

Numerical investigation on performance of three-fluid re-circulating type heat exchanger

Kavadiki Veerabhadrapa^{1*}, Tejaswini K², Chandragowda M³ and Bheemsha Arya⁴

^{1,4}*Department of Mechanical Engineering, B.M.S. College of Engineering, P.B. No. 1908, Bull Temple Road, Bengaluru – 560 019, Karnataka, India*

²*Department of Computer Science and Business Systems, B.M.S. College of Engineering, P.B. No. 1908, Bull Temple Road, Bengaluru – 560 019, Karnataka, India*

³*Department of Mechanical Engineering, Rao Bahadur Y Mahabaleswarappa Engineering College, Cantonment, Ballari – 583 104, Karnataka, India*

**Corresponding author: kavadiki.mech@bmsce.ac.in*

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ABSTRACT

Three-fluid heat exchangers are extensively used in refrigeration, air conditioning, chemical industries, milk pasteurization, cryogenics industries and energy recovery applications due to their higher heat transfer rates. The focus of this study is to analyse performance of a three-fluid re-circulating heat exchanger which is a new type for numerical study. Two possible configurations have been developed for the analysis. Cold fluid is being re-circulated and exchange heat between them. Finite element model for this type of heat exchanger is developed based on the detailed geometry and the specific working conditions. With the help of detailed geometry FEM model of the three-fluid re-circulated type heat exchanger is analysed. Non-Dimensional parameters are introduced which makes the analysis more versatile. Heating of the cold fluid has been identified as the main objective of the heat exchanger. Effect of non-dimensional parameters on the performance of three-fluid re-circulated type heat exchanger is studied. This paper gives valuable inputs towards better understanding of this type of heat exchangers.

Keywords: Three-fluid, Re-circulating type, Heat exchanger, Effectiveness, Finite element method.

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1. INTRODUCTION

Three-fluid heat exchangers are used in several applications such as heat recovery systems, cryogenic systems, power generation, transport, chemical and petro chemical industries, aerospace etc. They are also used in system deals with ammonia gas synthesis, purification and liquefaction of hydrogen, helium separation and air separation units [13]. Initial study on counter-current parallel flow three-fluid heat exchanger problem was solved by Morley [1] in 1933. Sekulic and Shah [10, 11, 13] have provided wide literature on steady state behaviour of three-fluid heat exchanger. Dushan and Sekulic [10] presented the analytical solutions to the three-fluid heat exchanger for four possible flow arrangements using Laplace transforms technique. Solrlie [2] developed general theory of two temperature effectiveness of three-fluid heat exchanger for parallel and counter flow arrangement, Aulds and Barron [6] extended work by Sorlie [2] to develop analytical relationship between the design variables for three-fluid heat exchanger with three thermal communications. Barron and Yeh [8] obtained the

temperature distribution and effectiveness for three-fluid heat exchanger for counter flow arrangement under steady state condition considering effect of longitudinal wall conduction. Barron [9] has developed a mathematical model where one of the fluids in a two-fluid heat exchanger is interacting with the ambient. Willis and Chapman [7] presented the performance of three-fluid cross-flow heat exchanger and the effectiveness is determined as a function of heat exchanger size for sets of fixed operating conditions. Many researchers [3, 12] have presented an analytical method for a multiple streams for one-dimensional heat exchanger to determine the temperature distribution along the length of the heat exchangers of all streams with an assumption that no multiple eigenvalues to the solution. Others [4–5] presented explicit/iterative stream direction-dependent results for this class of three-fluid heat exchanger assuming that multiple eigenvalues do not exist. Shrivastava and Ameel [14, 15] have developed a three-fluid heat exchanger with three thermal communications model for all possible thermal interactions and flow

**Author for Correspondence: kavadiki.mech@bmsce.ac.in*

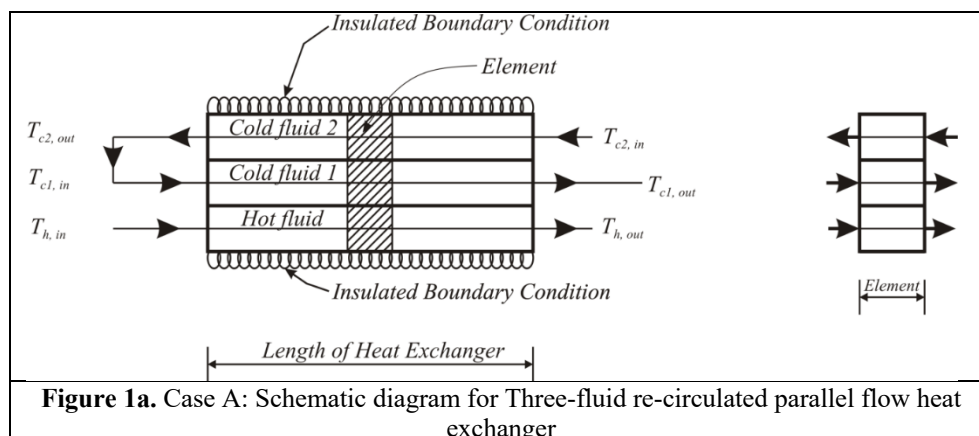
arrangements with insulated boundaries. The performance of three-fluid heat exchanger for co-current and counter-current flow arrangements is investigated numerically using FEM and compared with classical two-fluid heat exchanger by Saeid and Seetharamu [16] which is restricted to insulated conditions. effectiveness and degradation factor. Krishna et al. [17, 18] Investigated the thermal performance of three-fluid heat exchanger (cryogenic applications) with three thermal communications with and without longitudinal wall conduction effect. Quadir et al. [19, 20 and 32] have recently evaluated the performance of a triple concentric pipe heat exchanger investigated numerically using finite element method under steady state conditions for various flow arrangements for insulated as well as non-insulated boundary conditions and they conducted the experiments on three-fluid (triple concentric tube) heat exchanger under steady state conditions for different fluid flow arrangements. They presented their results in terms of temperature distributions of the three-fluids along the length of the heat exchanger. Veerabhadrapa et al. [21, 22] transient temperature response of a counter-flow three-fluid heat exchanger with two and three thermal communications are investigated numerically by using finite element method. Abdalla et al. [23] have presented experimental and CFD investigations of triple tube heat exchanger and they are considered for four different flow patterns of triple concentric tube heat exchanger. Based on their experiment they are predicted correlations of Nusselt number, friction factor and effectiveness and finally they concluded that triple concentric tube (three-fluid) heat exchanger is more effective and energy saving when compared with double concentric tube heat exchanger.

Jyothiprakash et al. [24, 25] investigated the performance of three-fluid cross-cocurrent type heat exchanger under heat-in-leak condition from the surrounding using finite element method and also they are focused on optimization of three-fluid cross-flow plate-fin heat exchanger by considering the objective function on maximizing the hot fluid effectiveness and minimizing the number of entropy generation units. Recently Vishnudas et al. [26, 27] investigated the counter-current, parallel flow three-fluid both two and three thermal communications cryogenic heat exchanger presents improved effectiveness due to the Joule-Thomson pressure drop.

In this paper authors present analysis on thermal performance of a three-fluid re-circulated type heat exchanger where in instead of having three-fluids flowing in three separate channels to enhance heat transfer, one of the fluids is being re-circulated. This type of configuration is used to heat a fluid to higher temperature and cool it down as its re-circulated which typically occurs during pasteurization of milk or bayonet heat exchangers. Non-dimensional variables are introduced to make the analysis versatile and effects of these non-dimensional parameters on the performance of three channel heat exchanger is studied.

2. MATHEMATICAL MODEL

A three-fluid heat exchanger with cold fluid being re-circulated has been considered for the analysis and is shown in Fig.1a and Fig.1b. The hot fluid is flowing in middle channel; cold fluid flowing in the outer channel is re-circulated. In one configuration outer cold fluid entry is in same direction as hot fluid (Fig. 1a) and in other it flows in counter direction (Fig. 1b).



