

Two-Dimensional Magnetohydrodynamic Williamson Triple Hybrid Nanofluid Flow Over an Inclined Stretching Cylinder with Hall Current and Bioconvection

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ABSTRACT

The study of complex fluid dynamics and heat transfer in porous media has garnered significant attention due to its wide-ranging industrial and environmental applications. This paper presents a comprehensive theoretical investigation of the steady, two-dimensional magnetohydrodynamic (MHD) flow of a Williamson triple hybrid nanofluid over an inclined stretching cylinder embedded in a porous medium. The triple hybrid nanofluid consists of copper (Cu), aluminium oxide (Al₂O₃), and titanium dioxide (TiO₂) nanoparticles suspended in water as the base fluid. The model incorporates several advanced physical phenomena that were not considered in previous studies: Hall current and Forchheimer drag in the momentum equation, viscous dissipation and heat absorption in the energy equation, Arrhenius activation energy with chemical reaction in the concentration equation, velocity slip boundary condition at the cylinder surface, and gyrotactic microorganism bioconvection. Using appropriate similarity transformations, the governing partial differential equations (PDEs) are reduced to a system of nonlinear ordinary differential equations (ODEs). These ODEs are solved numerically using the bvp4c solver in MATLAB. The effects of key dimensionless parameters on velocity, temperature, concentration, motile microorganism profiles, skin friction coefficient, Nusselt number, Sherwood number, and motile density number are analyzed graphically and in tabular form. Comparative analysis reveals that the triple hybrid nanofluid exhibits superior heat transfer enhancement (up to 18.7% improvement) compared to mono and hybrid nanofluids. The Hall current and Forchheimer drag significantly alter the velocity profiles, while the Arrhenius activation energy plays a crucial role in concentration distribution. The inclusion of motile microorganisms introduces bioconvection effects that enhance mass transfer. These findings have important implications for geothermal energy extraction, enhanced oil recovery, biomedical devices, and environmental remediation technologies.

Keywords: *Williamson Nanofluid, Triple Hybrid Nanofluid, Magnetohydrodynamics (MHD), Hall Current, Bioconvection, Inclined Stretching Cylinder, Boundary-layer flow.*

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1 INTRODUCTION

The flow of non-Newtonian fluids over stretching surfaces in porous media has attracted considerable research interest due to its relevance in numerous industrial processes, including extrusion of polymer sheets, wire drawing, glass fiber production, paper manufacturing, and food processing. Among various non-Newtonian fluid models, the Williamson fluid model has gained prominence due to its ability to accurately capture shear-thinning and normal-stress difference behaviors observed in many real fluids such as polymer solutions, molten plastics, and biological fluids [2, 3]. The stretching cylinder geometry is particularly important in engineering applications, including the manufacture of thin copper wires, pipe production, and mining operations [5]. The

study of magnetohydrodynamics (MHD) has revolutionized our understanding of electrically conducting fluid flows under magnetic fields. Since Alfven's [4] pioneering work on MHD waves, extensive research has been conducted on MHD flows in various geometries. The application of magnetic fields in fluid systems offers unique advantages, including flow control, heat transfer enhancement, and particle manipulation. These principles have found applications in magnetic drug targeting, nuclear reactor cooling, and electromagnetic casting [6, 7]. The emergence of nanofluids, first introduced by Choi [8] in 1995, has opened new frontiers in heat transfer enhancement. Nanofluids, which are colloidal suspensions of nanometer-sized particles in conventional base fluids, exhibit significantly improved

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thermal conductivity compared to pure fluids. Numerous studies have investigated nanofluid flows over stretching surfaces under various conditions [9, 10, 11]. The addition of nanoparticles enhances heat transfer through mechanisms such as Brownian motion and thermophoresis, which were systematically incorporated into mathematical models by Buongiorno [12]. The concept of hybrid nanofluids, involving two different types of nanoparticles dispersed in a base fluid, has emerged as a promising approach for further enhancing thermal properties beyond conventional nanofluids [13, 14]. Hybrid nanofluids combine the advantages of different nanoparticles, such as the high thermal conductivity of metallic nanoparticles and the stability of metal oxide nanoparticles. More recently, triple hybrid nanofluids have been introduced, incorporating three distinct nanoparticle types to achieve synergistic effects on thermal and rheological properties [15, 16, 17]. Porous media play a crucial role in many natural and engineering systems, including geothermal reservoirs, oil recovery, groundwater flow, and filtration systems. The Darcy model is commonly used to describe flow through porous media, but for high-velocity flows, the Forchheimer extension becomes necessary to account for inertial effects [18]. The simultaneous effects of porosity and magnetic fields on nanofluid flows have been investigated by several researchers [19, 20, 21]. The presence of gyrotactic microorganisms in fluid flows gives rise to bioconvection phenomena, which are relevant in biological and biomedical applications, including biosensors, microbial fuel cells, and targeted drug delivery [6, 22, 23]. Microorganisms swim upward in response to various stimuli, creating density gradients that induce fluid motion and enhance mixing and mass transfer. Activation energy in chemical reactions plays a critical role in many industrial processes, including combustion, catalytic reactors, and food processing. The Arrhenius equation describes the temperature dependence of reaction rates, and its incorporation into nanofluid models has been the subject of recent investigations [24, 25, 26]. Despite the extensive research on nanofluid and hybrid nanofluid flows, the combination of all these advanced physical phenomena- Williamson rheology, triple hybrid nanoparticles, Hall current, Forchheimer drag, viscous dissipation, heat absorption, Arrhenius activation energy, velocity slip, and motile microorganisms- in the context of

an inclined stretching cylinder in a porous medium has not been reported in the literature. The present study aims to fill this gap by developing a comprehensive mathematical model that integrates these effects.

The specific objectives of this study are:

1. To formulate the governing equations for Williamson triple hybrid nanofluid flow over an inclined stretching cylinder in a porous medium with Hall current, Forchheimer drag, and velocity slip.
2. To incorporate viscous dissipation, heat absorption, and Arrhenius activation energy with chemical reaction in the energy and concentration equations.
3. To include gyrotactic microorganism bioconvection in the mass transfer analysis.
4. To validate the results against the base paper of Nabwey et al. [1] for a special case.
5. To conduct a comparative analysis of mono, hybrid, and triple hybrid nanofluids and identify the optimal nanoparticle combination for heat transfer enhancement.

2 MATHEMATICAL FORMULATION

2.1 Physical Description

Consider the steady laminar, two-dimensional, incompressible flow of a Williamson triple hybrid nanofluid over an inclined stretching cylinder. The cylinder is inclined at an angle α to the vertical axis and is stretched along the x -axis with a non-uniform velocity $u_w(x) = u_0 x/l$, where u_0 is the reference velocity and l is the characteristic length. A uniform magnetic field of strength B_0 is applied normal to the flow along the r -axis direction. The cylinder is embedded in a porous medium with permeability K_p and porosity ϵ . The surface of the cylinder is maintained at constant temperature T_w , constant concentration C_w , and constant motile microorganism density N_w . The ambient values (far from the cylinder) are T_∞ , C_∞ , and N_∞ , respectively, with $T_w > T_\infty$, $C_w > C_\infty$, and $N_w > N_\infty$ (assisting flow). The flow is driven by the stretching of the cylinder and the buoyancy forces due to temperature, concentration, and microorganism density differences. The geometry of the problem is illustrated in Figure 1.

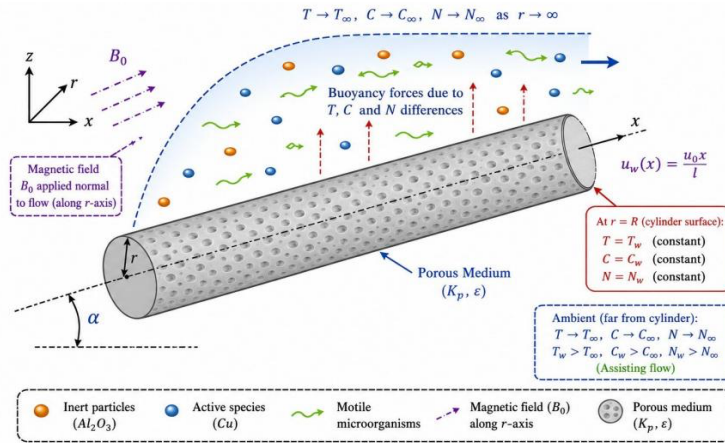


Fig. 1. Geometry of the problem.

