RESEARCH PAPER

Structural Analysis of Separated Sets and Compactness Using M-Open Sets in n-Cylindrical Neutrosophic Topological Spaces: Application to Drug Delivery

R. Vijayalakshmi [1], A. Sugapriya [2], K. Bhagyasri [3] and S. Priya [4]

[1,2,4]Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.
Email: viji_lakshmi80@rediffmail.com, sugamaths24@gmail.com, pre9433@gmail.com
[1] Arignar Anna Government Arts College, Namakkal - 637 002, India.
[3] Department of Mathematics, National College (Autonomous), Tiruchirappalli -620 001, TamilNadu, India, affliated to Bharathidasan University.
Email: bhagyasriumashankar@gmail.com
[4] Department of Mathematics, M. Kumarasamy College of Engineering, Karur- 639 113, TamilNadu, India. Email: pre9433@gmail.com
Corresponding Authors: K. Bhagyasri, S. Priya

Abstract: In this paper, we explore several topological concepts within the framework of n-cylindrical neutrosophic topological spaces. In particular, we examine n-cylindrical neutrosophic M-separated sets and M-compact spaces. We establish fundamental properties and theorems related to separated sets and compactness under these generalized notions. As a real-life illustration of the proposed framework, we further develop an n-cylindrical neutrosophic separation measure, based on the closeness of an alternative's aggregated profile to the extremal n-$CyNS$'s 0_{CyN} and 1_{CyN}, and apply it to classify and rank five transdermal drug delivery patches in drug delivery technology.

Keywords and phrases: n-cylindrical neutrosophic M-open set, n-cylindrical neutrosophic M-separated set and n-cylindrical neutrosophic M-compact spaces.

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1 Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [30], Chang [9] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy sets (ifs), was introduced by Atanassov in 1986 [4]. Building on this, Coker [10] introduced the concept of intuitionistic fuzzy topological spaces ($ifts$) based on ifs. Jeon et al. [12] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [21, 22], a new direction in uncertainty modeling emerged. Salama and Alblowi [17] introduced neutrosophic crisp sets and neutrosophic topological spaces (Nts), extending $ifts$ and incorporating degrees of membership, indeterminacy, and non-membership for each element. Neutrosophic has since formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [3] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [27] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [18] introduced the concept of n-cylindrical neutrosophic sets (abbreviated as n-$CyNS$), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic sets (ns), n-$CyNs$ represents the most extensive generalization of fuzzy sets (fs). In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [20] introduced the notion of n-CyN continuity for functions between two n-cylindrical fuzzy neutrosophic topological spaces (n-$CyNts$). They also defined the n-CyN interior (n-$CyNint$) and n-CyN closure (n-$CyNcl$) of subsets within n-$CyNts$.

In a related development, El-Maghrabi and Al-Juhani [11] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [16] also found M-open sets in nano topological spaces. Vadivel et al. [23, 2, 25] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [13] and Vadivel et al. [26] introduced M-open sets in fuzzy and neutrosophic nano topological spaces. Anandhi et al. [1, 2, 28, 29] introduced n-Cylindrical neutrosophic M open sets in n-Cylindrical neutrosophic topological spaces.

Beyond their theoretical development, the notion of separation is also of direct practical value: many real-world selection problems reduce to measuring how far a candidate's evaluation profile is from an ideal acceptable profile and from an ideal unacceptable profile, which is conceptually close to the idea of separated sets studied in this paper. Drug delivery technology offers a natural setting for such a problem, since a transdermal patch or nanocarrier is judged against several criteria (permeation efficiency, dosage accuracy, irritation resistance, mechanical stability, patient comfort, and so on) by expert panels whose assessments carry positive, neutral and negative evidence simultaneously. Motivated by this, in the present paper we also demonstrate how a separation (distance) measure built on the $n\text{-}CyNS$ framework, using the extremal sets 0_{CyN} and 1_{CyN} as reference points, can be applied to classify and rank candidate drug delivery patches.

The subsequent sections of this paper are organized as follows: Section 2 provides a brief review of fundamental definitions related to intuitionistic fuzzy sets (ifs's), neutrosophic sets (NS's), n-cylindrical neutrosophic sets ($n\text{-}CyNS$'s), and mappings on n-cylindrical neutrosophic topological spaces ($n\text{-}CyNts$). Sections 3 and 4 introduce the concepts of $n\text{-}CyNM$-separated sets and $n\text{-}CyNM$-compact spaces within $n\text{-}CyNts$, along with an exploration of their fundamental properties supported by illustrative examples. Section 5 presents a real-life application of the $n\text{-}CyNS$ framework, based on a separation measure, to the classification and ranking of transdermal drug delivery patches. The paper is concluded in section 6.

2 Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 *[30] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: A \to [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A, for all $x \in X$.*

Definition 2.2 *[9] A fuzzy topological space (briefly, fts) is a pair (X,τ), where X is any set and τ is a family of fuzzy sets in X satisfying following axioms:*

1. $\phi, X \in \tau$,
2. If $A, B \in \tau$, then $A \cap B$ in τ,
3. If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 *[4] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x) : x \in X\rangle\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\alpha_A(x)$ and $\gamma_A(x)$ satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$ $ifs(X)$ denotes the set of all the ifs's on X.*

Definition 2.4 *[22] An neutrosophic set A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral and $\gamma_A(x)$ is the degree of negative membership. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.*

Definition 2.5 *[17] An neutrosophic topology (Nt) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:*

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\cup G_i \in \tau_N$, for all $\{G_i : i \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called a neutrosophic topological spaces (briefly, Nts) and any neutrosophic set in τ is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_X are called neutrosophic open sets. A neutrosophic set F is closed if and only if $C(F)$ is neutrosophic open.

Definition 2.6 *[18] An n-cylindrical neutrosophic set (briefly, $n\text{-}CyNS$) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, is an integer. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components and $\beta_A(x)$ is 100% independent.*

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-cylindrical neutrosophic number (briefly, n - $CyNN$) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 *[18] Let $\{A_i : i \in I\}$ be an* *arbitrary family of n-$CyNS$ in X. Then,* $\cap A_i = \{\langle x, inf(\alpha_{A_i}(x)), inf(\beta_{A_i}(x)),\ sup(\gamma_{A_i}(x))\rangle : x \in X\}$, $\cup A_i = \{\langle x, sup(\alpha_{A_i}(x)), sup(\beta_{A_i}(x)), inf(\gamma_{A_i}(x))\rangle : x \in X\}$.

Definition 2.8 *[18]* $0_{CyN} = \{\langle x, 0,0,1\rangle : x \in X\}$ *and* $1_{CyN} = \{\langle x, 1,1,0\rangle : x \in X\}$

Definition 2.9 *[18]* **(The Basic Connectives)** *Let $\tau_{CyN}(X)$ denote the family of all n-$CyNS$'s on X.*

1. **Inclusion:** For every two $A, B \in \tau_{CyN}(X)$, the inclusion of two n-$CyNS$'s A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \le \alpha_B(x)$ and $\beta_A(x) \le \beta_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$) and ($A \subseteq B$ and $B \subseteq A$).
2. **Union:** For every two $A, B \in \tau_{CyN}(X)$, the union of two n-$CyNS$'s A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.
3. **Intersection:** For every two $A, B \in \tau_{CyN}(X)$, the intersection of two n-$CyNS$'s A and B is $A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.
4. **Complementary:** For every $A \in \tau_{CyN}(X)$, the complement of an n-$CyNS$ A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x),\ \alpha_A(x)\rangle : x \in X\}$.

Remark 2.1 *[18]*

1. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
2. $A \cup B = B \cup A$ & $A \cap B = B \cap A$,
3. $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$,
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,
5. $A \cap A = A$ & $A \cup A = A$,
6. De Morgan's Law for A & B ie., $(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$.

Definition 2.10 *[20] An n-cylindrical neutrosophic topology (briefly, n-$CyNt$) on a non-empty set X is a family, τ_{CyN}, of n-$CyNS$ in X which satisfies the following conditions:*

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. $A_1 \cap A_2 \in \tau_{CyN}$,
3. $\cup A_i \in \tau_{CyN}$, for any arbitrary family $A_i \in \tau_{CyN}, i \in I$.

The pair (X, τ_{CyN}) is called an n-cylindrical neutrosophic topological Spaces (briefly, n-$CyNts$) and any n-$CyNS$ belongs to τ_{CyN} is called an n-cylindrical neutrosophic open set (briefly, n-$CyNos$) and the complement of n-$CyNos$ is called n-cylindrical neutrosophic closed set (briefly, n-$CyNcs$) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-$CyNts$ and the topology containing all the n-CyN subsets is called discrete n-$CyNts$.

Remark 2.2 *[20] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-$CyNts$ as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.*

Definition 2.11 *[20] Let A and B be two n-cylindrical neutrosophic subsets of an n-$CyNts$. B is called neighbourhood of A if there exists an n-$CyNos$, O such that $A \subset O \subset B$.*

Proposition 2.1 *[20] $A \subset X$ is n-cylindrical neutrosophic open in (X, τ_{CyN}) if and only if it carries a neighbourhood of its subsets.*

Definition 2.12 *[20] Let (X, τ_{CyN}) be an n-$CyNts$ and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic interior (briefly, n-$CyNint$) is defined as the n-CyN union of all n-CyN open subsets of X. ie, n-$CyNint(A) = \cup\ \{G : G \in \tau_{CyN}$ and $G \subseteq A\}$.*

Clearly, n-$CyNint(A)$ is the biggest n-$CyNos$ that is contained by A.

