophic sets* A_1, A_2, A_3 *and* A_4 *in* X *are defined as*

$A_1 = \{\langle x_1, 0.1850738,0.50,0.1250732\rangle, \langle x_2, 0.1650736,0.50,0.1450734\rangle\}$,

$A_2 = \{\langle x_1, 0.1950739,0.50,0.1150731\rangle, \langle x_2, 0.1750737,0.50,0.1350733\rangle\}$,

$A_3 = \{\langle x_1, 0.1150731,0.50,0.1950739\rangle, \langle x_2, 0.1350733,0.50,0.1750737\rangle\}$,

$A_4 = \{\langle x_1, 0.1850738,0.50,0.1250732\rangle, \langle x_2, 0.1750737,0.50,0.1350733\rangle\}$.

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ be a n-cylindrical neutrosophic topology on X. Let $\mathcal{S} = A_2$. If $\mathcal{L} = A_1$ and $\mathcal{M} = A_3$ are $n\text{-}CyNMos$'s, then $\mathcal{S}$ is

1. $n\text{-}CyNMC_2\mathcal{C}on$ but not $n\text{-}CyNMC_1\mathcal{C}on$,
2. $n\text{-}CyNMC_3\mathcal{C}on$ but not $n\text{-}CyNMC_1\mathcal{C}on$,
3. $n\text{-}CyNMC_4\mathcal{C}on$ but not $n\text{-}CyNMC_1\mathcal{C}on$.

Example 3.4 *Let* $X = \{x_1, x_2\}$ *and the* n*-cylindrical neutrosophic sets* A_1, A_2, A_3, A_4, A_5 *and* A_6 *in* X *are defined as*

$A_1 = \{\langle x_1, 0.1250732,0.50,0.1850738\rangle, \langle x_2, 0.1450734,0.50,0.1650736\rangle\}$,

$A_2 = \{\langle x_1, 0.1150731,0.50,0.1950739\rangle, \langle x_2, 0.1350733,0.50,0.1750737\rangle\}$,

$A_3 = \{\langle x_1, 1,1,0\rangle, \langle x_2, 0,0,1\rangle\}$,

$A_4 = \{\langle x_1, 0,0,1\rangle, \langle x_2, 1,1,0\rangle\}$,

$A_5 = \{\langle x_1, 0.1150731,0.50,1\rangle, \langle x_2, 0.1350733,0.50,0.1450734\rangle\}$,

$A_6 = \{\langle x_1, 0.1250732,0.50,0.1850738\rangle, \langle x_2, 0.1650736,0.50,0.1450734\rangle\}$.

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, A_1, A_2\}$ be a n-cylindrical neutrosophic topology on X. Let $\mathcal{S} = A_5$. If $\mathcal{L} = A_3$ and $\mathcal{M} = A_6$ are $n\text{-}CyNMos$'s, then $\mathcal{S}$ is $n\text{-}CyNMC_4\mathcal{C}on$ but not $n\text{-}CyNMC_3\mathcal{C}on$.

Example 3.5 *Let* $X = \{x_1, x_2\}$ *and the* n*-cylindrical neutrosophic sets* A_1, A_2, A_3, A_4 *and* A_5 *in* X *are defined as*

$A_1 = \{\langle x_1, 0.1250732,0.50,0.1850738\rangle, \langle x_2, 0.1450734,0.50,0.1650736\rangle\}$,

$A_2 = \{\langle x_1, 0.1150731,0.50,0.1950739\rangle, \langle x_2, 0.1350733,0.50,0.1750737\rangle\}$,

$A_3 = \{\langle x_1, 0,0,1\rangle, \langle x_2, 0.1350733,0.50,0.1450734\rangle\}$,

$A_4 = \{\langle x_1, 1,0,0\rangle, \langle x_2, 0.1750737,0,1\rangle\}$,

$A_5 = \{\langle x_1, 0.1250732,0.50,0.1850738\rangle, \langle x_2, 0,0.50,1450734\rangle\}$.

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, A_1, A_2\}$ be a n-cylindrical neutrosophic topology on X. Let $\mathcal{S} = A_3$. If $\mathcal{L} = A_5$ and $\mathcal{M} = A_4$ are $n\text{-}CyNMos$'s, then $\mathcal{S}$ is $n\text{-}CyNMC_4\mathcal{C}on$ but not $n\text{-}CyNMC_2\mathcal{C}on$.

Definition 3.3 *Let* (X, τ_{CyN}) *be a* $n\text{-}CyNts$ *is a* $n\text{-}CyN$ *(resp.* $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, *and* $n\text{-}CyNM$*)* C_5*-disconnected (briefly,* $n\text{-}CyNC_5\mathcal{D}Con$ *(resp.* $n\text{-}CyN\delta C_5\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}C_5\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}C_5\mathcal{D}Con$, $n\text{-}CyN\theta C_5\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}C_5\mathcal{D}Con$, $n\text{-}CyNeC_5\mathcal{D}Con$, *and* $n\text{-}CyNMC_5\mathcal{D}Con$*)) if there exists* $n\text{-}CyN$ *subset* $\mathcal{S}$ *in* X *which is both* $n\text{-}CyNo$ *(resp.* $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, *and* $n\text{-}CyNMo$*) sets and* $n\text{-}CyNc$ *(resp.* $n\text{-}CyN\delta c$, $n\text{-}CyN\delta\mathcal{P}c$, $n\text{-}CyN\delta\mathcal{S}c$, $n\text{-}CyN\theta c$, $n\text{-}CyN\theta\mathcal{S}c$, $n\text{-}CyNec$, *and* $n\text{-}CyNMc$*) sets in* X*, such that* $\mathcal{S} \neq 0_{CyN}$, $\mathcal{S} \neq 1_{CyN}$. *If* X *is not* $n\text{-}CyNC_5\mathcal{D}Con$ *(resp.* $n\text{-}CyN\delta C_5\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}C_5\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}C_5\mathcal{D}Con$,