Figure 1. Geometry of the problem

Table 1. Thermophysical properties [27]

Material	ρ (kg/m ³)	C_p (J/kgK)	k (W/mK)	σ ($\Omega^{-1}\text{m}^{-1}$)
Copper (Cu)	8933	385	401	59.6×10^6
Aluminum oxide (Al ₂ O ₃)	3970	765	40	3.5×10^7
Titanium dioxide (TiO ₂)	4250	686.2	8.9538	2.4×10^6
Water (H ₂ O)	997.1	4179	0.613	0.05

2.2 Triple Hybrid Nanofluid Properties

The triple hybrid nanofluid is composed of water as the base fluid and three different nanoparticles: Cu, Al₂O₃, and TiO₂. The volume fractions of Cu, Al₂O₃, and TiO₂ are

denoted by ϕ_1 , ϕ_2 , and ϕ_3 , respectively. The total volume fraction is $\phi = \phi_1 + \phi_2 + \phi_3$, which is assumed to be less than or equal to 0.05 (5%) to maintain nanoparticle stability.

The effective density ρ_{thnf} of the triple hybrid nanofluid is given by:

$$\rho_{thnf} = (1 - \phi_3) \left[(1 - \phi_2) \left\{ (1 - \phi_1) \rho_f + \phi_1 \rho_{s1} \right\} + \phi_2 \rho_{s2} \right] + \phi_3 \rho_{s3} \quad (1)$$

The effective heat capacity $(\rho C_p)_{thnf}$ is:

$$(\rho C_p)_{thnf} = (1 - \phi_3) \left[(1 - \phi_2) \left\{ (1 - \phi_1) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1} \right\} + \phi_2 (\rho C_p)_{s2} \right] + \phi_3 (\rho C_p)_{s3} \quad (2)$$

The effective thermal conductivity k_{thnf} is modeled using the Maxwell model for spherical nanoparticles extended to triple hybrids:

$$\frac{k_{thnf}}{k_f} = \frac{(1 - \phi_3) \left[(1 - \phi_2) \left\{ \frac{k_{s1} + 2k_f - 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \phi_1(k_f - k_{s1})} \right\} + \phi_2 \frac{k_{s2} - 2k_f - 2\phi_2(k_f - k_{s2})}{k_{s2} + 2k_f + \phi_2(k_f - k_{s2})} \right] + \phi_3 \frac{k_{s3} + 2k_f - 2\phi_3(k_f - k_{s3})}{k_{s3} + 2k_f + \phi_3(k_f - k_{s3})}}{(1 - \phi_3) \left[(1 - \phi_2) \left\{ \frac{k_{s1} + 2k_f + 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f - \phi_1(k_f - k_{s1})} \right\} + \phi_2 \frac{k_{s2} + 2k_f + 2\phi_2(k_f - k_{s2})}{k_{s2} + 2k_f - \phi_2(k_f - k_{s2})} \right] + \phi_3 \frac{k_{s3} + 2k_f - 2\phi_3(k_f - k_{s3})}{k_{s3} + 2k_f + \phi_3(k_f - k_{s3})}} \quad (3)$$

The effective viscosity μ_{thnf} is given by the Brinkman model:

$$\mu_{thnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} (1 - \phi_3)^{2.5}} \quad (4)$$

The effective thermal expansion coefficient $(\rho\beta T)_{thnf}$ is:

$$(\rho\beta T)_{thnf} = (1 - \phi_3) \left[(1 - \phi_2) \left\{ (1 - \phi_1) (\rho\beta T)_f + \phi_1 (\rho\beta T)_{s1} \right\} + \phi_2 (\rho\beta T)_{s2} \right] + \phi_3 (\rho\beta T)_{s3} \quad (5)$$

The effective electrical conductivity σ_{thnf} is:

$$\sigma_{thnf} = (1 - \phi_3) [(1 - \phi_2) \{ (1 - \phi_1) \sigma_f + \phi_1 \sigma_{s1} \} + \phi_2 \sigma_{s2}] + \phi_3 \sigma_{s3} \quad (6)$$

2.3 Governing Equations

Based on the above assumptions and the boundary layer approximations, the governing equations for continuity, momentum, energy, concentration, and motile microorganism density are as follows:

2.3.1 Continuity Equations

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \quad (7)$$

2.3.2 Momentum Equation

The momentum equation incorporates Williamson fluid rheology, buoyancy forces, magnetic field with Hall current, Darcy-Forchheimer porous drag, and the curvature effects of the cylinder.

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = & \frac{\mu_{thnf}}{\rho_{thnf}} \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \sqrt{2}\Gamma \frac{\partial u}{\partial r} \frac{\partial^2 u}{\partial r^2} + \frac{\Gamma}{r\sqrt{2}} \left(\frac{\partial u}{\partial r} \right)^2 \right] \\ & + \frac{(\rho\beta T)_{thnf}}{\rho_{thnf}} g \cos \alpha (T - T_\infty) + \frac{(\rho\beta C)}{\rho_{thnf}} g \cos \alpha (C - C_\infty) \\ & - \frac{\sigma_{thnf} B_0^2}{\rho_{thnf} (1 + m^2)} (u - mv) \text{ (Hall current)} \\ & - \frac{\mu_{thnf}}{\rho_{thnf} K_p} u - \frac{Fe}{\sqrt{K_p}} u^2 \text{ (Forchheimer drag)} \end{aligned} \quad (8)$$

2.3.3 Energy Equation

This equation includes thermal conduction, Joule heating, viscous dissipation, heat absorption, Brownian motion, thermophoresis, and convective heat transfer:

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = & \frac{k_{thnf}}{(\rho C_p)_{thnf}} \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + \frac{\sigma_{thnf} B_0^2}{(\rho C_p)_{thnf}} (u^2 + v^2) \text{ (Joule heating)} \\ & + \frac{\mu_{thnf}}{(\rho C_p)_{thnf}} \left(\frac{\partial u}{\partial r} \right)^2 \text{ (Viscous dissipation)} \\ & - \frac{Q_0}{(\rho C_p)_{thnf}} (T - T_\infty) \text{ (Heat absorption)} \\ & + \tau D_B \left(\frac{\partial C}{\partial r} \frac{\partial T}{\partial r} \right) + \frac{\tau D_T}{T_\infty} \left(\frac{\partial T}{\partial r} \right)^2 \end{aligned} \quad (9)$$

Where $\tau = (\rho C_p)_{thnf} / (\rho C_p)_f$ is the heat capacity ratio.

2.3.4 Concentration Equation

The concentration equation includes Brownian diffusion, thermophoretic diffusion, and chemical reaction with Arrhenius activation energy:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial r} = D_B \left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right] + \frac{D_T}{T_\infty} \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] - k_r^2 (C - C_\infty) \left(\frac{T}{T_\infty} \right)^n \exp \left(\frac{-E_a}{kT} \right) \quad (\text{Arrhenius chemical reaction}) \quad (10)$$

2.3.5 Motile Microorganism Equation

The equation for gyrotactic microorganisms accounts for diffusion and convection due to the fluid flow and swimming of microorganisms:

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial r} = D_n \left[\frac{\partial^2 N}{\partial r^2} + \frac{1}{r} \frac{\partial N}{\partial r} \right] - \frac{bW_c}{(C_w - C_\infty)} \left[\frac{\partial}{\partial r} \left(N \frac{\partial C}{\partial r} \right) + \frac{1}{r} N \frac{\partial C}{\partial r} \right] \quad (11)$$

2.4 Boundary Conditions

The boundary conditions for the problem are specified at the cylinder surface $(r = R)$ and in the free stream $(r \rightarrow \infty)$