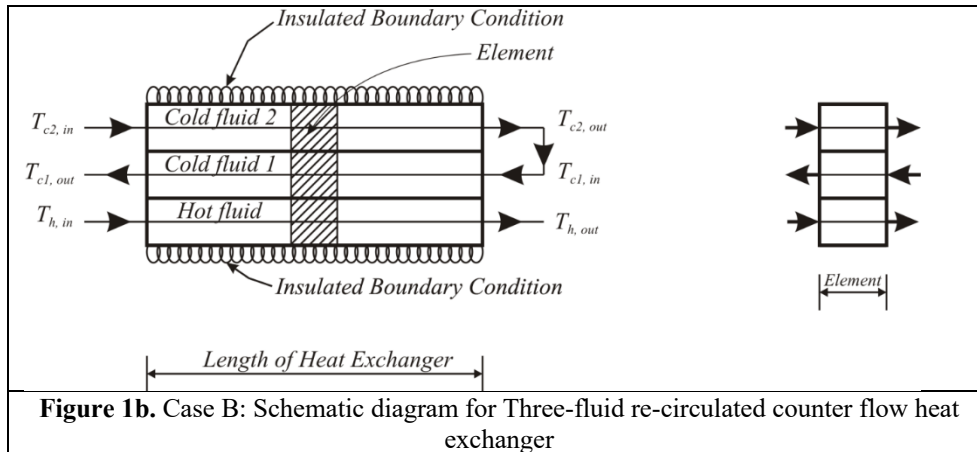


Figure 1b. Case B: Schematic diagram for Three-fluid re-circulated counter flow heat exchanger

To generalize the formulations, the following assumptions are made:

- (1) The mass flow rates of the fluids remain constant.
- (2) Longitudinal heat conduction through the pipes or the fluids is neglected.
- (3) Within a fluid stream the temperature distribution is uniform in the transverse direction and equal to the average temperature of the fluid.
- (4) There is no heat source or sink within the heat exchanger or in any of the fluids,
- (5) There is no phase change in the fluid streams.
- (6) Outer surfaces of the heat exchanger are insulated.
- (7) Heat transfer area is constant along the length of the heat exchanger.
- (8) There is no back pressure.
- (9) The flow is fully developed in all channels.

Heat exchanger is divided into number of elements as shown in Figure 1a and Figure 1b, and applying the conservation of energy principle for each of the streams the following equations are derived.

<i>Case A:</i>	
<i>Hot fluid:</i> $\frac{d\theta_h}{dx} + NTU_e R_1 (\theta_h - \theta_{c1}) = 0$	(1)
<i>Cold fluid:</i> $\frac{d\theta_{c2}}{dx} + NTU_e R_2 H (\theta_{c1} - \theta_{c2}) = 0$	(2)
<i>Re-circulated Cold fluid:</i> $\frac{d\theta_{c1}}{dx} - NTU_e (\theta_h - \theta_{c1}) + NTU_e H (\theta_{c1} - \theta_{c2}) = 0$	(3)
<i>Case B:</i>	
<i>Hot fluid:</i> $\frac{d\theta_h}{dx} + NTU_e R_1 (\theta_h - \theta_{c2}) = 0$	(4)
<i>Cold fluid:</i> $\frac{d\theta_{c2}}{dx} - NTU_e R_2 H (\theta_{c1} - \theta_{c2}) = 0$	(5)
<i>Re-circulated Cold fluid:</i> $\frac{d\theta_{c1}}{dx} + NTU_e (\theta_h - \theta_{c1}) - NTU_e H (\theta_{c1} - \theta_{c2}) = 0$	(6)

Following are the dimensionless variables that are introduced in this analysis:

$$NTU_e = \frac{U_1 P_1 L_e}{\dot{m}_{c1} C_{p c1}}; R_1 = \frac{\dot{m}_{c1} C_{p c1}}{\dot{m}_h C_{p h}}; R_2 = \frac{\dot{m}_{c1} C_{p c1}}{\dot{m}_{c2} C_{p c2}} = 1; H = \frac{U_2 P_2}{U_1 P_1}; \theta = \frac{T - T_{c2, in}}{T_{h, in} - T_{c2, in}}; \quad (7)$$

$$X = \frac{x}{L_e}; \quad L_e = \frac{L}{N}$$

R_2 remains unity for all the cases as the re-circulated cold fluid mass flow rate is same as that of cold fluid flowing in outer channel.

3. FINITE ELEMENT METHOD

The heat exchanger is divided into a number of elements. For all fluids a linear variation of temperature in each of the element is assumed as the differential equations are

first order. Applying the method of minimizing the weighted residual [28, 29] Eqs. (1) to (6) is reduced to as below:

<i>Case A:</i>	
$\int_0^1 W \left\{ \frac{d\theta_h}{dx} + NTU_e R_1 (\theta_h - \theta_{c1}) \right\} dx = 0$	(8)

$\int_0^1 W \left\{ \frac{d\theta_{c2}}{dX} + NTU_e R_2 H(\theta_{c1} - \theta_{c2}) \right\} dX = 0$	(9)
$\int_0^1 W \left\{ \frac{d\theta_{c1}}{dX} - NTU_e (\theta_h - \theta_{c1}) + NTU_e H(\theta_{c1} - \theta_{c2}) \right\} dX = 0$	(10)
<i>Case B:</i>	
$\int_0^1 W \left\{ \frac{d\theta_h}{dX} - NTU_e R_1 (\theta_h - \theta_{c2}) \right\} dX = 0$	(11)
$\int_0^1 W \left\{ \frac{d\theta_{c2}}{dX} + NTU_e R_1 (\theta_h - \theta_{c2}) + NTU_e R_2 H(\theta_{c1} - \theta_{c2}) \right\} dX = 0$	(12)
$\int_0^1 W \left\{ \frac{d\theta_{c1}}{dX} + NTU_e H(\theta_{c1} - \theta_{c2}) \right\} dX = 0$	(13)

W is the weighted function and as per subdomain collocation method its value is 1.

The fluid temperature at any point in each of the element is given by,

<i>Case A:</i>	
$\theta_h = N_1 \cdot \theta_{hi} + N_2 \cdot \theta_{ho}$	(14a)
$\theta_{c1} = N_1 \cdot \theta_{c1i} + N_2 \cdot \theta_{c1o}$	(14b)
$\theta_{c2} = N_2 \cdot \theta_{c2i} + N_1 \cdot \theta_{c2o}$	(14c)
<i>Case B:</i>	
$\theta_h = N_1 \cdot \theta_{hi} + N_2 \cdot \theta_{ho}$	(15a)
$\theta_{c2} = N_2 \cdot \theta_{c2i} + N_1 \cdot \theta_{c2o}$	(15b)
$\theta_{c1} = N_1 \cdot \theta_{c1i} + N_2 \cdot \theta_{c1o}$	(15c)

Where the prime denotes the element temperatures and the shape functions N_1 and N_2 are given by $N_1 = (1 - X)$ and $N_2 = X$; Substitution of these approximations in Eq. (8) to (13), the set of algebraic equations can be obtained. The discretized governing equations are written in matrix form for each element.

$$[K]\{\theta\} = \{f\} \quad (16)$$

Where [K] is the stiffness matrix (5×6) matrix for each element the details of the matrices are given in the Appendix for both case A and case B. Assembling the all element matrix in the solution domain leads to the global matrix. The resultant global stiffness matrix form is solved by Gauss-Jordan elimination method for the dimensionless temperatures along the length of the heat exchanger. $\{\theta\}$ Temperature matrix; $\{f\}$ is Load vector.

<i>Boundary conditions: Case A</i>	
$\theta_{hi} = 1$	(17a)
$\theta_{c2i} = 0$	(17b)
$\theta_c = \theta_{c2o} = \theta_{c1i}$	(17c)
<i>Boundary conditions: Case B</i>	
$\theta_{hi} = 1$	(18a)
$\theta_{c2i} = 0$	(18b)
$\theta_c = \theta_{c2o} = \theta_{c1i}$	(18c)

4. EFFECTIVENESS

The main objective of the three-fluid heat exchanger is to heating of hot fluid. The effectiveness is defined as the

ratio of the actual heat transfer rate to the maximum possible heat transfer rate.