Definition 2.13 *[20] Let (X, τ_{CyN}) be an n-$CyNts$ and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ is an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic closure (briefly, n-$CyNcl$) is defined as the n-CyN intersection of all n-CyN closed subsets of X. ie, n-$CyNcl(A) = \cap\ \{K : K \in \tau_{CyN}$ and $A \subseteq A\}$.*

Clearly, n-$CyNcl(A)$ is the smallest n-$CyNcs$ that contains A.

Definition 2.14 *[19] Let (X, τ_1) and (Y, τ_2) be two n-$CyNts$ and let $f : (X, \tau_1) \to (Y, \tau_2)$ be a n-CyN function. Then, f said to be n-CyN continuous (briefly, n-$CyNCts$) map if for any n-cylindrical neutrosophic subset A of X and for any neighbourhood $\mathfrak{V}$ of $f(A)$ there exists a neighbourhood $\mathfrak{U}$ of A such that $f(\mathfrak{U}) \subset \mathfrak{V}$.*

Definition 2.15 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and A be an n-$CyNS$. Then, A is said to be an n-CyN*

1. regular open set (briefly, n-$CyNros$), if $A = n$-$CyNint(n$-$CyNcl(A))$,
2. regular closed set (briefly, n-$CyNrcs$), if $A = n$-$CyNcl(n$-$CyNint(A))$.

Definition 2.16 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic δ-interior of A and the n-cylindrical neutrosophic δ-closure of A are denoted by n-$CyN\delta int(A)$ and n-$CyN\delta cl(A)$ are defined as* *follows:*

1. n - $CyN\delta int(A) = \bigcup \{G | G$ isan n-$CyNros$ and $G \subseteq A\}$,
2. n - $CyN\delta cl(A) = \bigcap \{K | K$ isan n-$CyNrcs$ and $A \subseteq K\}$.

Definition 2.17 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic θ-interior of A and the n-cylindrical neutrosophic θ-closure of A are denoted by n-$CyN\theta int(A)$ and n-$CyN\theta cl(A)$ are defined as follows:*

1. n - $CyN\theta int(A) = \bigcup \{n-CyNint(B) : B \subseteq A$ & B isa $n-CyNcs$ in $X\}$,
2. n - $CyN\theta cl(A) = \bigcap \{n-CyNcl(B) : A \subseteq B$ & B isa $n-CyNos$ in $X\}$.

Definition 2.18 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-$CyNS$ in X. A set A is said to be n-CyN*

1. δ-open set (briefly, n-$CyN\delta os$), if $A = n$-$CyN\delta int(A)$,
2. δ -pre open set (briefly, n-$CyN\delta\mathcal{P}os$), if $A \subseteq n$-$CyNint(n$-$CyN\delta cl(A))$,
3. δ -semi open set (briefly, n-$CyN\delta\mathcal{S}os$), if $A \subseteq n$-$CyNcl(n$-$CyN\delta int(A))$,
4. θ-open set (briefly, n-$CyN\theta os$), if $A = n$-$CyN\theta int(A)$,
5. θ -semi open set (briefly, n-$CyN\theta\mathcal{S}os$), if $A \subseteq n$-$CyNcl(n$-$CyN\theta int(A))$,
6. e-open set (briefly, n-$CyNeos$), if $A = n$ - $CyNcl(n$ - $CyN\delta int(A)) \cup n$-$CyNint(n$-$CyN\delta cl(A))$,
7. M-open set (briefly, n-$CyNMos$), if $A \subseteq n$ - $CyNcl(n$ - $CyN\theta int(A)) \cup n$-$CyNint(n$-$CyN\delta cl(A))$.

The complement of a n-$CyNMos$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$, n-$CyN\delta\mathcal{S}os$, n-$CyN\theta os$, n-$CyN\theta\mathcal{S}os$ & n-$CyNeos$) is called a n-$CyNM$ (resp. n-$CyN\delta$, n-$CyN\delta\mathcal{P}$, n-$CyN\delta\mathcal{S}$, n-$CyN\theta$, n-$CyN\theta\mathcal{S}$ & n-$CyNe$) closed set (briefly, n-$CyNMcs$ (resp. n-$CyN\delta cs$, n-$CyN\delta\mathcal{P}cs$, n-$CyN\delta\mathcal{S}cs$, n-$CyN\theta cs$, n-$CyN\theta\mathcal{S}cs$ & n-$CyNecs$)) in X.

The family of all n-$CyNMos$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$, n-$CyN\delta\mathcal{S}os$, n-$CyN\theta os$, n-$CyN\theta\mathcal{S}os$ & n-$CyNeos$) of X is denoted by n-$CyNMOS(X)$, (resp. n-$CyNMCS(X)$, n-$CyN\delta OS(X)$, n-$CyN\delta CS(X)$, n-$CyN\delta\mathcal{P}OS(X)$, n-$CyN\delta\mathcal{P}CS(X)$, n-$CyN\delta\mathcal{S}OS(X)$, n-$CyN\delta\mathcal{S}CS(X)$, n-$CyN\theta OS(X)$, n-$CyN\theta CS(X)$, n-$CyN\theta\mathcal{S}OS(X)$, n-$CyN\theta\mathcal{S}CS(X)$, n-$CyNeOS(X)$ & n-$CyNeCS(X)$).

Definition 2.19 *[1] Let (X, τ_{CyN}) be an n-$CyNts$ and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-$CyNS$ in X. Then, the n-CyN*

1. M-interior (resp. n-$CyN\delta$-interior, n-$CyN\delta\mathcal{P}$-interior, n-$CyN\delta\mathcal{S}$-interior, n-$CyN\theta$-interior, n-$CyN\theta\mathcal{S}$-interior & n-$CyNe$-interior) of A (briefly, n-$CyNMint(A)$ (resp. n-$CyN\delta int(A)$, n-$CyN\delta\mathcal{P}int(A)$, n-$CyN\delta\mathcal{S}int(A)$, n-$CyN\theta$ $int(A)$, n-$CyN\theta\mathcal{S}int(A)$ & n-$CyNeint(A)$) is defined by n-$CyNMint(A)$ (resp. n-$CyN\delta int(A)$, n-$CyN\delta\mathcal{P}int(A)$, n-$CyN\delta\mathcal{S}int(A)$, n-$CyN\theta int(A)$, n-$CyN\theta\mathcal{S}int(A)$ & n-$CyNeint(A)$) $= \bigcup \{G : G \subseteq A$ and G is a n-$CyNMos$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$, n-$CyN\delta\mathcal{S}os$, n-$CyN\theta os$, n-$CyN\theta\mathcal{S}os$ & n-$CyNeos$) in $X\}$,
2. M-closure (resp. n-$CyN\delta$-closure, n-$CyN\delta\mathcal{P}$-closure, n-$CyN\delta\mathcal{S}$-closure, n-$CyN\theta$-closure, n-$CyN\theta\mathcal{S}$-closure & n-$CyNe$-closure) of A (briefly, n-$CyNMcl(A)$ (resp. n-$CyN\delta cl(A)$, n-$CyN\delta\mathcal{P}cl(A)$, n-$CyN\delta\mathcal{S}cl(A)$, n-$CyN\theta cl(A)$, n-$CyN\theta\mathcal{S}cl(A)$ & n-$CyNecl(A)$) is defined by n-$CyNMcl(A)$ (resp. n-$CyN\delta cl(A)$, n-$CyN\delta\mathcal{P}cl(A)$, n-$CyN\delta\mathcal{S}cl(A)$, n-$CyN\theta cl(A)$, n-$CyN\theta\mathcal{S}cl(A)$ & n-$CyNecl(A)$) $= \bigcap \{K : K \subseteq A$ and K is a n-$CyNMcs$ (resp. n-$CyN\delta cs$, n-$CyN\delta\mathcal{P}cs$, n-$CyN\delta\mathcal{S}cs$, n-$CyN\theta cs$, n-$CyN\theta\mathcal{S}cs$ & n-$CyNecs$) in $X\}$.

Definition 2.20 *[2] Let (X, τ_1) and (Y, τ_2) be any two n-$CyNts$'s. A map $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-CyN*

1. δ -continuous map (briefly, n-$CyN\delta Cts$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyN\delta os$ in (X, τ_1),
2. δ -pre continuous map (briefly, n-$CyN\delta\mathcal{P}Cts$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyN\delta\mathcal{P}os$ in (X, τ_1),
3. δ -semi continuous map (briefly, n-$CyN\delta\mathcal{S}Cts$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyN\delta\mathcal{S}os$ in (X, τ_1),
4. θ -continuous map (briefly, n-$CyN\theta Cts$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyN\theta os$ in (X, τ_1),
5. θ -semi continuous map (briefly, n-$CyN\theta\mathcal{S}os$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyN\theta\mathcal{S}Cts$ in (X, τ_1),
6. e -continuous map (briefly, n-$CyNeCts$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyNeos$ in (X, τ_1),
7. M -continuous map (briefly, n-$CyNMCts$ map), if the inverse image of every n-$CyNos$ in (Y, τ_2) is a n-$CyNMos$ in (X, τ_1).