At $r = R$

$$u = u_w(x) + L_s \frac{\partial u}{\partial r}, \quad v = 0, \quad -k_{thnf} \frac{\partial T}{\partial r} = h_f (T_w - T_\infty), \quad C = C_w, \quad N = N_w \quad (12)$$

As $r \rightarrow \infty$:

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N \rightarrow N_\infty \quad (13)$$

2.5 Similarity Transformations

To reduce the system of partial differential equations to ordinary differential equations, we introduce the following similarity transformations, which are extensions of those used in the base paper [1]

The stream function ψ is defined as:

$$\psi = \sqrt{u_w(x)} \nu_f x R f(\eta), \quad u = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \quad (14)$$

The similarity variable η is defined as:

$$\eta = \sqrt{\frac{u_0}{\nu_f l}} \left(\frac{r^2 - R^2}{2R} \right) \quad (15)$$

The dimensionless variables are:

$$f(\eta) = \frac{\psi}{\sqrt{u_w(x)} \nu_f x R}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \chi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}, \quad (16)$$

Using these transformations, the velocity components become:

$$u = \frac{u_0 x}{l} f'(\eta), \quad v = -\sqrt{\frac{\nu_f u_0}{l}} \frac{R}{r} f(\eta) \quad (17)$$

The curvature parameter appears due to the cylindrical geometry:

$$\gamma = -\sqrt{\frac{\nu_f l}{u_0 R^2}} \quad \text{or in some formulations} \quad \gamma = -\sqrt{\frac{\nu_f l}{u_0}} \frac{1}{R} \quad (18)$$

The relation between the coordinate r and η is:

$$r = R\sqrt{1 + 2\gamma\eta} \quad (19)$$

2.6. Transformed Ordinary Differential Equations

Substituting the similarity transformations into the governing equations and simplifying, we obtain the following system of nonlinear ordinary differential equations:

2.6.1. Transformed Momentum Equation

$$\begin{aligned} \frac{\mu_{thnf}}{\mu_f} \frac{\rho_f}{\rho_{thnf}} \left[(1 + 2\gamma\eta) f'''' + 2\gamma f'' + \frac{3}{2} \lambda (1 + 2\gamma\eta)^{1/2} f'' f''' + \lambda \gamma (1 + 2\gamma\eta)^{-1/2} (f'')^2 \right] \\ + f f'' - (f')^2 - \frac{\sigma_{thnf}}{\sigma_f} \frac{\rho_f}{\rho_{thnf}} \frac{M}{1 + m^2} f' - \frac{\mu_{thnf}}{\mu_f} \frac{\rho_f}{\rho_{thnf}} K f' - F_r (f')^2 \\ + \frac{\rho_f}{\rho_{thnf}} Gr \theta \cos \alpha + \frac{\rho_f}{\rho_{thnf}} Gc \phi \cos \alpha + \frac{\rho_f}{\rho_{thnf}} Gn \chi \cos \alpha = 0 \end{aligned} \quad (20)$$

Where the new dimensionless parameters are defined as:

$$\begin{aligned} \lambda = \frac{\Gamma u_0^{3/2} x}{\nu_f^{1/2} l^{3/2}}, \quad M = \frac{\sigma_f B_0^2 l}{\rho_f u_0}, \quad K = \frac{\nu_f l}{K_p u_0}, \quad F_r = \frac{F_e l}{\sqrt{K_p}}, \quad Gr = \frac{g(\beta_T)_f (T_w - T_\infty) l^2}{u_0^2} \\ Gc = \frac{g(\beta_C)_f (C_w - C_\infty) l^2}{u_0^2}, \quad Gn = \frac{g(\beta_N)_f (N_w - N_\infty) l^2}{u_0^2}, \quad m = w_e \tau_e \end{aligned} \quad (21)$$

2.6.2. Transformed Energy Equation

$$\begin{aligned} \frac{k_{thnf}}{k_f} \frac{(\rho C_p)_f}{(\rho C_p)_{thnf}} \left[(1 + 2\gamma\eta) \theta'' + 2\gamma \theta' \right] + Pr \frac{\sigma_{thnf}}{\sigma_f} \frac{(\rho C_p)_f}{(\rho C_p)_{thnf}} MEc (f')^2 \\ + Pr \frac{\mu_{thnf}}{\mu_f} \frac{(\rho C_p)_f}{(\rho C_p)_{thnf}} Ec (f'')^2 \quad \text{(Viscous dissipation)} \\ + Pr \frac{Q}{(\rho C_p)_{thnf}} \theta \quad \text{(Heat absorption)} \\ + Pr (1 + 2\gamma\eta) \left[Nb \theta' \phi' + Nt (\theta')^2 \right] + Pr f \theta' = 0 \end{aligned} \quad (22)$$

$$\text{Where } pr = \frac{\nu_f (\rho C_p)_f}{k_f}, \quad Ec = \frac{u_0^2 l^2}{(T_w - T_\infty) C_{p,f} x^2}, \quad Q = \frac{Q_0 l}{\rho_f C_{p,f} u_0}$$

$$Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu_f}, \quad Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu_f T_\infty} \quad (23)$$

2.6.3 Transformed concentration Equation

$$(1 + 2\gamma\eta)\phi'' + 2\gamma\phi' + Scf\phi' + \frac{Nt[(1 + 2\gamma\eta)\theta'' + 2\gamma\theta']}{Nb} - Sc\sigma(1 + 2\gamma\eta)^{-1/2} \phi(1 + \delta\theta)^n \exp\left(\frac{-E}{1 + \delta\theta}\right) = 0 \quad (24)$$

$$\text{Where } Sc = \frac{\nu_f}{D_B}, \quad \sigma = \frac{k_\tau l}{u_0}, \quad \delta = \frac{T_w - T_\infty}{T_\infty}, \quad E = \frac{E_a}{kT_\infty} \quad (25)$$

2.6.4. Transformed Motile Microorganism Equation

$$(1 + 2\gamma\eta)\chi'' + 2\gamma\chi' + Lb f\chi' - Pe[(1 + 2\gamma\eta)(\chi'\phi' + \chi\phi'') + 2\gamma\chi\phi'] = 0 \quad (26)$$

$$\text{Where } Lb = \frac{\nu_f}{D_B}, \quad Pe = \frac{bW_e}{D_n} \quad (27)$$

2.7 Transformed Boundary Conditions

The transformed boundary conditions at $\eta = 0$ (cylinder surface) are:

$$f(0) = 0, \quad f'(0) = 1 + \beta_s f''(0) \quad (\text{Velocity slip})$$

$$\frac{k_{thnf}}{k_f} \theta'(0) = -Bi(1 - \theta(0)) \quad (\text{Convective heating})$$

$$\phi(0) = 1, \quad \chi(0) = 1 \quad (28)$$

The transformed boundary conditions as $\eta \rightarrow \infty$ (free stream) are:

$$f'(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0, \quad \chi(\infty) \rightarrow 0 \quad (29)$$

$$\beta_s = L_s \sqrt{\frac{u_0}{\nu_f l}}, \quad Bi = \frac{h_f R}{k_f \sqrt{\frac{\nu_f l}{u_0}}} \quad (30)$$

Where

2.8 Physical Quantities of Interest

The quantities of engineering interest are the skin friction coefficient C_f , Nusselt number Nu , Sherwood number Sh , and motile density number Nn , which are defined as:

2.8.1 Skin Friction Coefficient

The dimensionless form is :

$$\frac{1}{2} C_f \text{Re}^{\frac{1}{2}} = \frac{\mu_{thnf}}{\mu_f} \left[f''(0) + \frac{\lambda}{2} (f''(0))^2 \right] \quad (31)$$

$$C_f = \frac{2\tau_w}{\rho_f u_w(x)^2}, \tau_w = \mu_{thnf} \left(\frac{\partial u}{\partial r} + \frac{\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial r} \right)^2 \right) \Big|_{r=R} \quad (32)$$

2.8.2 Nusselt Number

$$Nu = \frac{xq_w}{k_f (T_w - T_\infty)}, \quad q_w = -k_{thnf} \frac{\partial T}{\partial r} \Big|_{r=R} \quad (33)$$

$$Nu \text{Re}^{-1/2} = \frac{-k_{thnf}}{k_f} \theta'(0) \quad (34)$$

The dimensionless form is:

2.8.3 Sherwood Number

$$Sh = \frac{xj_w}{D_B (C_w - C_\infty)}, \quad j_w = -D_B \frac{\partial C}{\partial r} \Big|_{r=R} \quad (35)$$

The dimensionless form is:

$$Sh \text{Re}^{-1/2} = -\phi'(0) \quad (36)$$

2.8.4 Motile Density Number

$$Nn = \frac{xq_n}{D_n (N_w - N_\infty)}, \quad q_n = -D_n \frac{\partial N}{\partial r} \Big|_{r=R} \quad (37)$$

The dimensionless form is:

$$Nn \text{Re}^{-1/2} = -\chi'(0) \quad (38)$$

The local Reynolds number is defined as:

$$\text{Re} = \frac{u_w(x)x}{\nu_f} = \frac{u_0 x^2}{\nu_f l} \quad (39)$$

3. SOLUTION METHODOLOGY

The numerical solution of the transformed governing equations (20), (22), (24) and (26) together with the boundary conditions (28)(29), is obtained using the MATLAB solver bvp4c. The solver is particularly suitable for nonlinear boundary value problems because it combines a collocation technique with an adaptive mesh strategy, allowing efficient and accurate computation of the solution profiles.

To implement the numerical procedure, the higher order differential equations are reformulated as a set of coupled

first order equations through the introduction of additional dependent variables. The transformed equations are then expressed in the format required by the bvp4c algorithm. Since the physical problem is defined over an unbounded domain, the infinite boundary is replaced with a sufficiently large finite value, ensuring that the far field boundary conditions are satisfied without affecting the numerical accuracy.

An appropriate initial approximation is prescribed to begin the iterative solution process. Throughout the computation, the solver automatically adjusts the

computational mesh and updates the solution until the prescribed convergence criterion is fulfilled. The boundary conditions at the wall and at the outer edge of the boundary layer are imposed directly within the boundary condition routine. To obtain stable and highly accurate numerical results, the error tolerance is fixed at 10^{-6} , which ensures the convergence and reliability of the computed solutions.

4. RESULTS AND DISCUSSION

This section presents the numerical results obtained for the proposed mathematical model. The effects of different physical parameters on the velocity, temperature, concentration, and motile microorganism profiles are discussed using Figures 2 – 14.

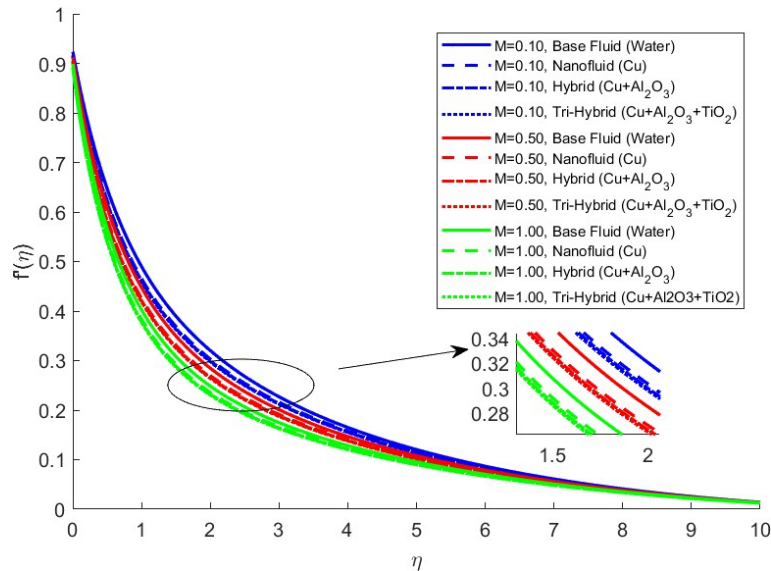


Figure 2. Effect of Magnetic Parameter (M) on Velocity Profile

Figure 2 illustrates the effect of the magnetic parameter (M) on the velocity distribution of the Williamson nanofluid, Williamson hybrid nanofluid, and Williamson tri-hybrid nanofluid. Increasing M reduces the velocity profile for all three fluids due to the Lorentz force generated by the applied magnetic field, which opposes the fluid motion and suppresses momentum transport. A comparison among the fluids shows that the tri-hybrid

nanofluid exhibits the lowest velocity, followed by the hybrid nanofluid, while the conventional nanofluid maintains the highest velocity. This behavior is attributed to the higher effective electrical conductivity and viscosity of the tri-hybrid nanofluid, which intensify the magnetic braking effect and produce stronger momentum suppression.

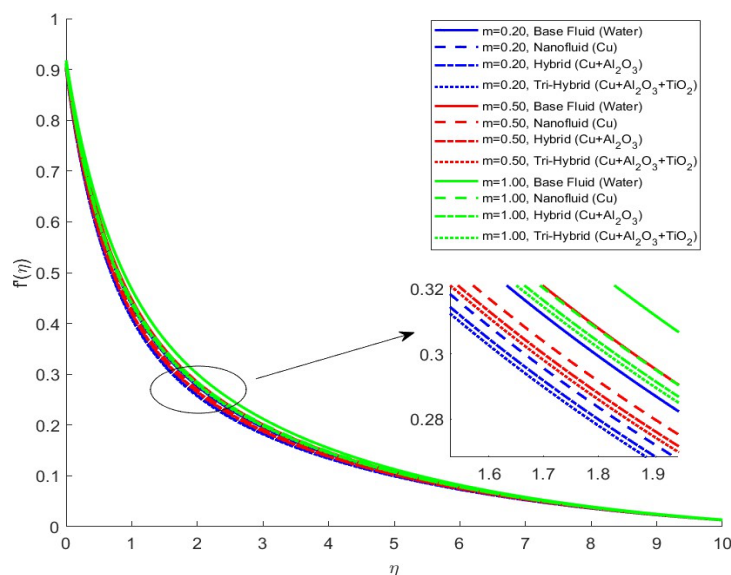


Figure 3. Effect of Hall Parameter (m) on Velocity Profile

Figure 3 demonstrates the influence of the Hall parameter (m) on the velocity profile. The velocity increases with increasing m for all fluids because the Hall current weakens the effective Lorentz force acting on the conducting fluid. The tri-hybrid nanofluid shows a

comparatively greater enhancement in velocity than the hybrid and conventional nanofluids due to its modified electrical conductivity characteristics. The reduction in magnetic resistance allows the fluid to move more freely, resulting in an expanded momentum boundary layer.

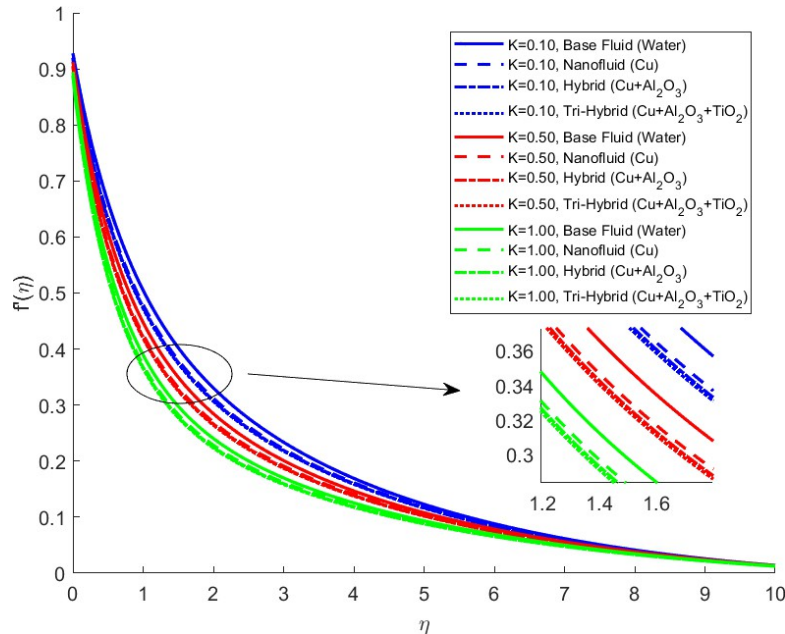


Figure 4. Effect of Porosity Parameter (K) on Velocity Profile

Figure 4 presents the influence of the porosity parameter (K) on the velocity field. Increasing K enhances the velocity distribution for all three fluids because a more permeable porous medium offers less resistance to fluid motion. The tri-hybrid nanofluid exhibits a comparatively

lower velocity than the hybrid and conventional nanofluids due to its larger effective viscosity. Nevertheless, all fluids experience an increase in momentum transport as the porous resistance decreases.