$\epsilon = \frac{q_{act}}{q_{max}}$	(19)
The actual heat transfer rate is: $q_{act} = \dot{m}_h C_{ph} (T_{hi} - T_{ho})$	(20)
Maximum heat transfer rate is: $q_{max} = \dot{m}_h C_{p,h} (T_{hi} - T_{c2i})$	(21)

5. RESULTS AND DISCUSSION

5.1 Validation of proposed methodology

Saied et al. [16] methodology is applied to re-circulated type three-fluid heat exchanger. Three-fluid heat exchanger for parallel-flow and counter-flow arrangement

is validated with experimental values of Abdalla et al. [23] under steady state condition as shown in Figures 2a and 2b. The numerically obtained values deviated with that of experimental results and can be attributed due to the steady state condition. Numerically obtained values agree well with that of Saied et al. [16] as shown in Figure 2c.

assumptions mentioned in mathematical modeling section. The Effectiveness of three-fluid heat exchanger for parallel-flow arrangement is investigated under

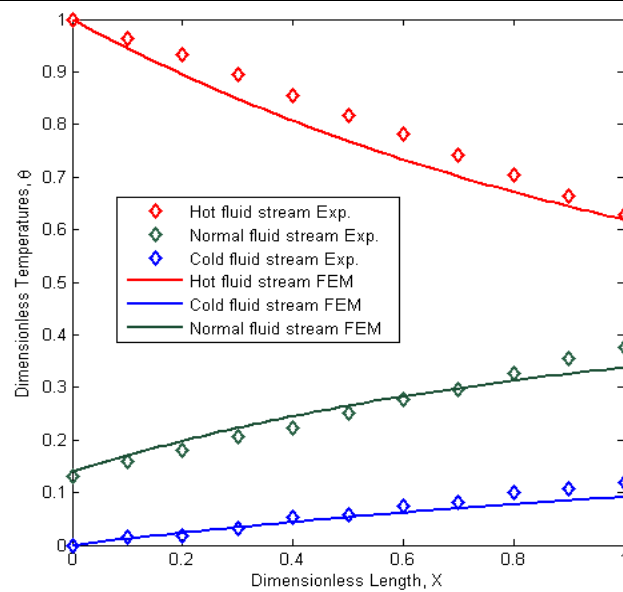


Figure 2a. Temperature variation of parallel-flow three-fluid streams along the length of the heat exchanger by [23], $R_1=1$ $R_2=1000$, $H=1$, $\theta_{h,in}=1$, $\theta_{c1,in}=0$, $\theta_{c2,in}=0$ [14]

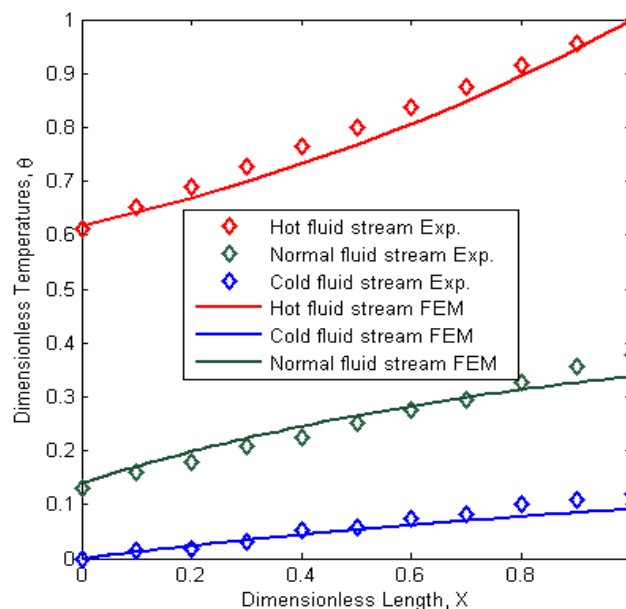
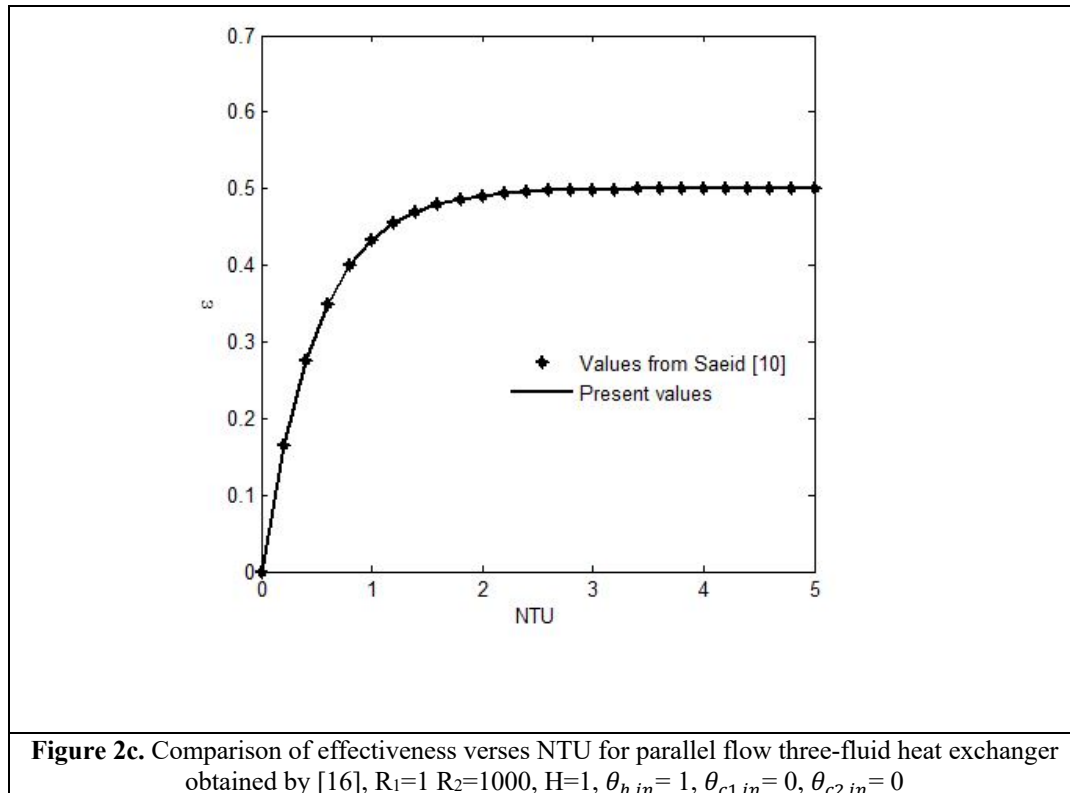


Figure 2b. Temperature variation of counter-flow three-fluid streams along the length of the heat exchanger by [23], $R_1=1$ $R_2=1000$, $H=1$, $\theta_{h,in}=1$, $\theta_{c1,in}=0$, $\theta_{c2,in}=0$



5.2 Parametric Study

Figure 3a shows the variation of the cold fluid effectiveness plotted against NTU for various values of the capacity ratio (R_1) ranging from 0 to 10. Cold fluid effectiveness increases sharply in the beginning and attains a constant value with NTU. For a given value of NTU, the cold fluid effectiveness decreases as the value of the R_1 increases. It is also observed that for higher values of R_1 the constant value of effectiveness occurs at very small value of NTU. As the value of R_1 reduces, the value at

which the constant value of the effectiveness is achieved as slightly higher values of NTU. It can be observed from the figure that for values of NTU beyond 2 cold fluid effectiveness is 0.6 when R_1 is increase to 0.5 the value of cold fluid effectiveness beyond the value of NTU is equals to 0.5 that there is a reduction of cold fluid effectiveness 16.16% when R_1 changes from 0 to 0.5. The value of cold fluid effectiveness for R_1 is equals to 1 is 0.3, when R_1 is increase to 10 its value is about 0.09. Thus there is reduction of 70%.