Definition 2.21 *[2] Let (X, τ_1) and (Y, τ_2) be any two n-$CyNts$'s. A map $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be a n-CyN*

1. δ -irresolute map (briefly, n-$CyN\delta Irr$ map), if $f^{-1}(B)$ is a n-$CyN\delta os$ in (X, τ_1) for every n-$CyN\delta os$ B of (Y, τ_2),

2. δ-pre irresolute map (briefly, $n\text{-}CyN\delta\mathcal{P}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{P}os$ in (X,τ_1) for every $n\text{-}CyN\delta\mathcal{P}os$ B of (Y,τ_2),

3. δ-semi irresolute map (briefly, $n\text{-}CyN\delta\mathcal{S}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\delta\mathcal{S}os$ in (X,τ_1) for every $n\text{-}CyN\delta\mathcal{S}os$ B of (Y,τ_2),

4. θ-irresolute map (briefly, $n\text{-}CyN\theta Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\theta os$ in (X,τ_1) for every $n\text{-}CyN\theta os$ B of (Y,τ_2),

5. θ-semi irresolute map (briefly, $n\text{-}CyN\theta\mathcal{S}Irr$ map), if $f^{-1}(B)$ is a $n\text{-}CyN\theta\mathcal{S}os$ in (X,τ_1) for every $n\text{-}CyN\theta\mathcal{S}os$ B of (Y,τ_2),

6. e-irresolute map (briefly, $n\text{-}CyNeIrr$ map), if $f^{-1}(B)$ is a $n\text{-}CyNeos$ in (X,τ_1) for every $n\text{-}CyNeos$ B of (Y,τ_2),

7. M-irresolute map (briefly, $n\text{-}CyNMIrr$ map), if $f^{-1}(B)$ is a $n\text{-}CyNMos$ in (X,τ_1) for every $n\text{-}CyNMos$ B of (Y,τ_2).

Definition 2.22 *[29] Let (X,τ_{CyN}) be a $n\text{-}CyNts$. Let $A=\{\langle\alpha_A(x),\beta_A(x),\gamma_A(x)\rangle: x\in X\}$ and $B=\{\langle\alpha_B(x),\beta_B(x),\ \gamma_B(x)\rangle: x\in X\}$ are $n\text{-}CyNS$'s in X. Then, (X,τ_{CyN}) is $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$ and $n\text{-}CyNM$) disconnected (briefly, $n\text{-}CyN\mathcal{D}Con$ (resp. $n\text{-}CyN\delta\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyNe\mathcal{D}Con$ and $n\text{-}CyNM\mathcal{D}Con$)) if there exists $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$ and $n\text{-}CyNMo$) sets A, B in X, $A\neq 0_{CyN}$, $B\neq 0_{CyN}$, such that $A\cup B=1_{CyN}$ and $A\cap B=0_{CyN}$. That is, $\alpha_A\vee\alpha_B=1_{CyN}$, $\beta_A\vee\beta_B=1_{CyN}$, and $\gamma_A\wedge\gamma_B=0_{CyN}$; $\alpha_A\wedge\alpha_B=0_{CyN}$, $\beta_A\wedge\beta_B=0_{CyN}$, and $\gamma_A\vee\gamma_B=1_{CyN}$.*

If X is not $n\text{-}CyN\mathcal{D}Con$ (resp. $n\text{-}CyN\delta\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}\mathcal{D}Con$, $n\text{-}CyNe\mathcal{D}Con$ and $n\text{-}CyNM\mathcal{D}Con$) then it is said to be $n\text{-}CyN$ (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, θ, $\theta\mathcal{S}$, e and M) connected (briefly, $n\text{-}CyN\mathcal{C}on$ (resp. $n\text{-}CyN\delta\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}\mathcal{C}on$, $n\text{-}CyNe\mathcal{C}on$ and $n\text{-}CyNM\mathcal{C}on$)).

3 $\boldsymbol{n}$-Cylindrical neutrosophic $\boldsymbol{M}$-separated sets

We will introduce n-cylindrical neutrosophic M-separated sets and look at some of its feature in this section.

Definition 3.1 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$. If A and B are non-empty $n\text{-}CyN$ subsets in X. Then, A and B are said to be $n\text{-}CyN$ (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, θ, $\theta\mathcal{S}$, e, and M) weakly separated (briefly, $n\text{-}CyN\mathcal{W}Sep$ (resp. $n\text{-}CyN\delta\mathcal{W}Sep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{W}Sep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{W}Sep$, $n\text{-}CyN\theta\mathcal{W}Sep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{W}Sep$, $n\text{-}CyNe\mathcal{W}Sep$, and $n\text{-}CyNM\mathcal{W}Sep$)) if $n\text{-}CyNcl(A)\subseteq B^c$ (resp. $n\text{-}CyN\delta cl(A)\subseteq B^c$, $n\text{-}CyN\delta\mathcal{P}cl(A)\subseteq B^c$, $n\text{-}CyN\delta\mathcal{S}cl(A)\subseteq B^c$, $n\text{-}CyN\theta cl(A)\subseteq B^c$, $n\text{-}CyN\theta\mathcal{S}cl(A)\subseteq B^c$, $n\text{-}CyNecl(A)\subseteq B^c$, and $n\text{-}CyNMcl(A)\subseteq B^c$) and $n\text{-}CyNcl(B)\subseteq A^c$ (resp. $n\text{-}CyN\delta cl(B)\subseteq A^c$, $n\text{-}CyN\delta\mathcal{P}cl(B)\subseteq A^c$, $n\text{-}CyN\delta\mathcal{S}cl(B)\subseteq A^c$, $n\text{-}CyN\theta cl(B)\subseteq A^c$, $n\text{-}CyN\theta\mathcal{S}cl(B)\subseteq A^c$, $n\text{-}CyNecl(B)\subseteq A^c$, and $n\text{-}CyNMcl(B)\subseteq A^c$).*

Definition 3.2 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$. If A and B are non-empty $n\text{-}CyN$ subsets in X. Then, A and B are said to be $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, and $n\text{-}CyNM$) separated (briefly, $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$)) if $n\text{-}CyNcl(A)\cap B=A\cap n\text{-}CyNcl(B)=0_{CyN}$ (resp. $n\text{-}CyN\delta cl(A)\cap B=A\cap n\text{-}CyN\delta cl(B)=0_{CyN}$, $n\text{-}CyN\delta\mathcal{P}cl(A)\cap B=A\cap n\text{-}CyN\delta\mathcal{P}cl(B)=0_{CyN}$, $n\text{-}CyN\delta\mathcal{S}cl(A)\cap B=A\cap n\text{-}CyN\delta\mathcal{S}cl(B)=0_{CyN}$, $n\text{-}CyN\theta cl(A)\cap B=A\cap n\text{-}CyN\theta cl(B)=0_{CyN}$, $n\text{-}CyN\theta\mathcal{S}cl(A)\cap B=A\cap n\text{-}CyN\theta\mathcal{S}cl(B)=0_{CyN}$, $n\text{-}CyNecl(A)\cap B=A\cap n\text{-}CyNecl(B)=0_{CyN}$, and $n\text{-}CyNMcl(A)\cap B=A\cap n\text{-}CyNMcl(B)=0_{CyN}$).*

Remark 3.1 *Any two disjoint non-empty $n\text{-}CyNc$ (resp. $n\text{-}CyN\delta c$, $n\text{-}CyN\delta\mathcal{P}c$, $n\text{-}CyN\delta\mathcal{S}c$, $n\text{-}CyN\theta c$, $n\text{-}CyN\theta\mathcal{S}c$, $n\text{-}CyNec$, and $n\text{-}CyNMc$) sets are $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$).*

Proof. Suppose, A and B are disjoint non-empty $n\text{-}CyNMcs$'s. Then, $n\text{-}CyNMcl(A)\cap B=A\cap n\text{-}CyNMcl(B)=A\cap B=0_{CyN}$. This shows that A and B are $n\text{-}CyNM\mathcal{S}ep$.

Similar situations exist in other scenarios.

Theorem 3.1 *Let (X,τ_{CyN}) be a $n\text{-}CyNts$. If A and B are non-empty $n\text{-}CyN$ subsets in X.*

1. If A and B are $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$) and $C\subseteq A$, $D\subseteq B$, then C and D are also $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$),
2. If A and B are both $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets and if $H=A\cap B^c$ and $G=B\cap A^c$, then H and G are $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$).

Proof.

1. Let A and B be $n\text{-}CyNM\mathcal{S}ep$ sets in $n\text{-}CyNts\ X$. Then, $n\text{-}CyNMcl(A) \cap B = 0_{CyN} = A \cap n\text{-}CyNMcl(B)$. Since, $C \subseteq A$ and $D \subseteq B$, then $n\text{-}CyNMcl(C) \subseteq n\text{-}CyNMcl(A)$ and $n\text{-}CyNMcl(D) \subseteq n\text{-}CyNMcl(B)$. This implies that, $n\text{-}CyNMcl(C) \cap D \subseteq n\text{-}CyNMcl(A) \cap B = 0_{CyN}$ and hence $n\text{-}CyNMcl(C) \cap D = 0_{CyN}$. Similarly, $n\text{-}CyNMcl(D) \cap C \subseteq n\text{-}CyNMcl(B) \cap A = 0_{CyN}$ and hence $n\text{-}CyNMcl(D) \cap C = 0_{CyN}$. Therefore, C and D are $n\text{-}CyNM\mathcal{S}ep$.

2. Let A and B both $n\text{-}CyNMo$ subsets in X. Then A^c and B^c are $n\text{-}CyNMcs$'s. Since $H \subseteq B^c$, then $n\text{-}CyNMcl(H) \subseteq n\text{-}CyNMcl(B^c) = B^c$ and so $n\text{-}CyNMcl(H) \cap B = 0_{CyN}$. Since, $G \subseteq B$, then $n\text{-}CyNMcl(H) \cap G \subseteq n\text{-}CyNMcl(H) \cap B = 0_{CyN}$. Thus, $n\text{-}CyNMcl(H) \cap G = 0_{CyN}$. Similarly, $n\text{-}CyNMcl(G) \cap H = 0_{CyN}$. Hence, H and G are $n\text{-}CyNM\mathcal{S}ep$.