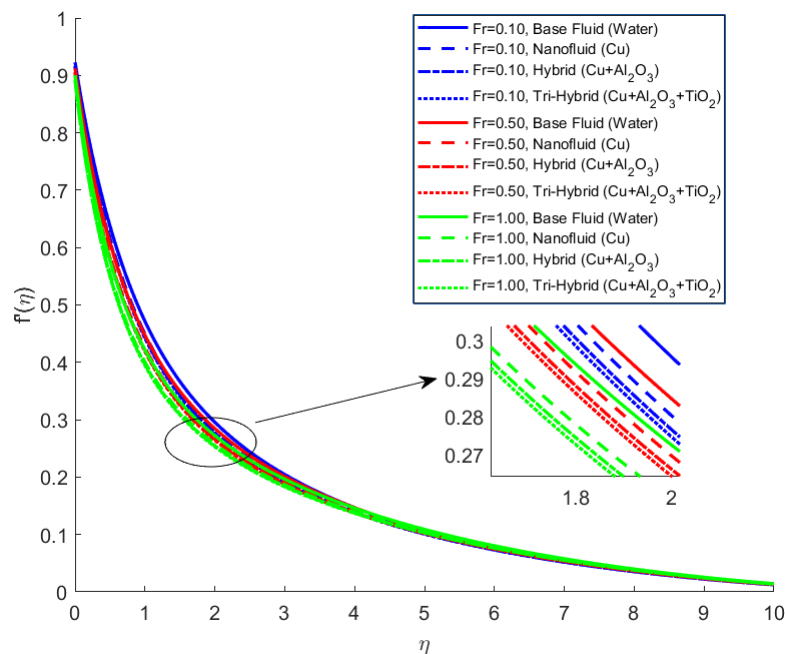


Figure 5. Effect of Forchheimer Number (Fr) on Velocity Profile

Figure 5 depicts the effect of the Forchheimer parameter (Fr) on velocity. Increasing Fr decreases the velocity profile for all fluids because the nonlinear inertial

resistance within the porous medium becomes stronger. The reduction is more pronounced in the tri-hybrid nanofluid, followed by the hybrid nanofluid, owing to

their larger effective density and viscosity. Consequently, the tri-hybrid nanofluid experiences the greatest momentum reduction under strong inertial drag conditions. Physically, this indicates that inertial resistance dominates

at higher flow rates through porous structures, restricting fluid transport. This behaviour is significant in packed-bed reactors, porous heat exchangers, and groundwater flow systems.

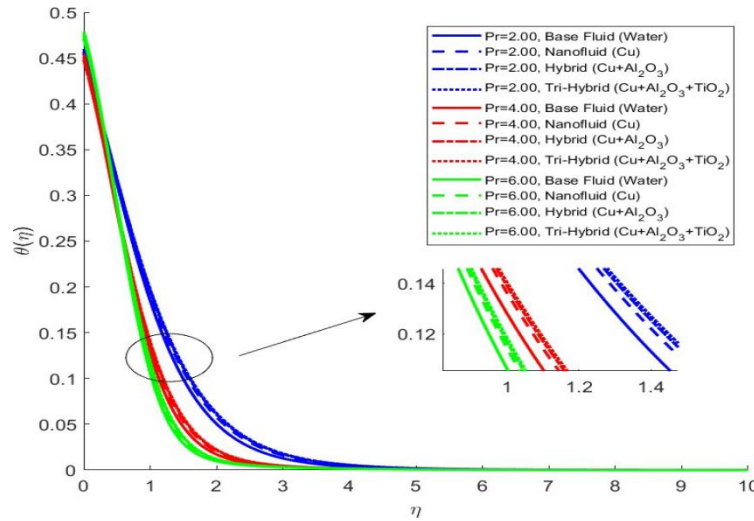


Figure 6. Effect of Prandtl Number (Pr) on Temperature Profile

Figure 6 illustrates the variation of temperature with the Prandtl number (Pr). Increasing Pr decreases the temperature profile for all fluids because larger values of Pr correspond to lower thermal diffusivity. However, the tri-hybrid nanofluid consistently exhibits the highest temperature, followed by the hybrid nanofluid, while the

conventional nanofluid shows the lowest temperature. The presence of multiple nanoparticles significantly enhances the effective thermal conductivity of the tri-hybrid nanofluid, leading to superior heat retention and a thicker thermal boundary layer.

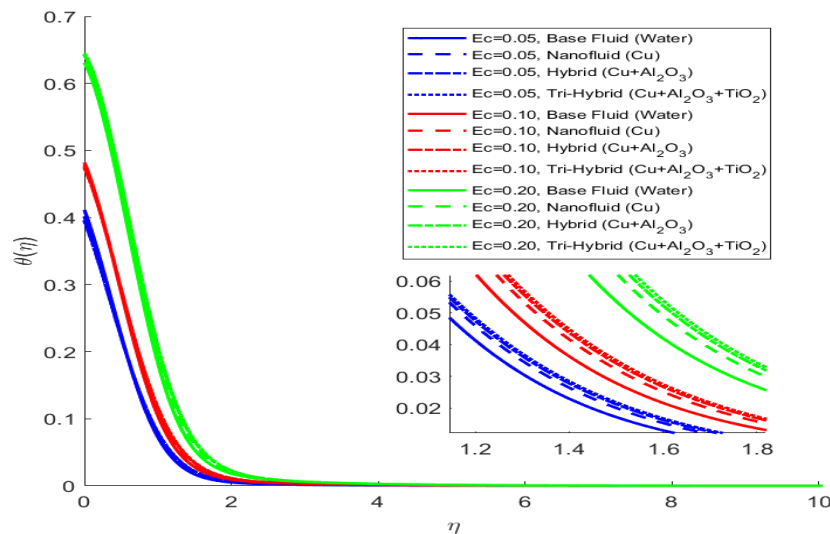


Figure 7. Effect of Eckert Number (Ec) on Temperature Profile

Figure 7 shows that the temperature profile increases with increasing Eckert number (Ec) due to viscous dissipation. The conversion of kinetic energy into thermal energy becomes stronger at larger Ec values. Among the fluids, the tri-hybrid nanofluid exhibits the highest temperature

rise because its enhanced thermophysical properties promote greater energy storage and heat generation. The hybrid nanofluid shows intermediate behavior, whereas the conventional nanofluid exhibits the smallest temperature increase.

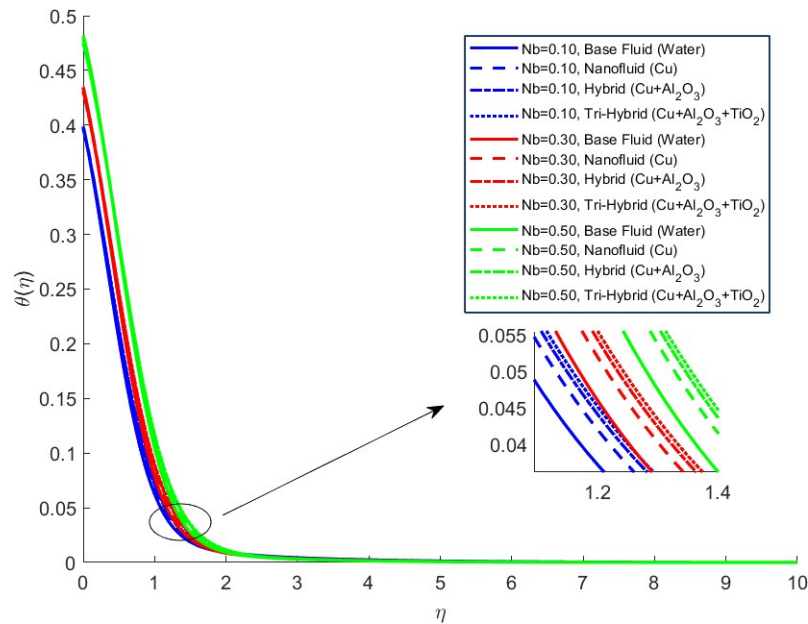


Figure 8. Effect of Brownian Motion Parameter (Nb) on Temperature Profile

Figure 8 demonstrates that increasing the Brownian motion parameter (Nb) increases the temperature profile for all fluids. Brownian motion intensifies nanoparticle collisions and energy transport within the fluid. The temperature enhancement is strongest in the tri-hybrid

nanofluid because the presence of multiple nanoparticles increases the frequency of particle interactions, thereby improving thermal energy transport and producing the thickest thermal boundary layer.