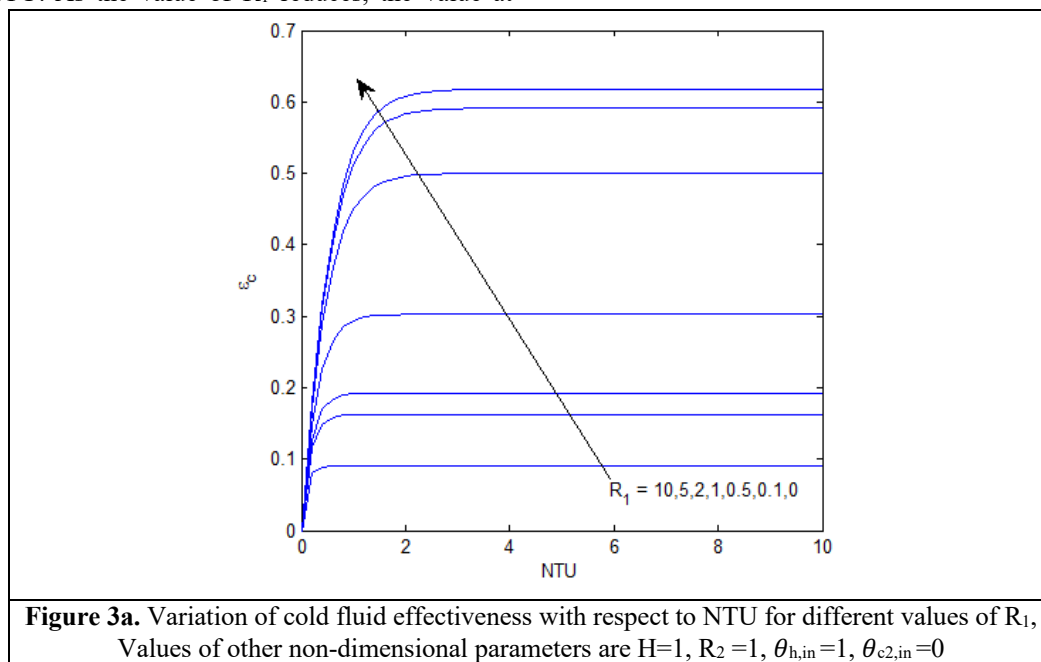


Figure 3b shows the variation of the cold fluid effectiveness plotted against NTU for various values of the capacity ratio (H) ranging from 0 to 10. Up to a value of about NTU is equals 0.2 the effectiveness is independent of the parameter H . For higher values of H effectiveness is smaller and constant value of effectiveness occurs at very small value of NTU of about 0.4. However for small

values of H (0) this goes up to about NTU equals 2. As H increases there is a reduction in cold fluid effectiveness when the value of H varies from 0 to 0.5 cold fluid effectiveness reduces by 10% from 0.5. When the value of H varies from 1 to 10 cold fluid effectiveness varies from 0.4 to 0.22 that is 45%.

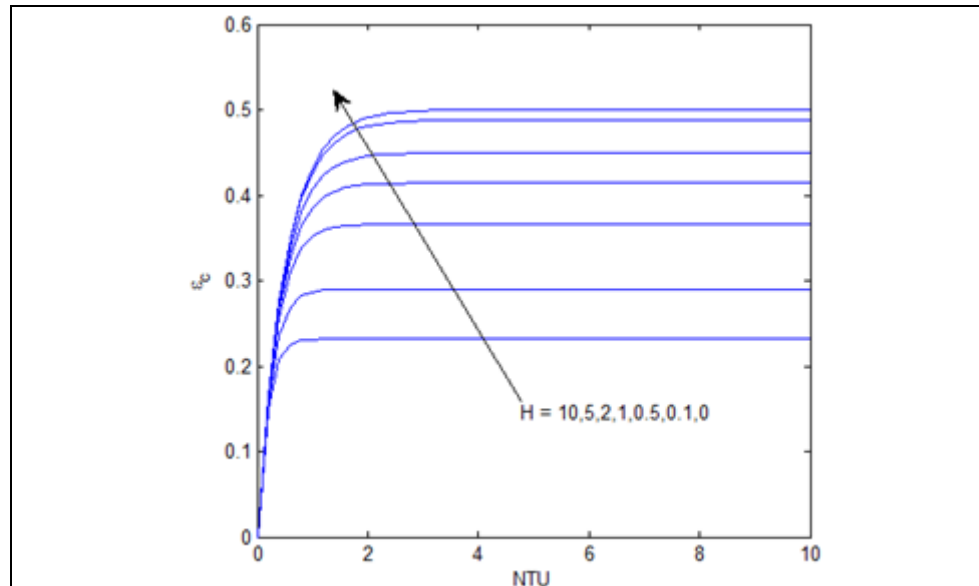


Figure 3b. Variation of cold fluid effectiveness with respect to NTU for different values of H , Values of other non-dimensional parameters are $R_1=1$, $R_2=1$, $\theta_{h,in}=1$, $\theta_{c2,in}=0$

Figure 3c shows the variation of temperature of fluids across the heat exchanger. It can be observed that hot fluid temperature decreases from inlet to the outlet. The cold fluid increases its temperature as it flows along the heat exchanger. The intermediate fluid increases as it flows

along the length and reaches a maximum and slightly reduces towards the end. When the NTU is changes from 1 to 2 the hot fluid outlet temperature changes marginally whereas the cold fluid outlet temperature increases marginally, when NTU is increases from 1 to 2.

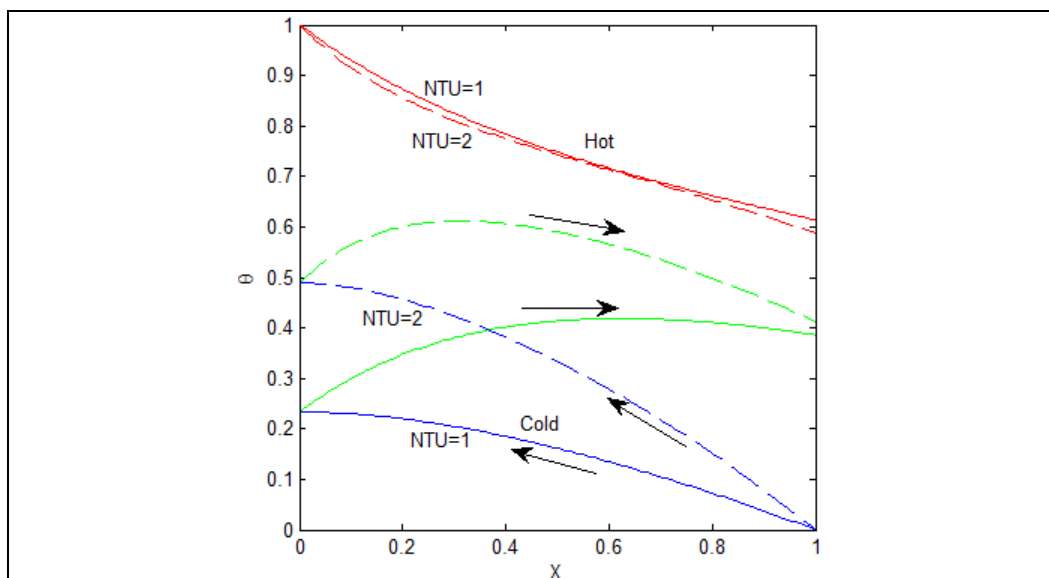


Figure 3c. Temperature profile of fluids across the heat exchanger at different value of NTU. $R_1=1$, $R_2=1$, $H=1$, $\theta_{h,in}=1$, $\theta_{c2,in}=0$

Figure 3d shows the variation of temperature of hot fluid across the heat exchanger. When H changes from 1 to 2

the hot fluid outlet temperature increases marginally. For the case of the cold fluid temperature as the value H

changes from 1 to 2 the cold fluid temperature very slightly as the parameter changes from 1 to 2.