Similar situations exist in other scenarios.

Theorem 3.2 *Let (X, τ_{CyN}) be a $n\text{-}CyNts$. If A and B are non-empty $n\text{-}CyN$ subsets in X are $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$) if and only if there exist $\mathcal{L}$ and $\mathcal{M}$ in $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) set in X, such that $A \subseteq \mathcal{L}$, $B \subseteq \mathcal{M}$ and $A \cap \mathcal{M} = 0_{CyN}$ and $B \cap \mathcal{L} = 0_{CyN}$.*

Proof. Let A and B be $n\text{-}CyNM\mathcal{S}ep$. Then, $A \cap n\text{-}CyNMcl(B) = 0_{CyN} = n\text{-}CyNMcl(A) \cap B$. Take, $M = (n\text{-}CyNMcl(A))^c$ and $\mathcal{L} = (n\text{-}CyNMcl(B))^c$. Then, $\mathcal{L}$ and $\mathcal{M}$ are $n\text{-}CyNMos$'s, such that $A \subseteq \mathcal{L}$, $B \subseteq \mathcal{M}$ and $A \cap \mathcal{M} = 0_{CyN}$ and $B \cap \mathcal{L} = 0_{CyN}$.

Conversely, let $\mathcal{L}$ and $\mathcal{M}$ be $n\text{-}CyNMos$'s such that $A \subseteq \mathcal{L}$, $B \subseteq \mathcal{M}$ and $A \cap \mathcal{M} = 0_{CyN}$, $B \cap \mathcal{L} = 0_{CyN}$. Then, $A \subseteq \mathcal{M}^c$ and $B \subseteq \mathcal{L}^c$ and $\mathcal{M}^c$ and $\mathcal{L}^c$ are $n\text{-}CyNMcs$'s. This implies, $n\text{-}CyNMcl(A) \subseteq n\text{-}CyNMcl(\mathcal{M}^c) = \mathcal{M}^c \subseteq B^c$ and $n\text{-}CyNMcl(B) \subseteq n\text{-}CyNMcl(\mathcal{L}^c) = \mathcal{L}^c \subseteq A^c$. That is, $n\text{-}CyNMcl(A) \subseteq B^c$ and $n\text{-}CyNMcl(B) \subseteq A^c$. Therefore, $A \cap n\text{-}CyNMcl(B) = 0_{CyN} = n\text{-}CyNMcl(A) \cap B$. Hence, A and B are $n\text{-}CyNM\mathcal{S}ep$.

Similar situations exist in other scenarios.

Proposition 3.1 *Each two $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\ \mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$) sets are always disjoint.*

Proof. Let A and B be $n\text{-}CyNM\mathcal{S}ep$. Then, $A \cap n\text{-}CyNMcl(B) = 0_{CyN} = n\text{-}CyNMcl(A) \cap B$. Now, $A \cap B \subseteq A \cap n\text{-}CyNMcl(B) = 0_{CyN}$. Therefore, $A \cap B = 0_{CyN}$ and hence A and B are disjoint.

Similar situations exist in other scenarios.

Theorem 3.3 *Let (X, τ_{CyN}) be a $n\text{-}CyNts$. Then, X is $n\text{-}CyN\mathcal{C}on$ (resp. $n\text{-}CyN\delta\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}\mathcal{C}on$, $n\text{-}CyNe\mathcal{C}on$, and $n\text{-}CyNM\mathcal{C}on$) iff $1_{CyN} \neq A \cup B$, where A and B are $n\text{-}CyN\mathcal{S}ep$ (resp. $n\text{-}CyN\delta\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{P}\mathcal{S}ep$, $n\text{-}CyN\delta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}ep$, $n\text{-}CyN\theta\mathcal{S}\mathcal{S}ep$, $n\text{-}CyNe\mathcal{S}ep$, and $n\text{-}CyNM\mathcal{S}ep$) sets.*

Proof. Assume that, X is a $n\text{-}CyNM\mathcal{C}on$ space. Suppose, $1_{CyN} = A \cup B$, where A and B are $n\text{-}CyNM\mathcal{S}ep$ sets. Then, $n\text{-}CyNMcl(A) \cap B = A \cap n\text{-}CyNMcl(B) = 0_{CyN}$. Since, $A \subseteq n\text{-}CyNMcl(A)$, we have $A \cap B \subseteq n\text{-}CyNMcl(A) \cap B = 0_{CyN}$. Therefore, $n\text{-}CyNMcl(A) \subseteq B^c = A$ and $n\text{-}CyNMcl(B) \subseteq A^c = B$. Hence, $A = n\text{-}CyNMcl(A)$ and $B = n\text{-}CyNMcl(B)$. Therefore, A and B are $n\text{-}CyNMcs$'s and hence $A = B^c$ and $B = A^c$ are disjoint $n\text{-}CyNMos$'s. Thus, $A \neq 0_{CyN}$, $B \neq 0_{CyN}$, such that $A \cup B = 1_{CyN}$ and $A \cap B = 0_{CyN}$, A and B are $n\text{-}CyNMos$'s. That is, X is not $n\text{-}CyNM\mathcal{C}on$, which is a contradiction to X is a $n\text{-}CyNM\mathcal{C}on$ space. Hence, 1_{CyN} is not the union of any two $n\text{-}CyNM\mathcal{S}ep$ sets.

Conversely, assume that 1_{CyN} is not the union of any two $n\text{-}CyNM\mathcal{S}ep$ sets. Suppose, X is not $n\text{-}CyNM\mathcal{C}on$. Then, $1_{CyN} = A \cup B$, where $A \neq 0_{CyN}$, $B \neq 0_{CyN}$, such that and $A \cap B = 0_{CyN}$, A and B are $n\text{-}CyNMos$'s in X. Since, $A \subseteq B^c$ and $B \subseteq A^c$, $n\text{-}CyNMcl(A) \cap B \subseteq B^c \cap B = 0_{CyN}$ and $A \cap n\text{-}CyNMcl(B) \subseteq A \cap A^c = 0_{CyN}$. That is, A and B are $n\text{-}CyNM\mathcal{S}ep$ sets. This is a contradiction. Therefore, X is $n\text{-}CyNM\mathcal{C}on$.

Similar situations exist in other scenarios.

Definition 3.3 *Let (X, τ_{CyN}) be an $n\text{-}CyNts$ and A be an $n\text{-}CyNS$. Then, A is said to be an $n\text{-}CyN$*

1. δ-regular open set (briefly, $n\text{-}CyN\delta ros$), if $A = n\text{-}CyN\delta int(n\text{-}CyN\delta cl(A))$,
2. δ-pre regular open set (briefly, $n\text{-}CyN\delta\mathcal{P}ros$), if $A = n\text{-}CyN\delta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(A))$,
3. δ-semi regular open set (briefly, $n\text{-}CyN\delta\mathcal{S}ros$), if $A = n\text{-}CyN\delta\mathcal{S}int(n\text{-}CyN\delta\mathcal{S}cl(A))$,
4. θ-regular open set (briefly, $n\text{-}CyN\theta ros$), if $A = n\text{-}CyN\theta int(n\text{-}CyN\theta cl(A))$,
5. θ-semi regular open set (briefly, $n\text{-}CyN\theta\mathcal{S}ros$), if $A = n\text{-}CyN\theta\mathcal{S}int(n\text{-}CyN\theta\mathcal{S}cl(A))$,

6. e-regular open set (briefly, $n\text{-}CyNeros$), if $A = n\text{-}CyNeint(n\text{-}CyNecl(A))$,

7. M-regular open set (briefly, $n\text{-}CyNMros$), if $A = n\text{-}CyNMint(n\text{-}CyNMcl(A))$.

The complement of a $n\text{-}CyNros$ (resp. $n\text{-}CyN\delta ros$, $n\text{-}CyN\delta\mathcal{P}ros$, $n\text{-}CyN\delta\mathcal{S}ros$, $n\text{-}CyN\theta ros$, $n\text{-}CyN\theta\mathcal{S}ros$, $n\text{-}CyNeros$, and $n\text{-}CyNMros$) is called a $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, and $n\text{-}CyNM$) closed set (briefly, $n\text{-}CyNrcs$ (resp. $n\text{-}CyN\delta rcs$, $n\text{-}CyN\delta\mathcal{P}rcs$, $n\text{-}CyN\delta\mathcal{S}rcs$, $n\text{-}CyN\theta rcs$, $n\text{-}CyN\theta\mathcal{S}rcs$, $n\text{-}CyNercs$, and $n\text{-}CyNMrcs$)) in X.

The family of all $n\text{-}CyNros$ (resp. $n\text{-}CyN\delta ros$, $n\text{-}CyN\delta\mathcal{P}ros$, $n\text{-}CyN\delta\mathcal{S}ros$, $n\text{-}CyN\theta ros$, $n\text{-}CyN\theta\mathcal{S}ros$, $n\text{-}CyNeros$, and $n\text{-}CyNMros$) of X is denoted by $n\text{-}CyN\mathcal{ROS}(X)$, (resp. $n\text{-}CyNM\mathcal{RCS}(X)$, $n\text{-}CyN\delta\mathcal{ROS}(X)$, $n\text{-}CyN\delta\mathcal{RCS}(X)$, $n\text{-}CyN\delta\mathcal{PROS}(X)$, $n\text{-}CyN\delta\mathcal{PRCS}(X)$, $n\text{-}CyN\delta\mathcal{SROS}(X)$, $n\text{-}CyN\delta\mathcal{SRCS}(X)$, $n\text{-}CyN\theta\mathcal{ROS}(X)$, $n\text{-}CyN\theta\mathcal{RCS}(X)$, $n\text{-}CyN\theta\mathcal{SROS}(X)$, $n\text{-}CyN\theta\mathcal{SRCS}(X)$, $n\text{-}CyNe\mathcal{ROS}(X)$, $n\text{-}CyNe\mathcal{RCS}(X)$, $n\text{-}CyNM\mathcal{ROS}(X)$, and $n\text{-}CyNM\mathcal{RCS}(X)$).