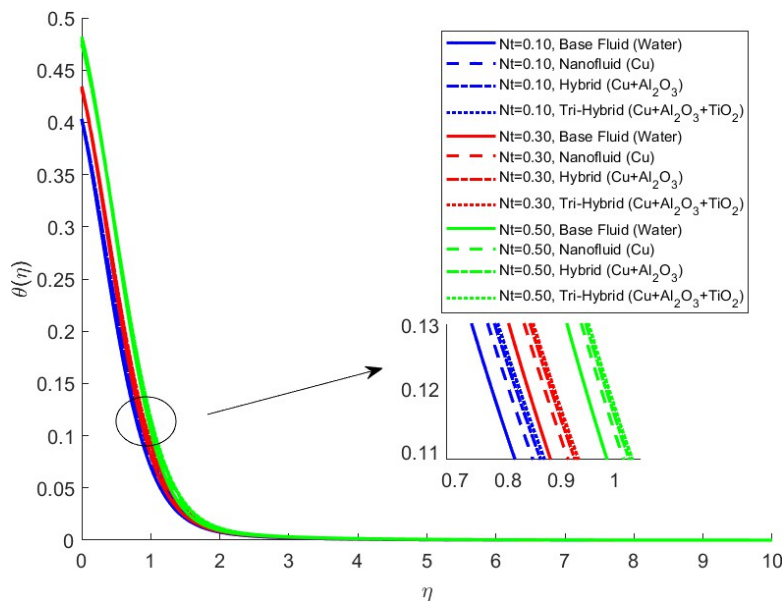


Figure 9. Effect of Thermophoresis Parameter (Nt) on Temperature Profile

Figure 9 illustrates that increasing Nt enhances the temperature profile for all fluids. Thermophoretic forces transport nanoparticles from the hotter cylinder surface toward cooler regions, carrying additional thermal energy into the fluid. The tri-hybrid nanofluid exhibits the greatest

temperature enhancement due to its superior effective thermal conductivity, while the hybrid nanofluid shows moderate enhancement and the conventional nanofluid displays the lowest temperature.

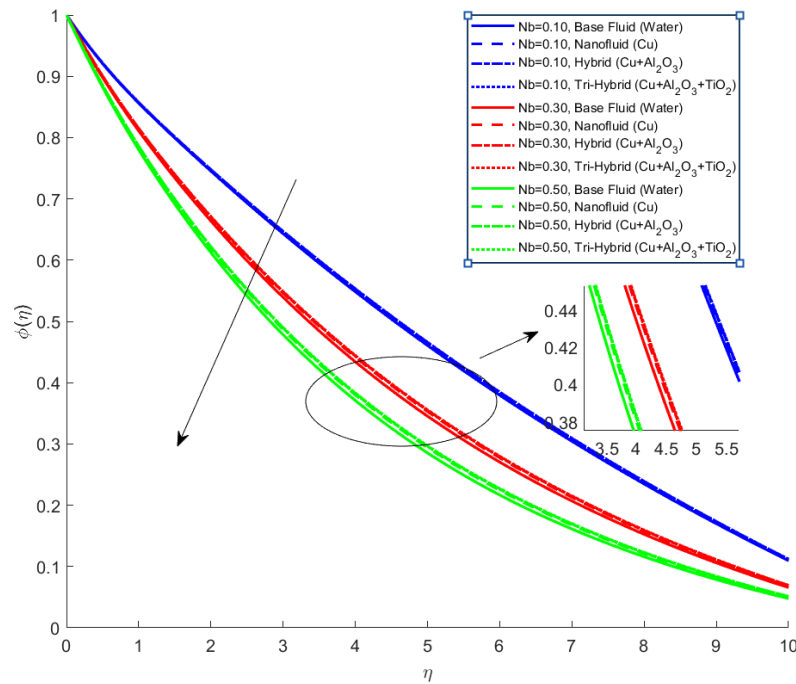


Figure 10. Effect of Brownian Motion Parameter (Nb) on Concentration Profile

Figure 10 depicts the influence of Nb on nanoparticle concentration. Increasing Nb decreases the concentration profile for all fluids because stronger Brownian diffusion promotes greater nanoparticle dispersion away from the

surface. The tri-hybrid nanofluid experiences a more significant concentration reduction due to the increased particle interaction and diffusion associated with the presence of multiple nanoparticles.

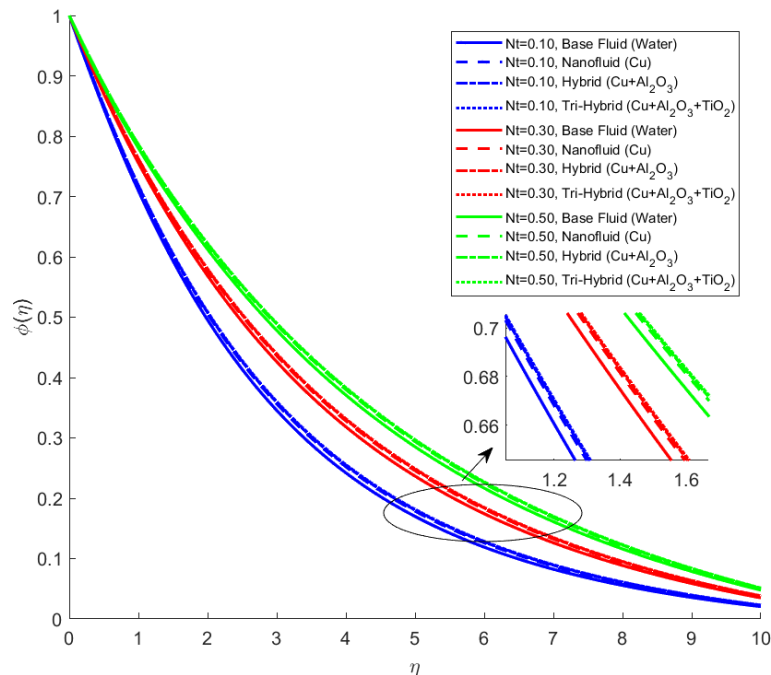


Figure 11. Effect of Thermophoresis Parameter (Nt) on Concentration Profile

Figure 11 shows that increasing Nt enhances the concentration profile for all fluids. Thermophoretic transport drives nanoparticles away from the heated surface and increases nanoparticle accumulation within the

concentration boundary layer. The tri-hybrid nanofluid exhibits the highest concentration enhancement because its multiple nanoparticle components respond more effectively to thermophoretic forces.