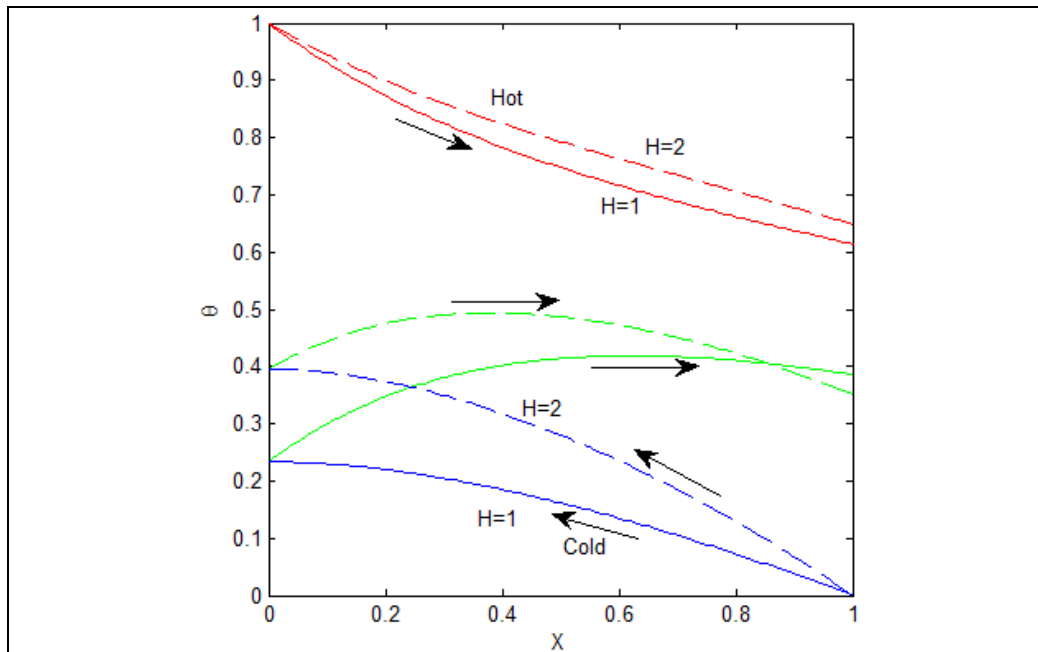


Figure 3d. Temperature profile of fluids across the heat exchanger at different value of H
 $R_1=1, R_2=1, NTU=1, \theta_{h,in}=1, \theta_{c2,in}=0$

Figure 3e shows the variation of the hot fluid temperature decreases from 0.71 to 0.51 and outlet temperature of cold fluid changes from 0.4 to 0.3. Figure 3e shows the variation of the hot fluid temperature decreases from 0.71 to 0.51 and outlet temperature of cold fluid changes from 0.4 to 0.3.

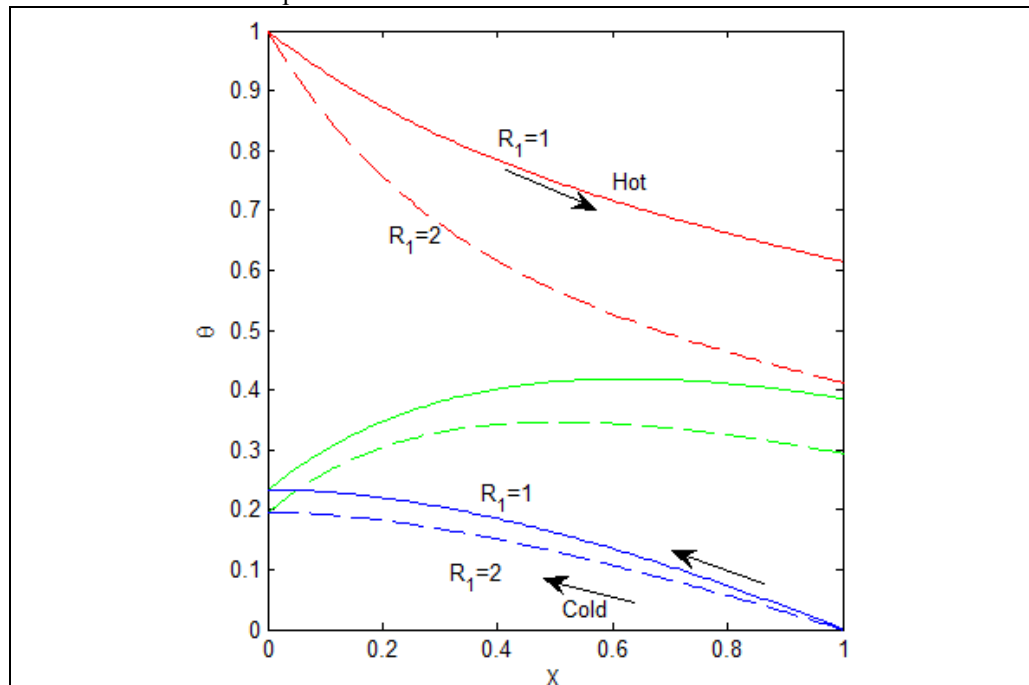


Figure 3e. Temperature profile of fluids across the heat exchanger at different value of R_1
 $H=1, R_2=1, NTU=1, \theta_{h,in}=1, \theta_{c2,in}=0$

Figure 4a shows variation cold fluid effectiveness versus NTU for different values of R_1 ranging from 0 to 10. The comments written for fig. 4.2a also applies to 4.3a.

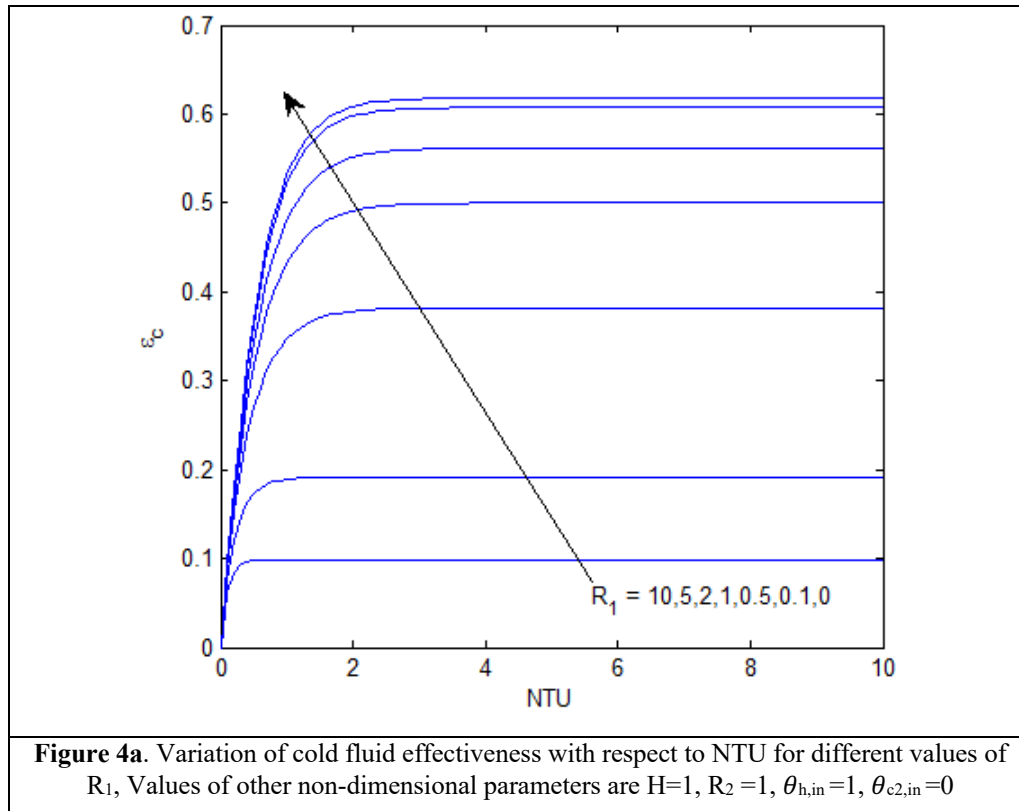


Figure 4b shows the variation of cold fluid effectiveness plotted against NTU for different values of H varying from 0 to 10. For a value of NTU less than 0.2, cold fluid effectiveness is independent of parameter H . For a value of $H=10$ cold fluid effectiveness becomes independent of

NTU beyond the value of $NTU=0.8$. For a given value of NTU, cold fluid effectiveness increases as the H reduces. For a value of $H=0$ cold fluid effectiveness increases with NTU. The cold fluid effectiveness becomes constant with NTU at a higher value of NTU as H goes on reducing.