n-$CyN\theta C_5\mathcal{D}Con$, n-$CyN\theta\mathcal{S}C_5\mathcal{D}Con$, n-$CyNeC_5\mathcal{D}Con$, and n-$CyNMC_5\mathcal{D}Con$) then it is said to be a n-CyN (resp. n-$CyN\delta$, n-$CyN\delta\mathcal{P}$, n-$CyN\delta\mathcal{S}$, n-$CyN\theta$, n-$CyN\theta\mathcal{S}$, n-$CyNe$, and n-$CyNM$) C_5-connected (briefly, n-$CyNC_5\mathcal{C}on$ (resp. n-$CyN\delta C_5\mathcal{C}on$, n-$CyN\delta\mathcal{P}C_5\mathcal{C}on$, n-$CyN\delta\mathcal{S}C_5\mathcal{C}on$, n-$CyN\theta C_5\mathcal{C}on$, n-$CyN\theta\mathcal{S}C_5\mathcal{C}on$, n-$CyNeC_5\mathcal{C}on$, and n-$CyNMC_5\mathcal{C}on$)).

Example 3.6

Let $X=\{x_1,x_2\}$ and the n-cylindrical neutrosophic sets A_1,A_2,A_3 and A_4 in X are defined as

$$A_1=\{\langle x_1,0.1650736,0.50,0.1450734\rangle,\langle x_2,0.1850738,0.50,0.1250732\rangle\},$$
$$A_2=\{\langle x_1,0.1750737,0.50,0.1350733\rangle,\langle x_2,0.1950739,0.50,0.1150731\rangle\},$$
$$A_3=\{\langle x_1,0.1350733,0.50,0.1750737\rangle,\langle x_2,0.1150731,0.50,0.1950739\rangle\},$$
$$A_4=\{\langle x_1,0.1750737,0.50,0.1350733\rangle,\langle x_2,0.1850738,0.50,0.1250732\rangle\}.$$

Then, we have $\tau_{CyN}=\{0_{CyN},1_{CyN},A_1,A_2,A_3,A_4\}$ be a n-cylindrical neutrosophic topology on X. Let $\mathcal{S}=A_3$ is n-$CyNC_5\mathcal{D}Con$ (resp. n-$CyN\delta C_5\mathcal{D}Con$, n-$CyN\delta\mathcal{P}C_5\mathcal{D}Con$, n-$CyN\delta\mathcal{S}C_5\mathcal{D}Con$, n-$CyN\theta C_5\mathcal{D}Con$, n-$CyN\theta\mathcal{S}C_5\mathcal{D}Con$, n-$CyNeC_5\mathcal{D}Con$, and n-$CyNMC_5\mathcal{D}Con$).

Theorem 3.1 *n-$CyNC_5\mathcal{C}on$ (resp. n-$CyN\delta C_5\mathcal{C}on$, n-$CyN\delta\mathcal{P}C_5\mathcal{C}on$, n-$CyN\delta\mathcal{S}C_5\mathcal{C}on$, n-$CyN\theta C_5\mathcal{C}on$, n-$CyN\theta\mathcal{S}\ C_5\mathcal{C}on$, n-$CyNeC_5\mathcal{C}on$, and n-$CyNMC_5\mathcal{C}on$)-ness implies n-$CyN\mathcal{C}on$ (resp. n-$CyN\delta\mathcal{C}on$, n-$CyN\delta\mathcal{P}\mathcal{C}on$, n-$CyN\delta\mathcal{S}\ \mathcal{C}on$, n-$CyN\theta\mathcal{C}on$, n-$CyN\theta\mathcal{S}\mathcal{C}on$, n-$CyNe\mathcal{C}on$, and n-$CyNM\mathcal{C}on$)-ness.*

Proof. Suppose that there exists non-empty n-$CyNMos$'s A and B, such that $A\cup B=1_{CyN}$ and $A\cap B=0_{CyN}$. In this case, we have $\alpha_A\vee\alpha_B=1_{CyN}$, $\beta_A\vee\beta_B=1_{CyN}$, and $\gamma_A\wedge\gamma_B=0_{CyN}$ and $\alpha_A\wedge\alpha_B=0_{CyN}$, $\beta_A\wedge\beta_B=0_{CyN}$, and $\gamma_A\vee\gamma_B=1_{CyN}$. Let $\mathcal{L}=\{x\in X:\alpha_A(x)>0_{CyN}\}$, $\mathcal{M}=\{x\in X:\alpha_B(x)=1_{CyN}\}$. If $x\in\mathcal{L}$, then $\alpha_A(x)=\beta_A(x)>0_{CyN}\Rightarrow\alpha_B(x)=\beta_B=0_{CyN}\Rightarrow\alpha_A(x)=\beta_A(x)=1_{CyN}\Rightarrow\gamma_A(x)=0_{CyN}\Rightarrow\gamma_B(x)=1_{CyN}$. If $x\in\mathcal{M}$, then $\alpha_B(x)=\beta_B(x)=1_{CyN}\Rightarrow\gamma_B(x)=0_{CyN}$. Hence, $\alpha_A=\beta_A=\gamma_B$ and $\gamma_A=\beta_B=\alpha_B$, that is $B=A^c$. Therefore, A is both n-$CyNMos$ and n-$CyNMcs$, and since $B\neq 0_{CyN}\Rightarrow A\neq 1_{CyN}$, $0_{CyN}\neq A\neq 1_{CyN}$. Thus, (X,τ_{CyN}) be a n-$CyNts$ is n-$CyNMC_5\mathcal{D}Con$. Similar situations exist in other scenarios.