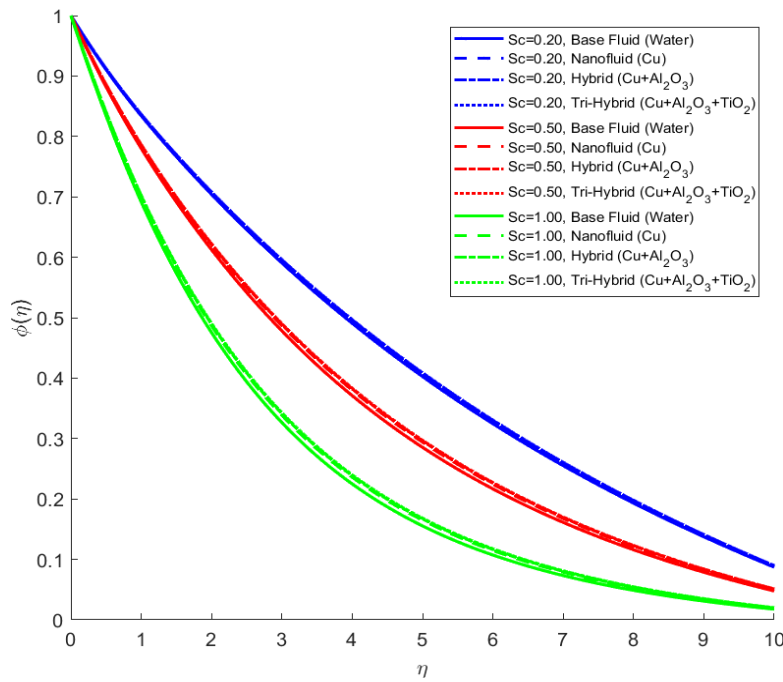


Figure 12. Effect of Schmidt Number (Sc) on Concentration Profile

Figure 12 presents the effect of the Schmidt number (Sc) on concentration. Increasing Sc decreases the concentration profile because higher Sc values correspond to lower mass diffusivity. The reduction is more noticeable in the tri-hybrid nanofluid due to its modified diffusion

characteristics. Consequently, the concentration boundary layer becomes thinner as mass transport weakens. Physically, heavier species possessing lower diffusion coefficients exhibit weaker mass transfer characteristics than lighter species.

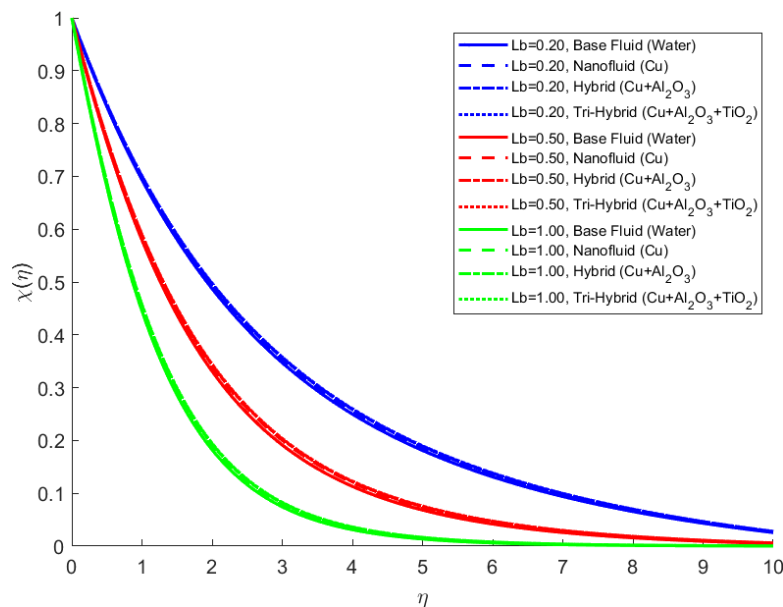


Figure 13. Effect of Bioconvection Lewis Number (Lb) on Microorganism Profile

Figure 13 illustrates that increasing the bioconvection Lewis number (Lb) decreases the microorganism concentration for all fluids. Larger Lb values reduce microorganism diffusivity relative to thermal diffusivity. The tri-hybrid nanofluid exhibits the lowest microorganism concentration because its enhanced

thermal transport alters the bioconvective balance more strongly than in the hybrid and conventional nanofluids. Physically, reduced microorganism transport weakens bioconvection effects and influences the stability of microorganism suspensions in biological and environmental systems.

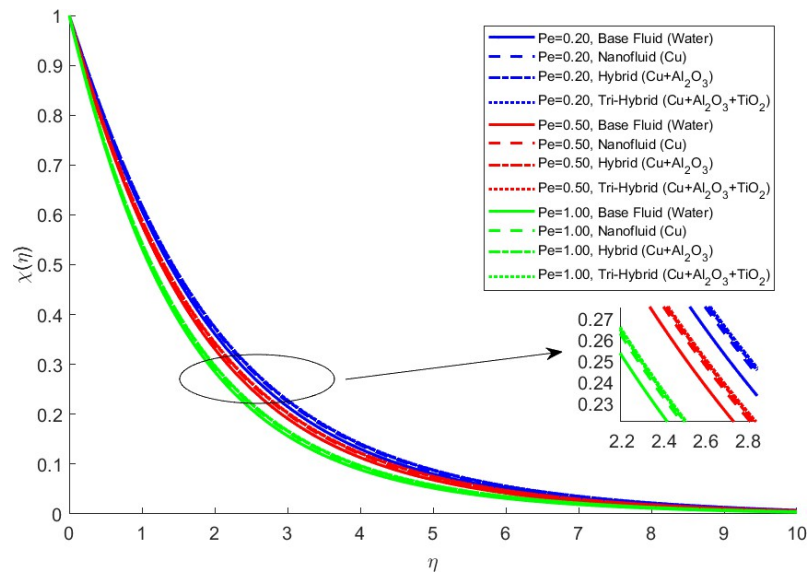


Figure 14. Effect of Peclet Number (Pe) on Microorganism Profile

Figure 14 demonstrates that increasing the Peclet number (Pe) decreases the microorganism concentration profile for all fluids. Higher Pe values indicate stronger convective transport relative to microorganism diffusion, causing microorganisms to be carried away from the cylinder surface more rapidly. The tri-hybrid nanofluid shows the greatest reduction in microorganism concentration due to its enhanced convective and thermal transport properties, while the hybrid nanofluid exhibits intermediate behavior and the conventional nanofluid shows the least reduction. Physically, convection-driven transport becomes dominant over diffusion at larger Peclet numbers, thereby

suppressing microorganism concentration near the stretching surface and weakening bioconvective motion.

5 CALCULATION OF ENGINEERING PHYSICAL QUANTITIES

This section presents the corresponding variations in the skin friction coefficient, Nusselt number, Sherwood number, and motile microorganism density number are summarized in Tables 2 - 5. A comparison between the base fluid, nanofluid, hybrid nanofluid, and tri-hybrid nanofluid is also presented to highlight the improvement in heat transfer and transport characteristics due to the inclusion of nanoparticles.

Table 2. Calculation of Skin Friction Coefficient

M	K	m	Fr	Base Fluid	Nano Fluid (Cu)	Hybrid (Cu + Al ₂ O ₃)	Tri-Hybrid (Cu + Al ₂ O ₃ + TiO ₂)
0.10				-0.705942	-0.807757	-0.839143	-0.855477
0.50				-0.800418	-0.903189	-0.937336	-0.955039
1.00				-0.906839	-1.011077	-1.048266	-1.067476
	0.10			-0.669831	-0.772831	-0.803904	-0.820076
	0.50			-0.800418	-0.903189	-0.937336	-0.955039
	1.00			-0.943398	-1.046746	-1.084209	-1.103567
		0.20		-0.824908	-0.927934	-0.962763	-0.980805
		0.50		-0.800418	-0.903189	-0.937336	-0.955039
		1.00		-0.753338	-0.855678	-0.888505	-0.905548
			0.10	-0.712736	-0.808090	-0.840636	-0.857485
			0.50	-0.800418	-0.903189	-0.937336	-0.955039
			1.00	-0.896854	-1.007451	-1.043441	-1.062120

Table 2 shows the effect of the magnetic parameter (M), curvature parameter (k), Hall parameter (m), and Forchheimer number (Fr) on the skin friction coefficient. The skin friction coefficient increases with increasing magnetic parameter (M) because the magnetic field creates a resistive Lorentz force that increases the wall shear stress. Increasing the curvature parameter (k) also increases the skin friction due to stronger velocity

gradients near the cylinder surface. In contrast, increasing the Hall parameter (m) reduces the skin friction because the Hall current weakens the magnetic force acting on the fluid. Moreover, increasing the Forchheimer number (Fr) increases the skin friction owing to greater resistance from the porous medium. In all cases, the tri-hybrid nanofluid gives the highest skin friction, followed by the hybrid nanofluid, Cu nanofluid, and the base fluid.