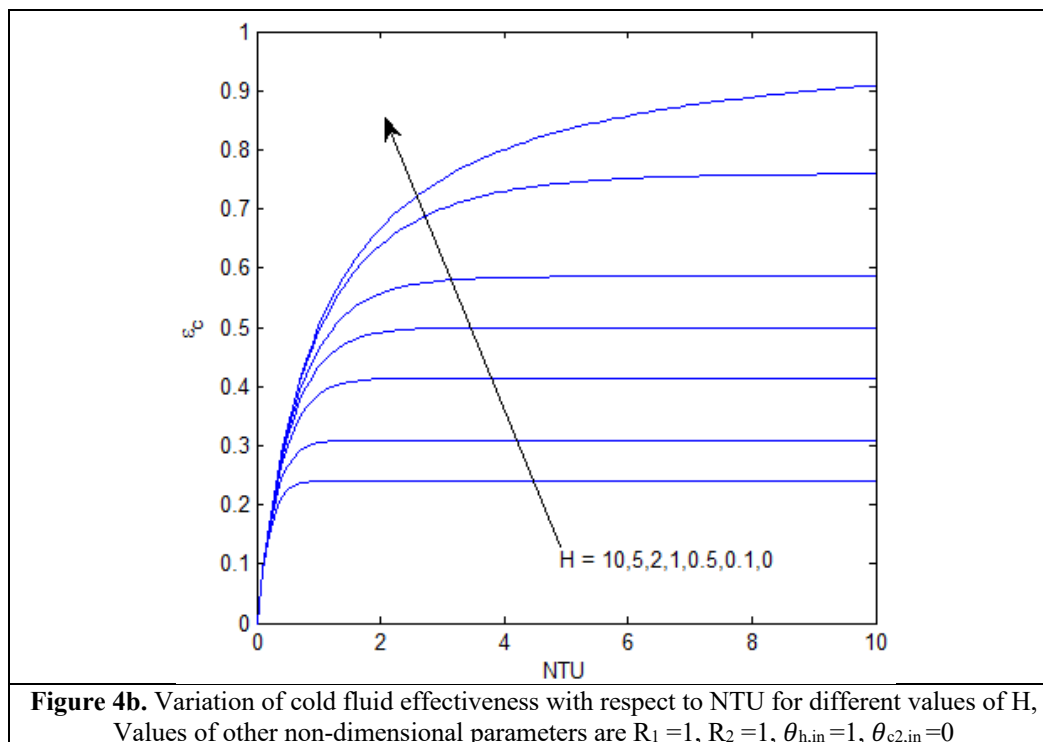


Figure 4c shows the temperature profile of fluids across the heat exchanger for different values of NTU. For a change of NTU from 1 to 2 the hot fluid outlet temperature

reduces from 0.57 to 0.51. The cold fluid outlet temperature increases from 0.43 to 0.49 when NTU

increases from 1 to 2. This effect may be due to increase in heat transfer coefficient for a given heat exchanger.

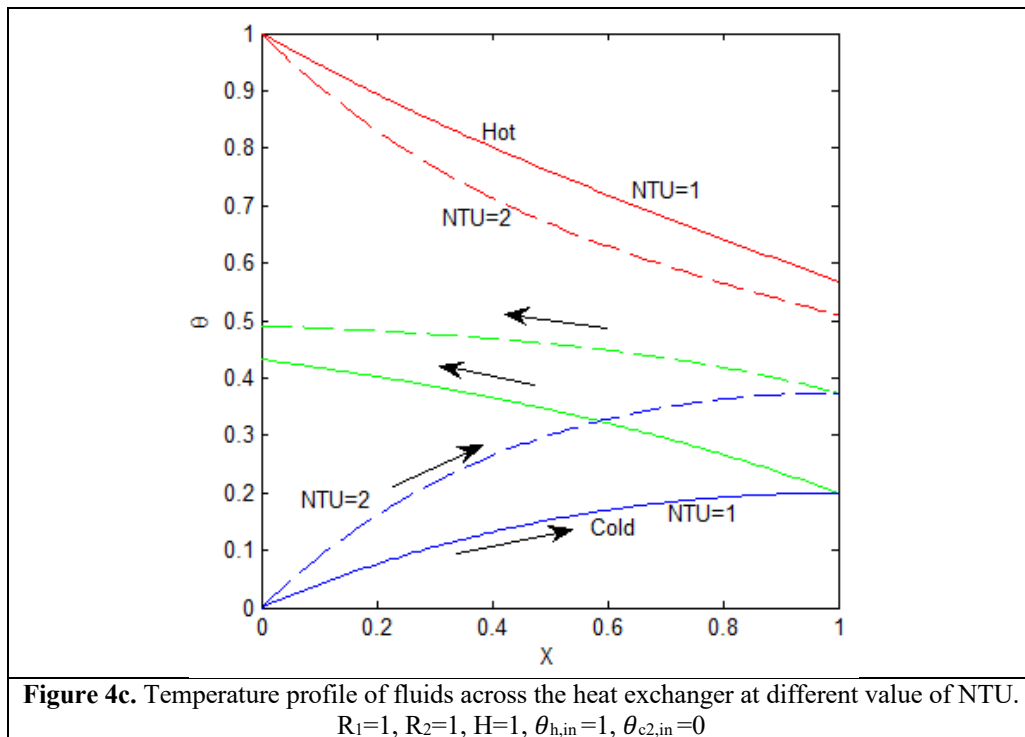


Figure 4d shows the temperature profile of fluids across the heat exchanger at different values of H . hot fluid outlet temperature slightly increases when the value of H changes from 1 to 2. Cold fluid outlet temperature slightly

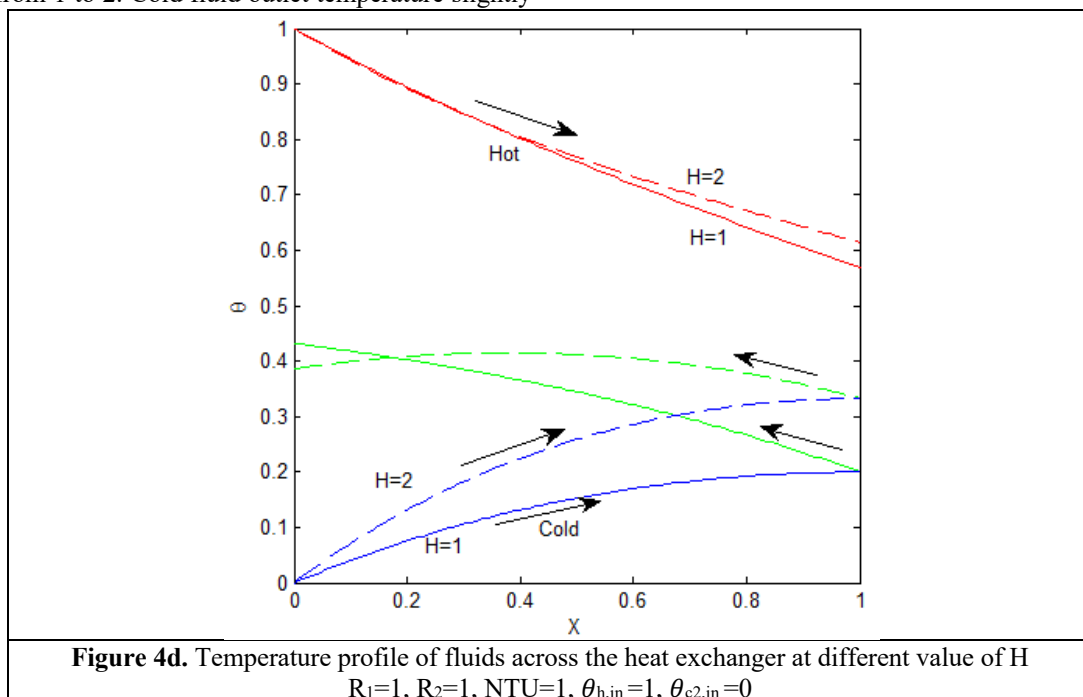


Figure 4e shows temperature profile of fluids across the heat exchanger at different values of R_1 . The hot fluid outlet temperature reduces when R_1 is changing from 1 to 2. The cold fluid outlet temperature also reduces when R_1 is changed from 1 to 2.