However, as the following example demonstrates, the opposite may not be true.

Example 3.7 *Let $X=\{x_1,x_2\}$ and the n-cylindrical neutrosophic sets A_1,A_2,A_3 and A_4 in X are defined as*

$$A_1=\{\langle x_1,0.1750737,0.50,0.1350733\rangle,\langle x_2,0.1950739,0.50,0.1150731\rangle\},$$
$$A_2=\{\langle x_1,0.1450734,0.50,0.1650736\rangle,\langle x_2,0.1250732,0.50,0.1850738\rangle\},$$
$$A_3=\{\langle x_1,0.1350733,0.50,0.1750737\rangle,\langle x_2,0.1250732,0.50,0.1850738\rangle\},$$
$$A_4=\{\langle x_1,0.1350733,0.50,0.1750737\rangle,\langle x_2,0.1150731,0.50,0.1950739\rangle\}.$$

Then, we have $\tau_{CyN}=\{0_{CyN},1_{CyN},A_1,A_2,A_3,A_4\}$ be a n-cylindrical neutrosophic topology on X. Let A_1 and A_2 are n-$CyNMos$'s. Then, X is n-$CyNM\mathcal{C}on$ but not n-$CyNMC_5\mathcal{C}on$.

Theorem 3.2 *Let $f:(X,\tau_1)\to(Y,\tau_2)$ be a n-$CyNCts$ (resp. n-$CyN\delta Cts$, n-$CyN\delta\mathcal{P}Cts$, n-$CyN\delta\mathcal{S}Cts$, n-$CyN\theta Cts$, n-$CyN\theta\mathcal{S}Cts$, n-$CyNeCts$, and n-$CyNMCts$) map is a surjection. If X is n-$CyN\mathcal{C}on$ (resp. n-$CyN\delta\mathcal{C}on$, n-$CyN\delta\mathcal{P}\mathcal{C}on$, n-$CyN\delta\mathcal{S}\ \mathcal{C}on$, n-$CyN\theta\mathcal{C}on$, n-$CyN\theta\mathcal{S}\mathcal{C}on$, n-$CyNe\mathcal{C}on$, and n-$CyNM\mathcal{C}on$), then so is Y.*

Proof. Assume that Y is not n-$CyNM\mathcal{C}on$, then there exists non-empty n-$CyNMos$'s $\mathcal{L}$ and $\mathcal{M}$ in Y, such that $\mathcal{L}\cup\mathcal{M}=1_{CyN}$ and $\mathcal{L}\cap\mathcal{M}=0_{CyN}$. Since, f is n-$CyNMCts$ map, $A=f^{-1}(\mathcal{L})\neq 0_{CyN}$, $B=f^{-1}(\mathcal{M})\neq 0_{CyN}$, which are n-$CyNMos$'s in X and $f^{-1}(\mathcal{L})\cup f^{-1}(\mathcal{M})=f^{-1}(1_{CyN})=1_{CyN}$, which implies $A\cup B=1_{CyN}$. Also, $f^{-1}(\mathcal{L})\cap f^{-1}(\mathcal{M})=f^{-1}(0_{CyN})=0_{CyN}$, which implies $A\cap B=0_{CyN}$. Thus, X is n-$CyNM\mathcal{D}Con$, which is a contradiction to our hypothesis. Hence, Y is n-$CyNM\mathcal{C}on$.

Similar situations exist in other scenarios.

Corollary 3.1 *Let $f:(X,\tau_1)\to(Y,\tau_2)$ be a n-$CyNCts$ (resp. n-$CyN\delta Cts$, n-$CyN\delta\mathcal{P}Cts$, n-$CyN\delta\mathcal{S}Cts$, n-$CyN\theta Cts$, n-$CyN\theta\mathcal{S}Cts$, n-$CyNeCts$, and n-$CyNMCts$) map is a surjection. If X is n-$CyNC_5\mathcal{C}on$ (resp. n-$CyN\delta C_5\mathcal{C}on$, n-$CyN\delta\mathcal{P}C_5\mathcal{C}on$, n-$CyN\delta\mathcal{S}C_5\mathcal{C}on$, n-$CyN\theta C_5\mathcal{C}on$, n-$CyN\theta\mathcal{S}C_5\mathcal{C}on$, n-$CyNeC_5\mathcal{C}on$, and n-$CyNMC_5\mathcal{C}on$), then so is Y.*