Proposition 3.2 *Let (X, τ_{CyN}) be an $n\text{-}CyNts$.*

1. Every $n\text{-}CyNros$ (resp. $n\text{-}CyN\delta ros$, $n\text{-}CyN\delta\mathcal{P}ros$, $n\text{-}CyN\delta\mathcal{S}ros$, $n\text{-}CyN\theta ros$, $n\text{-}CyN\theta\mathcal{S}ros$, $n\text{-}CyNeros$, and $n\text{-}CyNMros$) is $n\text{-}CyNos$ (resp. $n\text{-}CyN\delta os$, $n\text{-}CyN\delta\mathcal{P}os$, $n\text{-}CyN\delta\mathcal{S}os$, $n\text{-}CyN\theta os$, $n\text{-}CyN\theta\mathcal{S}os$, $n\text{-}CyNeos$, and $n\text{-}CyNMos$),
2. Every $n\text{-}CyNrcs$ (resp. $n\text{-}CyN\delta rcs$, $n\text{-}CyN\delta\mathcal{P}rcs$, $n\text{-}CyN\delta\mathcal{S}rcs$, $n\text{-}CyN\theta rcs$, $n\text{-}CyN\theta\mathcal{S}rcs$, $n\text{-}CyNercs$, and $n\text{-}CyNMrcs$) is $n\text{-}CyNcs$ (resp. $n\text{-}CyN\delta cs$, $n\text{-}CyN\delta\mathcal{P}cs$, $n\text{-}CyN\delta\mathcal{S}cs$, $n\text{-}CyN\theta cs$, $n\text{-}CyN\theta\mathcal{S}cs$, $n\text{-}CyNecs$, and $n\text{-}CyNMcs$).

Proof.

1. Let A be a $n\text{-}CyNMros$ in X. Then, $A = n\text{-}CyNMint(n\text{-}CyNMcl(A))$. Clearly, $n\text{-}CyNMint(n\text{-}CyNMcl(A))$ is $n\text{-}CyNMos$. Therefore, A is $n\text{-}CyNMos$. Other cases are similar.
2. Similar proof of (i).

Definition 3.4 *Let (X, τ_{CyN}) be a $n\text{-}CyNts$. Then, X is $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyNe$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyN\delta\mathcal{P}$, and $n\text{-}CyNM$) super disconnected (briefly, $n\text{-}CyNSuper\mathcal{D}Con$ (resp. $n\text{-}CyN\delta Super\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}Super\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}Super\mathcal{D}Con$, $n\text{-}CyN\theta Super\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}\ Super\mathcal{D}Con$, $n\text{-}CyNeSuper\mathcal{D}Con$, and $n\text{-}CyNMSuper\mathcal{D}Con$)) if there exists a $n\text{-}CyNros$ (resp. $n\text{-}CyN\delta ros$, $n\text{-}CyN\delta\mathcal{P}ros$, $n\text{-}CyN\delta\mathcal{S}ros$, $n\text{-}CyN\theta ros$, $n\text{-}CyN\theta\mathcal{S}\ ros$, $n\text{-}CyNeros$, and $n\text{-}CyNMros$) A in X such that $A \neq 0_{CyN}$ and $A \neq 1_{CyN}$.*

An $n\text{-}CyNts$ X is called $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, and $n\text{-}CyNM$) super connected (briefly, $n\text{-}CyNSuper\mathcal{C}on$ (resp. $n\text{-}CyN\delta Super\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}Super\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}Super\mathcal{C}on$, $n\text{-}CyN\theta\ \ Super\mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}Super\mathcal{C}on$, $n\text{-}CyNeSuper\mathcal{C}on$, and $n\text{-}CyNMSuper\ \ \mathcal{C}on$)) if X is not $n\text{-}CyNSuper\mathcal{D}Con$ (resp. $n\text{-}CyN\delta Super\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}Super\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{S}Super\mathcal{D}Con$, $n\text{-}CyN\theta Super\mathcal{D}Con$, $n\text{-}CyN\theta\mathcal{S}Super\mathcal{D}\ \ Con$, $n\text{-}CyNeSuper\mathcal{D}Con$, and $n\text{-}CyNMSuper\mathcal{D}Con$).

Example 3.1 *Let $X = \{x_1, x_2\}$ and the n-cylindrical neutrosophic sets A_1, A_2, A_3, A_4 and A_5 in X are defined as*

$A_1 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\}$,

$A_2 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$,

$A_3 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$,

$A_4 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$,

$A_5 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}$.

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, A_1, A_2, A_3, A_4\}$ be a n-cylindrical neutrosophic topology on X. Then,

1. A_1 is $n\text{-}CyNSuper\mathcal{D}Con$ (resp. $n\text{-}CyN\delta Super\mathcal{D}Con$, $n\text{-}CyN\delta\mathcal{P}Super\mathcal{D}Con$, $n\text{-}CyN\theta Super\mathcal{D}Con$, and $n\text{-}CyNMSuper\mathcal{D}Con$)
2. A_5 is $n\text{-}CyN\delta\mathcal{S}Super\mathcal{D}Con$ (resp. $n\text{-}CyN\theta\mathcal{S}Super\mathcal{D}Con$)

Theorem 3.4 *Let (X, τ_{CyN}) be a $n\text{-}CyNts$, the equivalents are as follows:*

1. X is $n\text{-}CyNSuper\mathcal{C}on$ (resp. $n\text{-}CyN\delta Super\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{P}Super\mathcal{C}on$, $n\text{-}CyN\delta\mathcal{S}Super\mathcal{C}on$, $n\text{-}CyN\theta Super\ \ \mathcal{C}on$, $n\text{-}CyN\theta\mathcal{S}Super\mathcal{C}on$, $n\text{-}CyNeSuper\mathcal{C}on$, and $n\text{-}CyNMSuper\ \mathcal{C}on$),
2. For each $n\text{-}CyNos$ (resp. $n\text{-}CyN\delta os$, $n\text{-}CyN\delta\mathcal{P}os$, $n\text{-}CyN\delta\mathcal{S}os$, $n\text{-}CyN\theta os$, $n\text{-}CyN\theta\mathcal{S}os$, $n\text{-}CyNeos$, and $n\text{-}CyNMos$) $\mathcal{L} \neq 0_{CyN}$ in X, we have $n\text{-}CyNcl(\mathcal{L}) = 1_{CyN}$ (resp. $n\text{-}CyN\delta cl(\mathcal{L}) = 1_{CyN}$, $n\text{-}CyN\delta\mathcal{P}cl(\mathcal{L}) = 1_{CyN}$, $n\text{-}CyN\delta\mathcal{S}cl(\mathcal{L}) = 1_{CyN}$, $n\text{-}CyN\theta cl(\mathcal{L}) = 1_{CyN}$, $n\text{-}CyN\theta\mathcal{S}cl(\mathcal{L}) = 1_{CyN}$, $n\text{-}CyNecl(\mathcal{L}) = 1_{CyN}$, and $n\text{-}CyNMcl(\mathcal{L}) = 1_{CyN}$),
3. For each $n\text{-}CyNcs$ (resp. $n\text{-}CyN\delta cs$, $n\text{-}CyN\delta\mathcal{P}cs$, $n\text{-}CyN\delta\mathcal{S}cs$, $n\text{-}CyN\theta cs$, $n\text{-}CyN\theta\mathcal{S}cs$, $n\text{-}CyNecs$, and $n\text{-}CyNMcs$) $\mathcal{L} \neq 1_{CyN}$ in X, we have $n\text{-}CyNint(\mathcal{L}) = 0_{CyN}$ (resp.