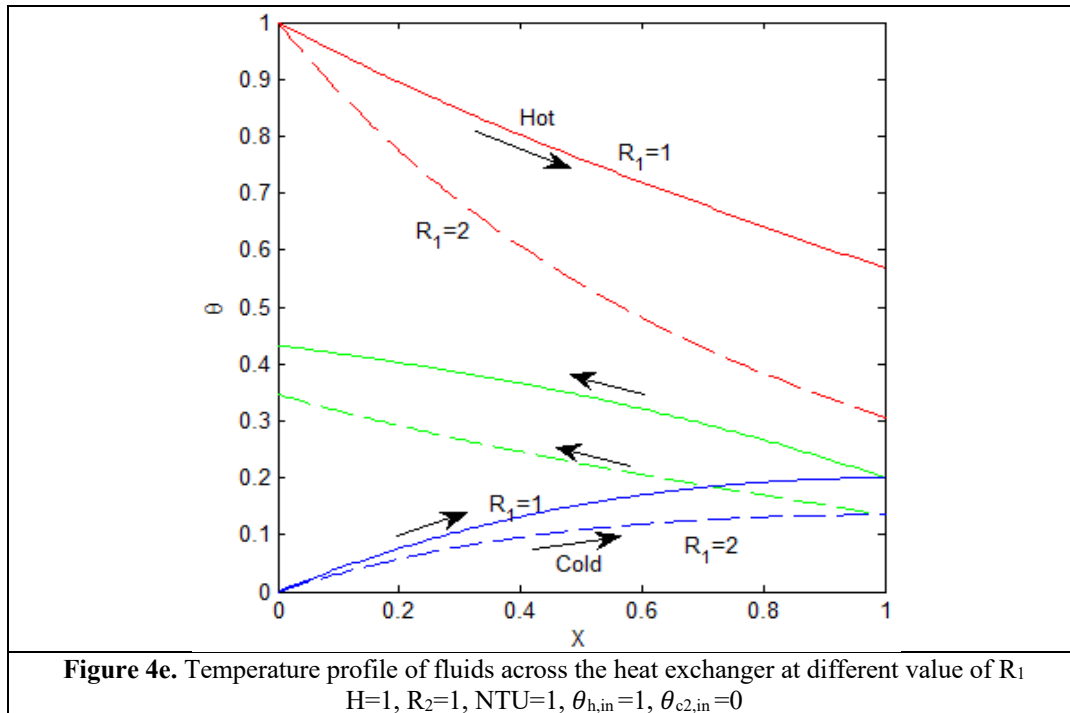


Figure 4e. Temperature profile of fluids across the heat exchanger at different value of R_1
 $H=1, R_2=1, NTU=1, \theta_{h,in}=1, \theta_{c,in}=0$

6. CONCLUSION

In the proposed study finite element method is used to study the performance of re-circulating type three-fluid heat exchanger. Heating of the cold fluid has been defined as the main objective of the heat exchanger. Non-dimensional parameters NTU , R_1 , R_2 and H are studied to analyse the performance of this type of heat exchanger. The temperature profiles of the cold and hot fluid, effectiveness are studied for the case A and Case B.

1. An increase in the value of H leads to decrease in the heat transfer resistance between the re-circulated cold fluid and outer channel cold fluids and/or increase in the heat transfer resistance between the hot and re-circulated cold fluid. There is increased temperature of the re-circulated cold fluid at its inlet due to increased H and as re-circulated fluid flows it receives less heat from the hot fluid and its temperature at exit drops due loss of more heat than received from hot fluid.

It is also observed that at higher values of NTU for any given H , the heat transfer from the hot fluid to the cold fluid will reach the maximum possible heat transfer and the effectiveness approaches its maximum value and remains constant with further increase in NTU . As H value increases the NTU at which effectiveness reaches constant value decreases. Maximum effectiveness in case A is 0.5 and in case B its above 0.9, So case B is preferred arrangement for heat exchanger with twice effectiveness as case A.

2. An increase in value of R_1 leads to an increase in ratio of mass flow rates of cold to that of hot fluids when all other parameters are kept constant. As mass flow rate of

cold fluid increases there is increase in its specific heat capacity due to which there is increased heat transfer, yet temperature of cold fluid reduces as heat lost by hot fluid is taken by larger mass of cold fluid. Increase in R_1 leads to decrease in cold fluid out temperature and its effectiveness. It is also observed that at higher values of NTU for any given R_1 , the heat transfer from the hot fluid to the cold fluid will reach the maximum possible heat transfer and the effectiveness approaches its maximum value and remains constant with further increase in NTU . As R_1 value increases the NTU at which effectiveness reaches constant value decreases. Maximum effectiveness in case A and case B is same. For a given R_1 and NTU effectiveness in case B is greater or equal than case A.

3. An increase in NTU will not result in any change in the thermal resistance between the hot and the cold fluids. This will only result in a decrease in the thermal capacity of the cold fluid. If the specific heat is assumed constant, then this result in a decrease in the mass flow rate of the cold fluid. Since R_1 is constant this results in the decrease in the flow rates of the hot and the as well. The effect of all this is steeper thermal gradients for all three fluids and increased differences between the inlet and exit temperatures, with increasing values of NTU

The validation of the finite element methodology and usage of non-dimensional parameters to present the results for the analysis establishes the versatility of this methodology, which can account for most real time situations with relative ease and arrive at accurate results. The results presented for the proposed methodology gives valuable inputs towards better understanding of the behaviour of the cold fluid in this class of heat exchangers.