Theorem 3.3 *Let $f:(X,\tau_1)\to(Y,\tau_2)$ be a n-$CyNIrr$ (resp. n-$CyN\delta Irr$, n-$CyN\delta\mathcal{P}Irr$, n-$CyN\delta\mathcal{S}Irr$, n-$CyN\theta Irr$, n-$CyN\theta\mathcal{S}Irr$, n-$CyNeIrr$, and n-$CyNMIrr$) map is a surjection. If X is n-$CyN\mathcal{C}on$ (resp. n-$CyN\delta\mathcal{C}on$, n-$CyN\delta\mathcal{P}\mathcal{C}on$, n-$CyN\delta\mathcal{S}\ \mathcal{C}on$, n-$CyN\theta\mathcal{C}on$, n-$CyN\theta\mathcal{S}\mathcal{C}on$, n-$CyNe\mathcal{C}on$, and n-$CyNM\mathcal{C}on$), then so is Y.*

Proof. Assume that Y is not n-$CyNM\mathcal{C}on$, then there exists non-empty n-$CyNMos$'s $\mathcal{L}$ and $\mathcal{M}$ in Y, such that $\mathcal{L}\cup\mathcal{M}=1_{CyN}$ and $\mathcal{L}\cap\mathcal{M}=0_{CyN}$. Since, f is n-$CyNMIrr$ mapping, $A=f^{-1}(\mathcal{L})\neq 0_{CyN}$, $B=f^{-1}(\mathcal{M})\neq 0_{CyN}$, which are n-$CyNMos$'s in X and $f^{-1}(\mathcal{L})\cup f^{-1}(\mathcal{M})=f^{-1}(1_{CyN})=1_{CyN}$, which implies $A\cup B=1_{CyN}$. Also, $f^{-1}(\mathcal{L})\cap f^{-1}(\mathcal{M})=f^{-1}(0_{CyN})=0_{CyN}$, which implies $A\cap B=0_{CyN}$. Thus, X is

$n\text{-}CyNM\mathcal{D}Con$, which is a contradiction to our hypothesis. Hence, Y is $n\text{-}CyNM\mathcal{C}on$.

Similar situations exist in other scenarios.

Corollary 3.2 *Let $f:(X,\tau_1)\to(Y,\tau_2)$ be a $n\text{-}CyNIrr$ (resp. $n\text{-}CyN\delta Irr$, $n\text{-}CyN\delta\mathcal{P}Irr$, $n\text{-}CyN\delta\mathcal{S}Irr$, $n\text{-}CyN\theta Irr$, $n\text{-}CyN\theta\mathcal{S}Irr$, $n\text{-}CyNeIrr$, and $n\text{-}CyNMIrr$) mapping is a surjection. If X is $n\text{-}CyNC_5\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_5\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_5\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_5\mathcal{C}on$, $n\text{-}CyN\theta C_5\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_5\mathcal{C}on$, $n\text{-}CyNeC_5\mathcal{C}on$, and $n\text{-}CyNMC_5\mathcal{C}on$), then so is Y.*

Theorem 3.4 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$ is $n\text{-}CyNC_5\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_5\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_5\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_5\mathcal{C}on$, $n\text{-}CyN\theta C_5\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_5\mathcal{C}on$, $n\text{-}CyNeC_5\mathcal{C}on$, and $n\text{-}CyNMC_5\mathcal{C}on$) if and only if there exists no nonempty $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets $\mathcal{L}$ and $\mathcal{M}$ in X such that $\mathcal{L}=\mathcal{M}^c$.*

Proof. Suppose that $\mathcal{L}$ and $\mathcal{M}$ are $n\text{-}CyNMos$'s in X such that $\mathcal{L}\neq 0_{CyN}$, $\mathcal{M}\neq 0_{CyN}$ and $\mathcal{L}=\mathcal{M}^c$. Since, $\mathcal{L}=\mathcal{M}^c$, $\mathcal{M}^c$ is a $n\text{-}CynMos$'s and $\mathcal{M}$ is $n\text{-}CyNMcs$ and $\mathcal{L}\neq 0_{CyN}$ implies $\mathcal{M}\neq 1_{CyN}$. But this is a contradiction to the fact that X is a $n\text{-}CyNMC_5\mathcal{C}on$.

Conversely, let $\mathcal{L}$ be a both $n\text{-}CyNMos$ and $\mathcal{M}$ is $n\text{-}CyNMcs$ in X such that $\mathcal{L}\neq 0_{CyN}$, $\mathcal{L}\neq 1_{CyN}$. Now, take $\mathcal{L}^c=\mathcal{M}$ is a $n\text{-}CyNMos$ and $\mathcal{L}\neq 1_{CyN}$ which implies $\mathcal{L}^c=\mathcal{M}\neq 0_{CyN}$ which is a contradiction. Hence, X is a $n\text{-}CyNMC_5\mathcal{C}on$.

Similar situations exist in other scenarios.

Theorem 3.5 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$ is $n\text{-}CyN\mathcal{C}on$ (resp. $n\text{-}CyN\delta\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}\mathcal{C}on$, $n\text{-}CyNe\mathcal{C}on$, and $n\text{-}CyNM\mathcal{C}on$) iff there exists no non-empty $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets $\mathcal{L}$ and $\mathcal{M}$ in X such that $\mathcal{L}=\mathcal{M}^c$.*

Proof. Necessity: Let $\mathcal{L}$ and $\mathcal{M}$ are two $n\text{-}CyNMos$'s in X such that $\mathcal{L}\neq 0_{CyN}$, $\mathcal{M}\neq 0_{CyN}$ and $\mathcal{L}=\mathcal{M}^c$. Therefore, $\mathcal{M}^c$ is a $n\text{-}CyNMcs$. Since, $\mathcal{L}\neq 0_{CyN}$, $\mathcal{M}\neq 1_{CyN}$. This implies $\mathcal{M}$ is a proper $n\text{-}CyN$ subset which is both $n\text{-}CyNMos$ and $n\text{-}CyNMcs$ in X. Hence, X is not a $N\text{-}CyNM\mathcal{C}on$ space. But this is a contradiction to our hypothesis. Thus, there exist no non-empty $n\text{-}CyNMos$'s $\mathcal{L}$ and $\mathcal{M}$ in X, such that $\mathcal{L}=\mathcal{M}^c$.