$n\text{-}CyN\delta int(\mathcal{L}) = 0_{CyN}$, $n\text{-}CyN\delta\mathcal{P}int(\mathcal{L}) = 0_{CyN}$, $n\text{-}CyN\delta\mathcal{S}int(\mathcal{L}) = 0_{CyN}$, $n\text{-}CyN\theta int(\mathcal{L}) = 0_{CyN}$, $n\text{-}CyN\theta\mathcal{S}int(\mathcal{L}) = 0_{CyN}$, $n\text{-}CyNeint(\mathcal{L}) = 0_{CyN}$, and $n\text{-}CyNMint(\mathcal{L}) = 0_{CyN}$),

4. There exists no $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) subsets $\mathcal{L}$ and $\mathcal{M}$ in X, such that $\mathcal{L} \neq 0_{CyN}$, $\mathcal{M} \neq 0_{CyN}$, and $\mathcal{L} \subseteq \mathcal{M}^c$,

5. There exists no $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) subsets $\mathcal{L}$ and $\mathcal{M}$ in X, such that $\mathcal{L} \neq 0_{CyN}$, $\mathcal{M} \neq 0_{CyN}$, $\mathcal{M} = (n\text{-}CyNcl(\mathcal{L}))^c$ (resp. $\mathcal{M} = (n\text{-}CyN\delta cl(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\delta\mathcal{P}cl(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\delta\mathcal{S}cl(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\theta cl(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\theta\mathcal{S}cl(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyNecl(\mathcal{L}))^c$, and $\mathcal{M} = (n\text{-}CyNMcl(\mathcal{L}))^c$), and $\mathcal{L} = (n\text{-}CyNcl(\mathcal{M}))^c$ (resp. $\mathcal{L} = (n\text{-}CyN\delta cl(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\delta\mathcal{P}cl(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\delta\mathcal{S}cl(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\theta cl(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\theta\mathcal{S}cl(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyNecl(\mathcal{M}))^c$, and $\mathcal{L} = (n\text{-}CyNMcl(\mathcal{M}))^c$),

6. There exists no $n\text{-}CyNc$ (resp. $n\text{-}CyN\delta c$, $n\text{-}CyN\delta\mathcal{P}c$, $n\text{-}CyN\delta\mathcal{S}c$, $n\text{-}CyN\theta c$, $n\text{-}CyN\theta\mathcal{S}c$, $n\text{-}CyNec$, and $n\text{-}CyNMc$) subsets $\mathcal{L}$ and $\mathcal{M}$ in X, such that $\mathcal{L} \neq 1_{CyN}$, $\mathcal{M} \neq 1_{CyN}$, $\mathcal{M} = (n\text{-}CyNint(\mathcal{L}))^c$ (resp. $\mathcal{M} = (n\text{-}CyN\delta int(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\delta\mathcal{P}int(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\delta\mathcal{S}int(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\theta int(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyN\theta\mathcal{S}int(\mathcal{L}))^c$, $\mathcal{M} = (n\text{-}CyNeint(\mathcal{L}))^c$, and $\mathcal{M} = (n\text{-}CyNMint(\mathcal{L}))^c$) and $\mathcal{L} = (n\text{-}CyNint(\mathcal{M}))^c$ (resp. $\mathcal{L} = (n\text{-}CyN\delta int(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\delta\mathcal{P}int(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\delta\mathcal{S}int(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\theta int(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyN\theta\mathcal{S}int(\mathcal{M}))^c$, $\mathcal{L} = (n\text{-}CyNeint(\mathcal{M}))^c$, and $\mathcal{L} = (n\text{-}CyNMint(\mathcal{M}))^c$).

Proof. (i) $\Rightarrow$ (ii) Assume that there exists a $n\text{-}CyNMos$ $\mathcal{L} \neq 0_{CyN}$, such that $n\text{-}CyNMcl(\mathcal{L}) \neq 1_{CyN}$. Now, take $\mathcal{M} = n\text{-}CyNMint(n\text{-}CyNMcl(\mathcal{L}))$. Then, $\mathcal{M}$ is proper $n\text{-}CyNMros$ in X which contradiction that X is $n\text{-}CyNMSuperCon$.

(ii) $\Rightarrow$ (iii) Let $\mathcal{L} \neq 1_{CyN}$ be a $n\text{-}CyNMcs$ in X. If $\mathcal{M} = \mathcal{L}^c$, then $\mathcal{M}$ is $n\text{-}CyNMos$ in X and $\mathcal{M} \neq 0_{CyN}$. Hence, $n\text{-}CyNMcl(\mathcal{L}) = 1_{CyN}$, $(n\text{-}CyNMcl(\mathcal{M}))^c = 0_{CyN} \Rightarrow n\text{-}CyNMint(\mathcal{M}^c) = 0_{CyN} \Rightarrow n\text{-}CyNMint(\mathcal{L}) = 0_{CyN}$.

(iii) $\Rightarrow$ (iv) Let $\mathcal{L}$ and $\mathcal{M}$ be $n\text{-}CyNMos$'s in X, such that $\mathcal{L} \neq 0_{CyN} \neq \mathcal{M}$ and $\mathcal{L} \subseteq \mathcal{M}^c$. Since, $\mathcal{M}^c$ is $n\text{-}CyNMcs$ in X and $\mathcal{M} \neq 0_{CyN}$ implies $\mathcal{M}^c \neq 1_{CyN}$, we obtain $n\text{-}CyNMint(\mathcal{M}^c) = 0_{CyN}$. But, from $\mathcal{L} \subseteq \mathcal{M}^c$, $0_{CyN} \neq \mathcal{L} = n\text{-}CyNMint(\mathcal{L}) \subseteq n\text{-}CyNMint(\mathcal{M}^c) = 0_{CyN}$, which is a contradiction.

(iv) $\Rightarrow$ (i) Let $0_{CyN} \neq \mathcal{L} \neq 1_{CyN}$ be $n\text{-}CyNMros$ in X. If we take $\mathcal{M} = (n\text{-}CyNMcl(\mathcal{L}))^c$, we get $\mathcal{M} \neq 0_{CyN}$. Otherwise, we have $\mathcal{M} \neq 0_{CyN}$ implies $(n\text{-}CyNMcl(\mathcal{L}))^c = 0_{CyN}$. That implies $n\text{-}CyNMcl(\mathcal{L}) = 1_{CyN}$. That shows $\mathcal{L} = n\text{-}CyNMint(n\text{-}CyNMcl(\mathcal{L})) = n\text{-}CyNMint(1_{CyN}) = 1_{CyN}$. But this is to a contradiction to $\mathcal{L} \neq 1_{CyN}$.

Further, $\mathcal{L} \subseteq \mathcal{M}^c$, this is also a contradiction.

(i) $\Rightarrow$ (v) Let $\mathcal{L}$ and $\mathcal{M}$ be $n\text{-}CyNMos$'s in X, such that $\mathcal{L} \neq 0_{CyN} \neq \mathcal{M}$ and $\mathcal{M} = (n\text{-}CyNMcl(\mathcal{L}))^c$, $\mathcal{L} = (n\text{-}CyNMint(\mathcal{M}))^c$. Now, $n\text{-}CyNMint(n\text{-}CyNMcl(\mathcal{L})) = n\text{-}CyNMint(\mathcal{M}^c) = (n\text{-}CyNMcl(\mathcal{M}))^c = \mathcal{L}$ and $\mathcal{L} \neq 0_{CyN}$, $\mathcal{L} \neq 1_{CyN}$. Suppose not, if $\mathcal{L} = 1_{CyN}$, then $1_{CyN} = (n\text{-}CyNMcl(\mathcal{M}))^c$ implies $0 = n\text{-}CyNMcl(\mathcal{M}) \Rightarrow \mathcal{M} = 0$. This is a contradiction.

(v) $\Rightarrow$ (i) Let $\mathcal{L}$ be $n\text{-}CyNMos$ in X, such that $\mathcal{L} = n\text{-}CyNMint(n\text{-}CyNMcl(\mathcal{L}))$, $0_{CyN} \neq \mathcal{L} \neq 1_{CyN}$. Now, $\mathcal{M} = (n\text{-}CyNMcl(\mathcal{L}))^c$ and $(n\text{-}CyNMcl(\mathcal{M}))^c = (n\text{-}CyNMcl(n\text{-}CyNMcl(\mathcal{L}))^c)^c = n\text{-}CyNMint(n\text{-}CyNMcl(\mathcal{L})) = \mathcal{L}$. This is a contradiction.

(v) $\Rightarrow$ (vi) Let $\mathcal{L}$ and $\mathcal{M}$ be $n\text{-}CyNMcs$ in X, such that $\mathcal{L} \neq 1_{CyN} \neq \mathcal{M}$. $\mathcal{M} = (n\text{-}CyNMint(\mathcal{L}))^c$, $\mathcal{L} = (n\text{-}CyNMint(\mathcal{M}))^c$. Taking, $C = \mathcal{L}^c$ and $D = \mathcal{M}^c$, C and D become $n\text{-}CyNMos$ in X and $C \neq 0_{CyN} \neq D$, $(n\text{-}CyNMcl(C))^c = (n\text{-}CyNMcl(\mathcal{L}^c))^c = ((n\text{-}CyNMint(\mathcal{L}))^c)^c = n\text{-}CyNMint(\mathcal{L}) = \mathcal{M}^c = D$ and similarly $(n\text{-}CyNMcl(D))^c = C$. But this is a contradiction.

(vi) $\Rightarrow$ (v) Similar as in above.

4 $\boldsymbol{n}$-Cylindrical neutrosophic $\boldsymbol{M}$-compact spaces

We will introduce n-cylindrical neutrosophic M-compact spaces and look at some of its feature in this section.