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REFERENCES

- [1] Morley, T. B. "Exchange of heat between three fluids." *Engineer* 155 (4029) (1933): 314-316.
- [2] T. Sorlie, Three-Fluid Heat Exchanger Design Theory-Counter and Parallel Flow Technical Report 54, Department of Mechanical Engineering, Stanford University, Stanford, California, 1962.
- [3] J. Wolf, General solution of the equations of parallel-flow multichannel heat exchangers, *Int. J. Heat Mass Transfer* 7 (1964) 901-919.
- [4] V.V.G. Krishnamurty, C.V. Rao, Heat transfer in three-fluid heat exchangers, *Indian J. Technol.* 2 (1964) 325-327.
- [5] V.V.G. Krishnamurty, Heat transfer in multi-fluid heat exchangers, *Indian J. Technol.* 4 (1966) 167-169.
- [6] Darrell D. Aulds, Randall F. Barron, Three-fluid heat exchanger effectiveness, *Int. J. Heat Mass Transf.* 10 (1967) 1457-1462.
- [7] N.C. Willis, A.J. Chapman, Analysis of three-fluid, cross flow heat exchanger, *Trans. ASME J. Heat Transfer* 90 (1968) 333-339.
- [8] Barron, R. L., and S. L. Yeh. "Longitudinal conduction in a three-fluid heat exchanger." *ASME Paper* 76 (1976) 2-7.
- [9] R.F. Barron, Effect of heat transfer from ambient on cryogenic heat transfer performance, *Advances in Cryogenic Engineering* 29 (1984) 265-272.
- [10] D.P. Sekulic, R.K. Shah, Thermal design theory of three-fluid heat exchangers, *Adv. Heat Transfer* 26 (1995) 219-329.
- [11] Dusan P. Sekulic, A compact solution of the parallel flow three-fluid heat exchanger problem, *Int. J Heat Mass Transfer* 37 (14) (1994) 2183-2187.
- [12] X. Luo, M. Li, W. Roetzel, A general solution for one-dimensional multi-stream heat exchangers and their networks, *Int. J. Heat Mass Tran.* 45 (2002) 2695-2705.
- [13] R.K. Shah, D.P. Sekulic, Fundamentals of heat exchanger design, John Wiley and Sons, Hoboken, 2003.
- [14] D. Shrivastava, T.A. Ameel, Three-fluid heat exchangers with three thermal communications Part A: General mathematical model, *International Journal of Heat and Mass Transfer* 47(2004) 3855-3865.
- [15] D. Shrivastava, T.A. Ameel, Three-fluid heat exchangers with three thermal communications Part B: Effectiveness evaluation, *International Journal of Heat and Mass Transfer* 47(2004) 3867-3875.
- [16] Nawaf H. Saaid, K.N. Seetharamu, Finite element analysis for co-current and counter-current parallel flow three-fluid heat exchanger, *International Journal Numerical Methods for Heat and Fluid Flow* 16 (2006) 324-337.
- [17] V. Krishna, P.G. Hegde, N. Subramanian, K.N. Seetharamu, Effect of ambient heat-in-leak on the performance of a three fluid heat exchanger, for cryogenic applications, using finite element method, *International Journal of Heat and Mass Transfer*, 55(2012) 5459-5470.
- [18] V. Krishna, S. Spoorthi, Pradeep G. Hegde, K.N. Seetharamu, Effect of longitudinal wall conduction on the performance of a three-fluid cryogenic heat exchanger with three thermal communications, *International Journal of Heat and Mass Transfer* 62 (2013) 567-577.
- [19] G.A. Quadir, Saqab S. Jarallah, N.J. Salman Ahmed, Irfan Anjum Badruddin, Experimental investigation of the performance of a triple concentric pipe heat exchanger, *International Journal of Heat and Mass Transfer* 62 (2013) 562-566.
- [20] G.A. Quadir, Irfan Anjum Badruddin, N.J. Salman Ahmed, Numerical investigation of the performance of a triple concentric pipe heat exchanger, *International Journal of Heat and Mass Transfer* 75 (2014) 165-172.
- [21] Veerabhadrapa K, Meghana HC, Krishna V, Seetharamu KN. Dynamic behaviour of a three fluid heat exchanger subjected to step change in inlet temperature of hot and cold fluids. *International Journal of Numerical Methods for Heat and Fluid*, 25 (2015) 1252-81.
- [22] Kavadike Veerabhadrapa, Gulabal Vinayakaraddy, K.N. Seetharamu, Pradeep G. Hegde, V. Krishna, Transient behavior of three-fluid heat exchanger with three thermal communications under step change in inlet temperature of fluids using finite element method *Applied Thermal Engineering* 108 (2016) 1390-1400
- [23] Abdalla Gomaa, M.A. Halim, Ashraf Mimi Elsaid, Experimental and numerical investigations of a triple concentric-tube heat exchanger, *Applied Thermal Engineering* 99 (2016) 1303-1315.
- [24] K.H. Jyothiprakash, Y.T. Krishnegowda, Krishna Venkataram, K.N. Seetharamu, Effect of ambient heat-in-leak on the performance of three-fluid cross-flow heat exchanger, *Int. J. Numer. Meth. Heat Fluid Flow* 28 (9) (2018) 2012-2035.
- [25] K.H. Jyothiprakash, J. Harshith, Abhimanyu Sharan, K.N. Seetharamu, Y.T. Krishnegowda, Thermodynamic Optimization of Three-Fluid Cross-Flow Heat Exchanger Using GA and PSO Heuristics, *Thermal Science and Engineering Progress* 11 (2019) 289-301
- [26] Vishnudas Chodankara, Aswatha, V. Krishna, K.N. Seetharamu, Improved effectiveness of a cryogenic counter-current parallel flow-Three fluid heat exchanger with three thermal communication due to Joule Thomson pressure drop, *Cryogenics* 106 (2020), 103046
- [27] Vishnudas Alias Vipul L., Chodankar, Aswatha, K.N. Seetharamu, Improved effectiveness of a cryogenic counter-current parallel flow-Three fluid

- heat exchanger with three thermal communication due to Joule Thomson pressure drop, International Journal of Thermal Sciences 172 (2022) 107267
- [28] Roland W. Lewis, Perumal Nithiarasu, Kankanhalli N. Seetharamu, Fundamentals of Finite Element Method for Heat and Fluid Flow' John Wiley and Sons, 2004.
- [29] P. Nithiarasu, R. W. Lewis, and K. N. Seetharamu, Fundamentals of the Finite Element Method for Heat and Mass Transfer, John Wiley and Sons, 2016.
- [30] F.P. Incropera, D.P. Dewitt, Fundamentals of Heat and Mass Transfer, fifth ed., Wiley India, New Delhi, 2010.
- [31] Ramesh KS, Dusan PS. Fundamentals of heat exchanger design. Hoboken, New Jersey: John Wiley & Sons, Inc.; 2003.
- [32] K.N. Seetharamu, G.A. Quadir, Z.A. Zainal, G.M. Krishnan, FEM analysis of multifluid heat exchangers." International Journal of Numerical Methods for Heat and Fluid 14 (2) (2004) 242-255

APPENDIX

Case A:

$$[K] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ \left(-1 + \frac{NTU_e R_1}{2}\right) & \left(-\frac{NTU_e R_1}{2}\right) & \left(1 + \frac{NTU_e R_1}{2}\right) & \left(-\frac{NTU_e R_2}{2}\right) & 0 \\ \left(-\frac{NTU_e}{2}\right) & \left(-1 + \frac{NTU_e}{2}\right) & \left(-\frac{NTU_e}{2}\right) & \left(1 + \frac{NTU_e}{2} + \frac{NTU_e H}{2}\right) & \left(-\frac{NTU_e H}{2}\right) \\ 0 & -1 & 0 & \left(\frac{NTU_e R_2 H}{2}\right) & \left(1 - \frac{NTU_e R_2 H}{2}\right) \end{bmatrix}$$

Where	$BC1 = 1$
	$BC2 = 0$
	$\theta_c = \theta_{c2o} = \theta_{c1i}$

$$\{\theta\} = \begin{Bmatrix} \theta_{hi} \\ \theta_c \\ \theta_{h0} \\ \theta_{c10} \\ \theta_{c2i} \end{Bmatrix} \quad \{f\} = \begin{Bmatrix} BC1 \\ 0 \\ 0 \\ 0 \\ BC2 \end{Bmatrix}$$

Case B:

$$[K] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \left(-1 + \frac{NTU_e R_1}{2}\right) & \left(-\frac{NTU_e R_1}{2}\right) & \left(1 + \frac{NTU_e R_1}{2}\right) & \left(-\frac{NTU_e R_2}{2}\right) & 0 \\ \left(\frac{NTU_e}{2}\right) & \left(-1 - \frac{NTU_e}{2}\right) & \left(\frac{NTU_e}{2}\right) & \left(1 - \frac{NTU_e}{2} - \frac{NTU_e H}{2}\right) & \left(\frac{NTU_e H}{2}\right) \\ 0 & -1 & 0 & \left(-\frac{NTU_e H R_2}{2}\right) & \left(1 + \frac{NTU_e H R_2}{2}\right) \end{bmatrix}$$

Where	$BC1 = 1$
	$BC2 = 0$
	$\theta_c = \theta_{c2o} = \theta_{c1i}$

$$\{\theta\} = \begin{Bmatrix} \theta_{hi} \\ \theta_c \\ \theta_{h0} \\ \theta_{c2i} \\ \theta_{c1o} \end{Bmatrix} \quad \{f\} = \begin{Bmatrix} BC1 \\ 0 \\ 0 \\ BC2 \\ 0 \end{Bmatrix}$$