Sufficiency: Let $\mathcal{L}$ be both $n\text{-}CyNMos$ and $n\text{-}CyNMcs$ in X such that $\mathcal{L}\neq 0_{CyN}$, $\mathcal{L}\neq 1_{CyN}$. Now, let $\mathcal{M}=\mathcal{L}^c$. Then, $\mathcal{M}$ is a $n\text{-}CyNMos$ and $\mathcal{M}\neq 1_{CyN}$. This implies $\mathcal{L}^c=\mathcal{M}\neq 0_{CyN}$, which is a contradiction to our hypothesis. Therefore, X is $n\text{-}CyNM\mathcal{C}on$.

Similar situations exist in other scenarios.

Theorem 3.6 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$ is $n\text{-}CyN\mathcal{C}on$ (resp. $n\text{-}CyN\delta\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}\mathcal{C}on$, $n\text{-}CyNe\mathcal{C}on$, and $n\text{-}CyNM\mathcal{C}on$) iff there exists no non-empty $n\text{-}CyN$ subsets $\mathcal{L}$ and $\mathcal{M}$ in X, such that $\mathcal{L}=\mathcal{M}^c$, $\mathcal{M}=(n\text{-}CyNcl(\mathcal{L}))^c$ (resp. $\mathcal{M}=(n\text{-}CyN\delta cl(\mathcal{L}))^c$, $\mathcal{M}=(n\text{-}CyN\delta\mathcal{P}cl(\mathcal{L}))^c$, $\mathcal{M}=(n\text{-}CyN\delta\mathcal{S}cl(\mathcal{L}))^c$, $\mathcal{M}=(n\text{-}CyN\theta cl(\mathcal{L}))^c$, $\mathcal{M}=(n\text{-}CyN\theta\mathcal{S}cl(\mathcal{L}))^c$, $\mathcal{M}=(n\text{-}CyNecl(\mathcal{L}))^c$, and $\mathcal{M}=(n\text{-}CyNMcl(\mathcal{L}))^c$) and $\mathcal{L}=(n\text{-}CyNcl(\mathcal{M}))^c$ (resp. $\mathcal{L}=(n\text{-}CyN\delta cl(\mathcal{M}))^c$, $\mathcal{L}=(n\text{-}CyN\delta\mathcal{P}cl(\mathcal{M}))^c$, $\mathcal{L}=(n\text{-}CyN\delta\mathcal{S}cl(\mathcal{M}))^c$, $\mathcal{L}=(n\text{-}CyN\theta cl(\mathcal{M}))^c$, $\mathcal{L}=(n\text{-}CyN\theta\mathcal{S}cl(\mathcal{M}))^c$, $\mathcal{L}=(n\text{-}CyNecl(\mathcal{M}))^c$, and $\mathcal{L}=(n\text{-}CyNMcl(\mathcal{M}))^c$).*

Proof. Necessity: Let $\mathcal{L}$ and $\mathcal{M}$ be two $n\text{-}CyN$ subsets in X such that $\mathcal{L}\neq 0_{CyN}$, $\mathcal{M}\neq 0_{CyN}$ and $\mathcal{L}=\mathcal{M}^c$, $\mathcal{M}=(n\text{-}CyNMcl(\mathcal{L}))^c$ and $\mathcal{L}=(n\text{-}CyNMcl(\mathcal{M}))^c$. Since, $(n\text{-}CyNMcl(\mathcal{L}))^c$ and $(n\text{-}CyNMcl(\mathcal{M}))^c$ are $n\text{-}CyNMos$'s in X, $\mathcal{L}$ and $\mathcal{M}$ are $n\text{-}CyNMos$ in X. This implies X is not a $n\text{-}CyNM\mathcal{C}on$ space, which is a contradiction. Therefore, there exists no non-empty $n\text{-}CyNMos$ $\mathcal{L}$ and $\mathcal{M}$ in X, such that $\mathcal{L}=\mathcal{M}^c$, $\mathcal{M}=(n\text{-}CyNMcl(\mathcal{L}))^c$ and $\mathcal{L}=(n\text{-}CyNMcl(\mathcal{M}))^c$.

Sufficiency: Let $\mathcal{L}$ be both $n\text{-}CyNMos$ and $n\text{-}CyNMcs$ in X such that $\mathcal{L}\neq 0_{CyN}$, $\mathcal{L}\neq 1_{CyN}$. Now, by taking $\mathcal{M}=\mathcal{L}^c$, we obtain a contradiction to our hypothesis. Hence, X is $n\text{-}CyNM\mathcal{C}on$ space.

Similar situations exist in other scenarios.