Definition 4.1 *Let (X, τ_{CyN}) be an $n\text{-}CyNts$. A collection B of $n\text{-}CyNo$ (resp. $n\text{-}CyN\delta o$, $n\text{-}CyN\delta\mathcal{P}o$, $n\text{-}CyN\delta\mathcal{S}o$, $n\text{-}CyN\theta o$, $n\text{-}CyN\theta\mathcal{S}o$, $n\text{-}CyNeo$, and $n\text{-}CyNMo$) sets in X is called a $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$, $n\text{-}CyN\theta\mathcal{S}$, $n\text{-}CyNe$, and $n\text{-}CyNM$) open cover (briefly, $n\text{-}CyNOCov$ (resp. $n\text{-}CyN\delta OCov$, $n\text{-}CyN\delta\mathcal{P}OCov$, $n\text{-}CyN\delta\mathcal{S}OCov$, $n\text{-}CyN\theta OCov$, $n\text{-}CyN\theta\mathcal{S}OCov$, $n\text{-}CyNeOCov$, and $n\text{-}CyNMOCov$)) of a subset B of X if $B \subseteq \cup\ \{L_\alpha : L_\alpha \in B\}$.*

Definition 4.2 *Let (X, τ_{CyN}) be an $n\text{-}CyNts$, then X is said to be $n\text{-}CyN$ (resp. $n\text{-}CyN\delta$, $n\text{-}CyN\delta\mathcal{P}$, $n\text{-}CyN\delta\mathcal{S}$, $n\text{-}CyN\theta$,*

n-$CyN\theta\mathcal{S}$, n-$CyNe$, and n-$CyNM$) compact (briefly, n-$CyN\mathcal{C}omp$ (resp. n-$CyN\delta\mathcal{C}omp$, n-$CyN\delta\mathcal{P}\mathcal{C}omp$, n-$CyN\delta\mathcal{S}\mathcal{C}omp$, n-$CyN\theta\mathcal{C}omp$, n-$CyN\theta\mathcal{S}\mathcal{C}omp$, n-$CyNe\mathcal{C}omp$, and n-$CyNM\mathcal{C}omp$)) if every n-$CyN\mathcal{O}\mathcal{C}ov$ (resp. n-$CyN\delta\mathcal{O}\mathcal{C}ov$, n-$CyN\delta\mathcal{P}\mathcal{O}\mathcal{C}ov$, n-$CyN\delta\mathcal{S}\mathcal{O}\mathcal{C}ov$, n-$CyN\theta\mathcal{O}\mathcal{C}ov$, n-$CyN\theta\mathcal{S}\mathcal{O}\mathcal{C}ov$, n-$CyNe\mathcal{O}\mathcal{C}ov$, and n-$CyNM\mathcal{O}\mathcal{C}ov$) of X has a finite subcover.

Definition 4.3 *Let (X,τ_{CyN}) be a n-$CyNts$. A subset A of X is said to be n-$CyN\mathcal{C}omp$ (resp. n-$CyN\delta\mathcal{C}omp$, n-$CyN\delta\mathcal{P}\mathcal{C}omp$, n-$CyN\delta\mathcal{S}\mathcal{C}omp$, n-$CyN\theta\mathcal{C}omp$, n-$CyN\theta\mathcal{S}\mathcal{C}omp$, n-$CyNe\mathcal{C}omp$, and n-$CyNM\mathcal{C}omp$) relative to X if every n-$CyN\mathcal{O}\mathcal{C}ov$ (resp. n-$CyN\delta\mathcal{O}\mathcal{C}ov$, n-$CyN\delta\mathcal{P}\mathcal{O}\mathcal{C}ov$, n-$CyN\delta\mathcal{S}\mathcal{O}\mathcal{C}ov$, n-$CyN\theta\mathcal{O}\mathcal{C}ov$, n-$CyN\theta\mathcal{S}\mathcal{O}\mathcal{C}ov$, n-$CyNe\mathcal{O}\mathcal{C}ov$, and n-$CyNM\mathcal{O}\mathcal{C}ov$) of X has a finite subcover.*

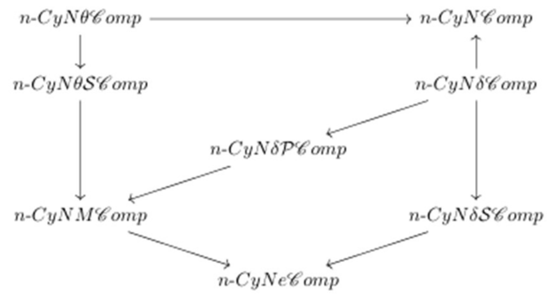
Theorem 4.1 *The statements are true, but the converse need not be true.*

1. Every n-$CyN\theta\mathcal{C}omp$ is a n-$CyN\mathcal{C}omp$,
2. Every n-$CyN\theta\mathcal{C}omp$ is a n-$CyN\theta\mathcal{S}\mathcal{C}omp$,
3. Every n-$CyN\theta\mathcal{S}\mathcal{C}omp$ is a n-$CyNM\mathcal{C}omp$,
4. Every n-$CyN\delta\mathcal{C}omp$ is a n-$CyN\mathcal{C}omp$,
5. Every n-$CyN\delta\mathcal{C}omp$ is a n-$CyN\delta\mathcal{P}\mathcal{C}omp$,
6. Every n-$CyN\delta\mathcal{C}omp$ is a n-$CyN\delta\mathcal{S}\mathcal{C}omp$,
7. Every n-$CyN\delta\mathcal{S}\mathcal{C}omp$ is a n-$CyNe\mathcal{C}omp$,
8. Every n-$CyN\delta\mathcal{P}\mathcal{C}omp$ is a n-$CyNM\mathcal{C}omp$,
9. Every n-$CyNM\mathcal{C}omp$ is a n-$CyNe\mathcal{C}omp$.

Proof. Only (viii) is proven; the others are similar.

[(viii)] Let X be n-$CyNM\mathcal{C}omp$ space. Suppose, X is not n-$CyN\delta\mathcal{P}\mathcal{C}omp$. Then, there exists a n-$CyN\delta\mathcal{P}\mathcal{O}\mathcal{C}ov$ B of X has no finite subcover. Since, every n-$CyN\delta\mathcal{P}os$ is n-$CyNMos$, then we have n-$CyNM\mathcal{O}\mathcal{C}ov$ B of X, which has no finite subcover. This is a contradiction to X is n-$CyNM\mathcal{C}omp$. Hence, X is n-$CyN\delta\mathcal{P}\mathcal{C}omp$.

Remark 4.1 *From the above mentioned results. We get the diagram below.*



Note: $A \to B$ denotes A implies B, but not conversely.

Theorem 4.2 *An n-$CyNc$ (resp. n-$CyN\delta c$, n-$CyN\delta\mathcal{P}c$, n-$CyN\delta\mathcal{S}c$, n-$CyN\theta c$, n-$CyN\theta\mathcal{S}c$, n-$CyNec$, and n-$CyNMc$) subset of a n-$CyN\mathcal{C}omp$ (resp. n-$CyN\delta\mathcal{C}omp$, n-$CyN\delta\mathcal{P}\mathcal{C}omp$, n-$CyN\delta\mathcal{S}\mathcal{C}omp$, n-$CyN\theta\mathcal{C}omp$, n-$CyN\theta\mathcal{S}\mathcal{C}omp$, n-$CyNe\mathcal{C}omp$, and n-$CyNM\mathcal{C}omp$) X is n-$CyN\mathcal{C}omp$ (resp. n-$CyN\delta\mathcal{C}omp$, n-$CyN\delta\mathcal{P}\mathcal{C}omp$, n-$CyN\delta\mathcal{S}\mathcal{C}omp$, n-$CyN\theta\mathcal{C}omp$, n-$CyN\theta\mathcal{S}\mathcal{C}omp$, n-$CyNe\mathcal{C}omp$, and n-$CyNM\mathcal{C}omp$) relative to X.*

Proof. Let A be a n-$CyNMc$ subset of a n-$CyNM\mathcal{C}omp$ X. Then, A^c is n-$CyNMos$ in X. Let $B=\{A_i : i \in I\}$ be a n-$CyNM\mathcal{O}\mathcal{C}ov$ of A. Then, $B \cup \{A^c\}$ is a n-$CyNM\mathcal{O}\mathcal{C}ov$ of X. Since, X is n-$CyNM\mathcal{C}omp$, it has a finite subcover say $\{P_1, P_2, \cdots P_P, A^c\}$. Then, $\{P_1, P_2, \dots, P_P\}$ is a finite n-$CyNM\mathcal{O}\mathcal{C}ov$. Thus, A is n-$CyNM\mathcal{C}omp$ relative to X.

Similar situations exist in other scenarios

Theorem 4.3 *Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a n-$CyNCts$ (resp. n-$CyN\delta Cts$, n-$CyN\delta\mathcal{P}Cts$, n-$CyN\delta\mathcal{S}Cts$, n-$CyN\theta Cts$, n-$CyN\theta\mathcal{S}Cts$, n-$CyNeCts$, and n-$CyNMCts$) map is a surjection and X be n-$CyN\mathcal{C}omp$ (resp. n-$CyN\delta\mathcal{C}omp$, n-$CyN\delta\mathcal{P}$ $\mathcal{C}omp$, n-$CyN\delta\mathcal{S}\mathcal{C}omp$, n-$CyN\theta\mathcal{C}omp$, n-$CyN\theta\mathcal{S}\mathcal{C}omp$, n-$CyNe\mathcal{C}omp$, and n-$CyNM\mathcal{C}omp$). Then, Y is n-$CyN\mathcal{C}omp$.*

Proof. Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a n-$CyNMCts$ map is a surjection and X be n-$CyNM\mathcal{C}omp$. Let $\{M_\alpha\}$ be a n-$CyNM\mathcal{O}\mathcal{C}ov$ for Y. Since, f is n-$CyNMCts$ map, $\{f^{-1}(M_\alpha)\}$ is a n-$CyNM\mathcal{O}\mathcal{C}ov$ of X. Since, X is n-$CyNM\mathcal{C}omp$, $\{f^{-1}(M_\alpha)\}$ contains a finite subcover, namely $\{f^{-1}(M_{\alpha_1}), f^{-1}(M_{\alpha_2}), \dots, f^{-1}(M_{\alpha_P})\}$. Since, f is surjection, $\{M_{\alpha_1}, M_{\alpha_2}, \dots, M_{\alpha_P}\}$ is a finite subcover for Y. Thus, Y is a n-$CyN\mathcal{C}omp$.