Definition 3.4 *Let (X,τ_{CyN}) be an $n\text{-}CyNts$ is $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, and $n\text{-}CyNM$) strongly connected (briefly, $n\text{-}CyN\mathcal{S}tCon$ (resp. $n\text{-}CyN\delta\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}t\ Con$, $n\text{-}CyN\theta\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyNe\mathcal{S}tCon$, and $n\text{-}CyNM\mathcal{S}tCon$)), if there exists a non-empty $n\text{-}CyNc$ (resp. $n\text{-}CyN\delta c$, $n\text{-}CyN\delta\mathcal{P}c$, $n\text{-}CyN\delta\mathcal{S}c$, $n\text{-}CyN\theta c$, $n\text{-}CyN\theta\mathcal{S}c$, $n\text{-}CyNec$, and $n\text{-}CyNMc$) sets A and B in X such that $alpha_A+\alpha_B\leq 1_{CyN}$, $\beta_A+\beta_B\leq 1_{CyN}$, and $\gamma_A+\gamma_B\geq 1_{CyN}$.*

In other words, a $n\text{-}CyNts$ X is $n\text{-}CyN\mathcal{S}tCon$ (resp. $n\text{-}CyN\delta\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyNe\mathcal{S}tCon$, and $n\text{-}CyNM\mathcal{S}tCon$), if there exists no non-empty $n\text{-}CyNc$ (resp. $n\text{-}CyN\delta c$, $n\text{-}CyN\delta\mathcal{P}c$, $n\text{-}CyN\delta\mathcal{S}c$, $n\text{-}CyN\theta c$, $n\text{-}CyN\theta\mathcal{S}c$, $n\text{-}CyNec$, and $n\text{-}CyNMc$) sets A and B in X such that $A\cap B=0_{CyN}$.

Example 3.8 *Let $X=\{x_1,x_2\}$ and the n-cylindrical neutrosophic sets A_1, A_2, A_3 and A_4 in X are defined as*

$$A_1=\{\langle x_1,0.1250732,0.50,0.1850738\rangle,\langle x_2,0.1450734,0.50,0.1650736\rangle\},$$

$$A_2=$$

$\{\langle x_1, 0.1150731,0.50,0.1950739\rangle, \langle x_2, 0.1350733,0.50,0.1750737\rangle\}$,

$$A_3 = \{\langle x_1, 1,1,0\rangle, \langle x_2, 0,0,1\rangle\},$$
$$A_4 = \{\langle x_1, 0,0,1\rangle, \langle x_2, 1,1,0\rangle\}.$$

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, A_1, A_2\}$ be a n-cylindrical neutrosophic topology on X. Let A_3 and A_4 are $n\text{-}CyNMcs$'s. Then, $n\text{-}CyNM\mathcal{S}tCon$.

Theorem 3.7 *Let (X, τ_{CyN}) be a $n\text{-}CyNts$ is $n\text{-}CyN\mathcal{S}tCon$ (resp. $n\text{-}CyN\delta\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{S}\ \mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyNe\mathcal{S}tCon$, and $n\text{-}CyNM\mathcal{S}tCon$), if there exists no non-empty $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets A and B in X, $A \neq 1_{CyN} \neq B$ such that $\alpha_A + \alpha_B \geq 1_{CyN}$.*

Proof. Let A and B be $n\text{-}CyNMos$'s in X, such that $A \neq 1_{CyN} \neq B$ and $\alpha_A + \alpha_B \geq 1_{CyN}$. If we take $C = A^c$ and $D = B^c$, then C and D become $n\text{-}CyNMcs$'s in X and $C \neq 0_{CyN} \neq D$, $\alpha_C + \alpha_D \leq 1_{CyN}$, a contradiction. Conversely, use a similar technique as above.

Similar situations exist in other scenarios.

Theorem 3.8 *Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a $n\text{-}CyNCts$ (resp. $n\text{-}CyN\delta Cts$, $n\text{-}CyN\delta\mathcal{P}Cts$, $n\text{-}CyN\delta\mathcal{S}Cts$, $n\text{-}CyN\theta Cts$, $n\text{-}CyN\theta\mathcal{S}Cts$, $n\text{-}CyNeCts$, and $n\text{-}CyNMCts$) map is a surjection. If X is $n\text{-}CyN\mathcal{S}tCon$ (resp. $n\text{-}CyN\delta\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyNe\mathcal{S}tCon$, and $n\text{-}CyNM\mathcal{S}tCon$), then so is Y.*

Proof. Assume that Y is not $n\text{-}CyNM\mathcal{S}tCon$, then there exists non-empty $n\text{-}CyNcs$'s $\mathcal{L}$ and $\mathcal{M}$ in Y, such that $\mathcal{L} \cup \mathcal{M} \neq 0_{CyN}$ and $\mathcal{L} \cap \mathcal{M} = 0_{CyN}$. Since, f is $n\text{-}CyNCts$ mapping, $A = f^{-1}(\mathcal{L}) \neq 0_{CyN}$, $B = f^{-1}(\mathcal{M}) \neq 0_{CyN}$, which are $n\text{-}CyNcs$'s in X and $f^{-1}(\mathcal{L}) \cap f^{-1}(\mathcal{M}) = f^{-1}(0_{CyN}) = 0_{CyN}$, which implies $A \cap B = 0_{CyN}$. Thus, X is $n\text{-}CyNM\mathcal{S}tCon$, which is a contradiction to our hypothesis. Hence, Y is $n\text{-}CyNM\mathcal{S}tCon$.

Similar situations exist in other scenarios.

Theorem 3.9 *Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a $n\text{-}CyNIrr$ (resp. $n\text{-}CyN\delta Irr$, $n\text{-}CyN\delta\mathcal{P}Irr$, $n\text{-}CyN\delta\mathcal{S}Irr$, $n\text{-}CyN\theta Irr$, $n\text{-}CyN\theta\mathcal{S}Irr$, $n\text{-}CyNeIrr$, and $n\text{-}CyNMIrr$) mapping is a surjection. If X is $n\text{-}CyN\mathcal{S}tCon$ (resp. $n\text{-}CyN\delta\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{S}\ \mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyNe\mathcal{S}tCon$, and $n\text{-}CyNM\mathcal{S}tCon$), then so is Y.*

Proof. Assume that Y is not $n\text{-}CyNM\mathcal{S}tCon$, then there exists non-empty $n\text{-}CyNMcs$'s $\mathcal{L}$ and $\mathcal{M}$ in Y, such that $\mathcal{L} \cap \mathcal{M} = 0_{CyN}$. Since, f is $n\text{-}CyNMIrr$ mapping, $A = f^{-1}(\mathcal{L}) \neq 0_{CyN}$, $B = f^{-1}(\mathcal{M}) \neq 0_{CyN}$, which are $n\text{-}CyNMcs$'s in X and $f^{-1}(\mathcal{L}) \cap f^{-1}(\mathcal{M}) = f^{-1}(0_{CyN}) = 0_{CyN}$, which implies $A \cap B = 0_{CyN}$. Thus, X is $n\text{-}CyNM\mathcal{S}tCon$, which is a contradiction to our hypothesis. Hence, Y is $n\text{-}CyNM\mathcal{S}tCon$.

Similar situations exist in other scenarios.

Remark 3.2 *$n\text{-}CyN\mathcal{S}tCon$ (resp. $n\text{-}CyN\delta\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}tCon$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}tCon$, $n\text{-}CyN\theta\mathcal{S}\ \mathcal{S}tCon$, $n\text{-}CyNe\mathcal{S}tCon$, and $n\text{-}CyNM\mathcal{S}tCon$) and $n\text{-}CyNC_5\mathcal{C}on$ (resp. $n\text{-}CyN\delta C_5\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}C_5\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}C_5\ \mathcal{C}on$, $n\text{-}CyN\theta C_5\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}C_5\mathcal{C}on$, $n\text{-}CyNeC_5\mathcal{C}on$, and $n\text{-}CyNMC_5\mathcal{C}on$) are independent.*

Example 3.9 *In Example 3.8, A_3 and A_4 are $n\text{-}CyNMos$'s. Then, $n\text{-}CyNM\mathcal{S}tCon$ but not $n\text{-}CyNMC_5\mathcal{C}on$.*

Example 3.10 *In Example 3.1, A_2 and A_3 are $n\text{-}CyNMos$'s. Then, X is $n\text{-}CyNMC_5\mathcal{C}on$ but not $n\text{-}CyNM\mathcal{S}tCon$.*

4 Real Life Application: Selection of an Optimal Nanocarrier in Drug Delivery Technology

Modern drug delivery technology depends heavily on the choice of a suitable nanocarrier system to transport a therapeutic agent to its target site while minimising side effects. In practice, an expert panel (pharmacologists, formulation scientists and clinicians) evaluates each candidate nanocarrier against several performance criteria, and the judgment of every expert typically carries three simultaneous components: the degree to which the carrier *satisfies* a criterion (positive evidence α), the degree of *hesitation/indeterminacy* in the assessment (neutral evidence β), and the degree to which the carrier *fails* the criterion (negative evidence γ). Since α and γ are dependent components while β is completely independent, and since α, γ are permitted to jointly range over a wider region than in an ordinary neutrosophic set (governed only by $\alpha^n(x) + \gamma^n(x) \leq 1$), an $n\text{-}CyNS$ is a natural and flexible tool to model this kind of expert evaluation, allowing α and γ to be simultaneously assigned larger values than classical neutrosophic sets would permit. We now use the $n\text{-}CyNS$ framework (with $n = 3$) to rank a set of nanocarrier systems and identify the most suitable one for targeted drug delivery.

Definition 4.1 *Let $A_j = \langle \alpha_j, \beta_j, \gamma_j \rangle$, $j = 1,2,\ldots,k$, be a collection of $n\text{-}CyNN$'s corresponding to the evaluation of an alternative under k criteria, and let $w = (w_1, w_2, \ldots, w_k)$ be a weight vector of the criteria with $w_j \in [0,1]$ and $\sum_{j=1}^{k} w_j = 1$. The n-cylindrical neutrosophic weighted averaging operator (briefly, $n\text{-}CyNWA$) aggregates $A_1, A_2, \ldots, A_k$ into a single $n\text{-}CyNN$ as*

$$n - CyNWA_w(A_1, A_2, \ldots, A_k) = \Big\langle \Big(1 - \prod_{j=1}^{k} (1 - \alpha_j^{n}$$

Remark 4.1 *If each A_j satisfies $0 \leq \beta_j \leq 1$ and $\alpha_j^n + \gamma_j^n \leq 1$, then the aggregated value $n\text{-}CyNWA_w(A_1, \ldots, A_k) = \langle \alpha, \beta, \gamma \rangle$ obtained from Definition 4.1 is again a valid $n\text{-}CyNN$, i.e., it satisfies $0 \leq \beta \leq 1$ and $\alpha^n + \gamma^n \leq 1$. Indeed, $\gamma^n = \prod_{j=1}^{k} (\gamma_j^n)^{w_j} \leq \prod_{j=1}^{k} (1 - \alpha_j^n)^{w_j} = 1 - \alpha^n$, since $\gamma_j^n \leq 1 - \alpha_j^n$ for every j and the weighted geometric mean is monotone.*

Definition 4.2 *The score of an $n\text{-}CyNN$ $A = \langle \alpha, \beta, \gamma \rangle$ is defined by*

$$S(A) = \frac{2 + \alpha^n - \beta - \gamma^n}{3}$$

For two $n\text{-}CyNN$'s A_1 and A_2, if $S(A_1) > S(A_2)$, then A_1 is preferred over A_2, written $A_1 \succ A_2$.