Similar situations exist in other scenarios

Theorem 4.4 *Let $f:(X,\tau_1) \to (Y,\tau_2)$ be a n-$CyNO$ (resp. n-$CyN\delta O$, n-$CyN\delta\mathcal{P}O$, n-$CyN\delta\mathcal{S}O$, n-$CyN\theta O$, n-$CyN\theta\mathcal{S}O$, n-$CyNeO$, and*

n-$CyNMO$) map and Y be n-$CyN\mathcal{C}omp$ (resp. n-$CyN\delta\mathcal{C}omp$, n-$CyN\delta\mathcal{P}$ $\mathcal{C}omp$, n-$CyN\delta\mathcal{S}\mathcal{C}omp$, n-$CyN\theta\mathcal{C}omp$, n-$CyN\theta\mathcal{S}\mathcal{C}omp$, n-$CyNe\mathcal{C}omp$, and n-$CyNM\mathcal{C}omp$). Then, X is n-$CyN\mathcal{C}omp$.

Proof. Let $f:(X,\tau_1)\to(Y,\tau_2)$ be a n-$CyNMO$ map and Y be n-$CyNM\mathcal{C}omp$. Let $\{M_\alpha\}$ be a n-$CyNMOCov$ for X. Since, f is n-$CyNMO$, $\{f(M_\alpha)$ is a n-$CyNMOCov$ of Y. Since, Y is n-$CyNM\mathcal{C}omp$, $\{f(M_\alpha)\}$ contains a finite sub n - $CyNMOCov$, namely $\{f(M_{\alpha_1}), f(M_{\alpha_2}), \dots, f(M_{\alpha_P})\}$. Then, $\{M_{\alpha_1}, M_{\alpha_2}, \dots, M_{\alpha_P}\}$ is a finite subcover for X. Thus, X is n-$CyN\mathcal{C}omp$.

5 Real Life Application: Classification of Transdermal Drug Delivery Patches via an $\boldsymbol{n}$-Cylindrical Neutrosophic Separation Measure

A recurring problem in drug delivery technology is to judge how close a candidate delivery system's overall performance lies to a clinically *ideal* profile and how far it lies from a clinically *unacceptable* profile. This is naturally modelled, within an n-$CyNts$, by measuring the separation of an alternative's aggregated evaluation from the two extremal n-$CyNS$'s $0_{CyN}=\langle 0,0,1\rangle$ and $1_{CyN}=\langle 1,1,0\rangle$ introduced in Definition 2.8: an alternative whose profile is well separated from 0_{CyN} and close to 1_{CyN} is preferable. We now formalise this idea and use it to rank a set of transdermal drug delivery patches.

Definition 5.1 *Let $A_j=\langle\alpha_j,\beta_j,\gamma_j\rangle$, $j=1,2,\dots,k$, be a collection of n-$CyNN$'s corresponding to the evaluation of an alternative under k criteria, and let $w=(w_1,w_2,\dots,w_k)$ be a weight vector of the criteria with $w_j\in[0,1]$ and $\sum_{j=1}^k w_j=1$. The n-cylindrical neutrosophic weighted averaging operator (briefly, n-$CyNWA$) aggregates $A_1,A_2,\dots,A_k$ into a single n-$CyNN$ as*

$$n-CyNWA_w(A_1,A_2,\dots,A_k)=\left\langle\left(1-\prod_{j=1}^k\left(1-\alpha_j^n\right)^{w_j}\right)^{1/n},\prod_{j=1}^k\beta_j^{w_j},\prod_{j=1}^k\gamma_j^{w_j}\right\rangle,$$

which is again a valid n-$CyNN$ whenever each A_j satisfies $0\le\beta_j\le 1$ and $\alpha_j^n+\gamma_j^n\le 1$.

Definition 5.2 *The separation (distance) between two n-$CyNN$'s $A=\langle\alpha_A,\beta_A,\gamma_A\rangle$ and $B=\langle\alpha_B,\beta_B,\gamma_B\rangle$ is defined by*

$$d(A,B)=\sqrt{\frac{1}{3}\left[(\alpha_A-\alpha_B)^2+(\beta_A-\beta_B)^2+(\gamma_A-\gamma_B)^2\right]}.$$

For an n-$CyNN$ A, the closeness coefficient of A relative to the extremal sets 1_{CyN} and 0_{CyN} is defined by

$$CC(A)=\frac{d(A,0_{CyN})}{d(A,1_{CyN})+d(A,0_{CyN})}.$$

Clearly $0\le CC(A)\le 1$; the closer $CC(A)$ is to 1, the better A is separated from the unacceptable profile 0_{CyN} and the closer it lies to the ideal profile 1_{CyN}. For two n-$CyNN$'s A_1 and A_2, if $CC(A_1)>CC(A_2)$, then A_1 is preferred over A_2, written $A_1\succ A_2$.

Example 5.1 *Let $X=\{P_1,P_2,P_3,P_4,P_5\}$ be a set of five candidate transdermal drug delivery patches, namely*

P_1: Microneedle patch, P_2: Hydrogel patch, P_3: Matrix-type patch, P_4: Reservoir patch, P_5: Iontophoretic patch,

evaluated against the criteria

C_1: Permeation efficiency, C_2: Dosage accuracy, C_3: Skin irritation resistance, C_4: Mechanical/adhesive stability, C_5: Patient comfort,

with weight vector $w=(0.25,0.25,0.20,0.15,0.15)$.

An expert panel records its assessment of each P_i under each C_j as an n-$CyNN$ $\langle\alpha,\beta,\gamma\rangle$ ($n=3$), presented in Table 5.1.

Table 1: Decision matrix of n-$CyNN$'s $\langle\alpha,\beta,\gamma\rangle$

	C_1	C_2	C_3	C_4	C_5
	⟨0.80,0.15	⟨0.75,0.20	⟨0.60,0.30	⟨0.70,0.25	⟨0.65,0.30
	⟨0.65,0.30	⟨0.60,0.35	⟨0.80,0.15	⟨0.75,0.20	⟨0.80,0.15
	⟨0.70,0.25	⟨0.70,0.25	⟨0.65,0.30	⟨0.80,0.15	⟨0.70,0.25
	⟨0.60,0.35	⟨0.65,0.30	⟨0.55,0.35	⟨0.60,0.35	⟨0.55,0.40
	⟨0.85,0.10	⟨0.80,0.15	⟨0.70,0.25	⟨0.65,0.30	⟨0.60,0.30

Every entry of Table 5.1 satisfies $0\le\beta\le 1$ and $\alpha^3+\gamma^3\le 1$, hence each is a valid n-$CyNN$ ($n=3$). Applying the n-$CyNWA_w$ operator of Definition 5.1 row-wise aggregates the criterion-wise evaluations of each P_i into a single n-$CyNN$, and Definition 5.2 then gives its separation from 1_{CyN} and 0_{CyN} together with the closeness coefficient $CC(P_i)$, as recorded in Table 5.1.

Table 2: Aggregated n-$CyNWA_w$ value, separation distances and closeness coefficient $CC(P_i)$

Patch	α	β	γ	$d(P_i,1_{CyN}$	$d(P_i,0_{CyN}$	$CC(P_i$
P_1 (Microneedle)	0.7236	0.2218	0.5626	0.5769	0.5047	0.4666
P_2 (Hydrogel)	0.7229	0.2302	0.5463	0.5680	0.5103	0.4733
P_3						

(Matrix-type)	0.7108	0.2402	0.5638	0.5712	0.5010	0.4673
P_4 (Reservoir)	0.5985	0.3436	0.6466	0.5803	0.4476	0.4355
P_5 (Iontophoretic)	0.7626	0.1848	0.5306	0.5781	0.5279	0.4773

Since $CC(P_5) > CC(P_2) > CC(P_3) > CC(P_1) > CC(P_4)$, we obtain the ranking

$$P_5 \succ P_2 \succ P_3 \succ P_1 \succ P_4,$$

so that P_5, the iontophoretic patch, is best separated from the unacceptable performance profile 0_{CyN} and is the most suitable transdermal drug delivery patch among the five candidates, followed closely by the hydrogel patch P_2; the reservoir patch P_4, being closest to 0_{CyN}, is the least suitable.

Remark 5.1 *Since $CC(A)$ compares the separation of an alternative from the two disjoint extremal n-$CyNS$'s 0_{CyN} and 1_{CyN}, this application is a direct practical counterpart of the separation notions studied in Section 3: a candidate that is n-CyN well separated from 0_{CyN}, in the sense of lying at a large distance from it, is precisely the type of alternative a decision maker seeks. This illustrates the practical relevance of n-$CyNS$ based separation structures to multi-criteria classification problems arising in drug delivery technology and, more broadly, in biomedical and pharmaceutical sciences.*

6 Conclusion

In this paper, we introduce the concepts of n-cylindrical neutrosophic M-connected and M-disconnected spaces within the framework of n-cylindrical neutrosophic topological spaces. Additionally, we define, n-cylindrical neutrosophic M-compactness and n-cylindrical neutrosophic M-separated sets. Further, to demonstrate the practical significance of this framework, we developed an n-cylindrical neutrosophic separation measure and closeness coefficient, built on the extremal sets 0_{CyN} and 1_{CyN}, and applied them to a real-life problem concerning the classification and ranking of transdermal drug delivery patches. Among the five candidate patches considered, the iontophoretic patch emerged as the most suitable choice, followed by the hydrogel patch, illustrating how the notion of separation in n-$CyNS$'s translates naturally into a practical decision-support tool for biomedical applications. It is anticipated that the findings presented herein will support and inspire further research in n-cylindrical neutrosophic topology, contributing to the development of a more comprehensive theoretical framework and promoting practical applications in various fields.

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