Example 4.1 *Let $X = \{D_1, D_2, D_3, D_4, D_5\}$ be a set of five candidate nanocarrier systems for a targeted anti-cancer drug delivery formulation, namely*

D_1: Liposomal nanocarriers, D_2: PLGA-based polymeric nanoparticles, D_3: Solid lipid nanoparticles,
D_4: Dendrimers, D_5: Micellar nanocarriers,

which are to be evaluated against the criteria

C_1: Drug encapsulation efficiency, C_2: Controlled/sustained release behaviour, C_3: Biocompatibility & biodegradability,
C_4: Cellular targeting specificity, C_5: Physicochemical storage stability,

with weight vector $w = (0.25, 0.25, 0.20, 0.20, 0.10)$ (reflecting that release control and encapsulation efficiency are considered slightly more critical than storage stability). An expert panel records its assessment of each D_i under each C_j as an $n\text{-}CyNN$ $\langle \alpha, \beta, \gamma \rangle$ ($n = 3$), presented in Table 4.1.

Table 1: Decision matrix of $n\text{-}CyNN$'s $\langle \alpha, \beta, \gamma \rangle$

	C_1	C_2	C_3	C_4	C_5
	⟨0.75,0.20	⟨0.65,0.30	⟨0.80,0.15	⟨0.60,0.35	⟨0.55,0.40
	⟨0.80,0.15	⟨0.85,0.10	⟨0.75,0.20	⟨0.80,0.15	⟨0.70,0.25
	⟨0.70,0.25	⟨0.60,0.30	⟨0.85,0.10	⟨0.55,0.35	⟨0.75,0.20
	⟨0.85,0.10	⟨0.70,0.25	⟨0.60,0.30	⟨0.75,0.20	⟨0.50,0.40
	⟨0.65,0.30	⟨0.75,0.20	⟨0.70,0.25	⟨0.65,0.30	⟨0.60,0.30

Every entry of Table 4.1 satisfies $0 \leq \beta \leq 1$ and $\alpha^3 + \gamma^3 \leq 1$, hence each is a valid $n\text{-}CyNN$ ($n = 3$). Applying the $n\text{-}CyNWA_w$ operator of Definition 4.1 row-wise aggregates the criterion-wise evaluations of each D_i into a single $n\text{-}CyNN$, and Definition 4.2 then yields its score, as recorded in Table 4.1.

Table 2: Aggregated $n\text{-}CyNWA_w$ value and score $S(D_i)$

Nanocarrier	α	β	γ	$S(D_i)$	Rank
D_1 (Liposomal)	0.7033	0.2505	0.5720	0.6368	4
D_2 (PLGA nanoparticles)	0.7987	0.1511	0.5055	0.7431	1
D_3 (Solid lipid NPs)	0.7119	0.2279	0.5771	0.6469	3
D_4 (Dendrimers)	0.7382	0.2067	0.5436	0.6783	2
D_5 (Micelles)	0.6860	0.2614	0.5679	0.6261	5

Since $S(D_2) > S(D_4) > S(D_3) > S(D_1) > S(D_5)$, we obtain the ranking

$$D_2 \succ D_4 \succ D_3 \succ D_1 \succ D$$

so that D_2, the PLGA-based polymeric nanoparticle, is the optimal nanocarrier for the targeted drug delivery formulation under consideration, followed by dendrimers.

Remark 4.2 *The independence of the neutral component β from α and γ in an $n\text{-}CyNS$, together with the relaxed constraint $\alpha^n + \gamma^n \leq 1$, allows the decision maker to record stronger simultaneous confidence and doubt about a nanocarrier's performance (e.g., a formulation may be strongly effective on one attribute such as encapsulation while still carrying non-trivial toxicity concerns) than is possible in an ordinary neutrosophic model. This illustrates the practical relevance of $n\text{-}CyNS$ based topological structures, of the kind developed in Sections 2 and 3, to multi-criteria decision-making problems arising in drug delivery technology and, more broadly, in biomedical and pharmaceutical sciences.*

5 Conclusion

In this paper, we introduce the concepts of n-cylindrical neutrosophic M-connected and M-disconnected spaces within the framework of n-cylindrical neutrosophic topological spaces. Further, to demonstrate the practical significance of this framework, we developed an n-cylindrical neutrosophic weighted averaging operator together with a score function, and applied them to a real-life multi-criteria decision-making problem concerning the selection of an optimal nanocarrier for targeted drug delivery. Among the five candidate nanocarrier systems considered, PLGA-based polymeric nanoparticles emerged as the most suitable choice, followed by dendrimers, illustrating how the flexibility of $n\text{-}CyNS$'s in modelling simultaneous positive, neutral and negative expert evidence makes them well-suited to biomedical decision-making problems of this kind. It is anticipated that the findings presented herein will support and inspire further research in n-cylindrical neutrosophic topology, contributing to the development of a more comprehensive theoretical framework and promoting practical applications in various fields.

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