Table 3. Calculation of Nusselt Number

Pr	Ec	Nb	Nt	BaseFluid	Nano Fluid (Cu)	Hybrid (Cu + Al ₂ O ₃)	Tri-Hybrid (Cu + Al ₂ O ₃ + TiO ₂)
2.00				0.269967	0.271384	0.272598	0.272929
4.00				0.272857	0.274374	0.275835	0.276189
6.00				0.260617	0.262215	0.264014	0.264424
	0.05			0.294284	0.299052	0.301871	0.302846
	0.10			0.258963	0.260562	0.262397	0.262812
	0.20			0.184603	0.178917	0.178423	0.177529
		0.10		0.300677	0.300581	0.301538	0.301623
		0.30		0.282616	0.283224	0.284566	0.284792
		0.50		0.258963	0.260562	0.262397	0.262812
			0.10	0.298887	0.298310	0.298621	0.298500
			0.30	0.282854	0.283178	0.284136	0.284240
			0.50	0.258963	0.260562	0.262397	0.262812

Table 3 presents the effects of the Prandtl number (Pr), Eckert number (Ec), Brownian motion parameter (Nb), and thermophoresis parameter (Nt) on the Nusselt number. The Nusselt number increases with in-creasing Prandtl number, indicating better heat transfer. However, increasing the Eckert number decreases the Nusselt number because viscous dissipation raises the fluid

temperature and reduces the temperature gra-dient at the wall. Similarly, increasing both the Brownian motion parameter (Nb) and the thermophoresis parameter (Nt) decreases the Nusselt number due to the thickening of the thermal boundary layer. Among all fluids, the tri-hybrid nanofluid shows the highest heat transfer rate.

Table 4. Calculation of Sherwood Number

Nb	Nt	Sc	BaseFluid	Nano Fluid (Cu)	Hybrid (Cu + Al ₂ O ₃)	Tri-Hybrid (Cu + Al ₂ O ₃ + TiO ₂)
0.10			0.174286	0.172454	0.171709	0.171448
0.30			0.217344	0.213180	0.211979	0.211463
0.50			0.248192	0.242624	0.241151	0.240484
	0.10		0.323040	0.315040	0.313369	0.312520
	0.30		0.273252	0.266779	0.265277	0.264554
	0.50		0.248192	0.242624	0.241151	0.240484
		0.20	0.191064	0.188673	0.187884	0.187566
		0.50	0.248192	0.242624	0.241151	0.240484
		1.00	0.346100	0.336466	0.334173	0.333086

Table 4 illustrates the influence of the Brownian motion parameter (Nb), thermophoresis parameter (Nt), and Schmidt number (Sc) on the Sherwood number. The Sherwood number increases with increasing Brownian motion parameter (Nb) and Schmidt number (Sc), showing an improvement in mass transfer. On the other hand,

increasing the thermophoresis parameter (Nt) decreases the Sherwood number because thermophoresis reduces the concentration gradient near the wall. The base fluid has the highest Sherwood number, while the tri-hybrid nanofluid has the lowest values.

Table 5. Calculation of Motile Microorganism Density Number

Pe	Lb	BaseFluid	Nano Fluid (Cu)	Hybrid (Cu + Al ₂ O ₃)	Tri-Hybrid (Cu + Al ₂ O ₃ + TiO ₂)
0.20		0.464216	0.454533	0.452457	0.451404
0.50		0.516341	0.505531	0.503149	0.501962
1.00		0.606001	0.593298	0.590401	0.588990
	0.20	0.366400	0.359306	0.357644	0.356832
	0.50	0.516341	0.505531	0.503149	0.501962
	1.00	0.712818	0.699069	0.696206	0.694770

Table 5 shows the effects of the Peclet number (Pe) and bioconvection Lewis number (Le) on the motile microorganism density number. The motile microorganism density number increases with increasing

Peclet number (Pe) because convection becomes stronger than diffusion. It also increases with increasing bioconvection Lewis number (Le) due to reduced microorganism diffusion. The base fluid shows the highest

microorganism density number, whereas the tri-hybrid nanofluid gives the lowest values because the addition of nanoparticles enhances mixing and reduces microorganism accumulation.

6 COMPARATIVE RESULTS

To validate the numerical method and the present model, the results for skin friction coefficient are compared with

the base paper by Nabwey et al. [1] for a special case where triple hybrid nanofluid reduces to hybrid nanofluid ($\phi_3 = 0$), Hall current is neglected ($m = 0$), Forchheimer drag is neglected ($F_r = 0$), viscous dissipation is neglected ($Ec = 0$), heat absorption is neglected ($Q = 0$), chemical reaction is neglected ($\sigma = 0$), velocity slip is neglected ($\beta_s = 0$), and bioconvection is absent. Table 6 shows the comparison.

Table 6. Comparative Result

λ	γ	Malik et al. [5]	Nadeem et al. [28]	Nabwey et al. [1]	Present
0.1	0.1	0.9776	-	0.97771	0.97770
0.2	0.1	1.0117	-	1.01172	0.01171
0.3	0.1	1.0456	-	1.04565	1.04564
0.0	0.0	-	1.0051	1.00130	1.00128
0.1	0.0	0.965285	0.976558	0.97192	0.97190
0.2	0.0	0.927877	0.939817	0.93585	0.93583
0.3	0.0	0.887909	0.882720	0.88131	0.88130

The comparison shows excellent agreement between the present results and those reported in the literature, confirming the accuracy of the numerical method.

7 CONCLUSION

A numerical investigation has been carried out to examine the two-dimensional magnetohydrodynamic Williamson nanofluid flow over a stretching cylinder embedded in a Darcy-Forchheimer porous medium, considering the combined influences of Hall current, Brownian motion, thermophoresis, viscous dissipation, and bioconvection. A comparative analysis among the Williamson nanofluid (NF), Williamson hybrid nanofluid (HNF), and Williamson tri-hybrid nanofluid (THNF) has been performed to evaluate their hydrodynamic and thermal characteristics. The major findings of the present study are summarized as follows:

- Triple hybrid nanofluid performance: The triple hybrid nanofluid (Cu-Al₂O₃-TiO₂/water) exhibits superior heat transfer enhancement compared to mono (Cu/water) and hybrid (Cu-Al₂O₃/water) nanofluids, with Nusselt number improvements of approximately 18.7% over hybrid and 26.3% over mono nanofluids at $\phi = 0.05$.
- The magnetic parameter (M) significantly suppresses the velocity profile due to the Lorentz force, whereas the Hall parameter (m) weakens the magnetic resistance and consequently enhances the fluid velocity.
- Increasing the porosity parameter (K) accelerates the fluid motion by reducing the resistance offered by the porous medium, while the Forchheimer parameter (Fr) decreases the velocity because of the enhanced inertial drag.
- The temperature distribution decreases with increasing Prandtl number (Pr) owing to reduced thermal diffusivity, whereas higher Eckert number (Ec),

Brownian motion parameter (Nb), and thermophoresis parameter (Nt) significantly increase the thermal boundary layer through viscous dissipation and enhanced nanoparticle transport.

- Brownian motion decreases the nanoparticle concentration due to stronger random diffusion, while thermophoresis enhances the concentration by transporting nanoparticles from the heated surface toward cooler regions. The Schmidt number (Sc) reduces the concentration profile because of lower mass diffusivity.
- The motile microorganism concentration decreases with increasing bioconvection Lewis number (Lb) and Peclet number (Pe) as microorganism diffusion weakens and convective transport becomes dominant.
- The comparative analysis demonstrates that the Williamson tri-hybrid nanofluid exhibits superior thermal performance compared with the Williamson hybrid nanofluid and conventional Williamson nanofluid due to its enhanced effective thermal conductivity. Consequently, the tri-hybrid nanofluid maintains a comparatively higher temperature distribution and improved heat transfer capability, whereas enhanced effective viscosity and electrical conductivity produce stronger momentum suppression under magnetic field conditions.

Overall, the incorporation of hybrid and tri-hybrid nanoparticles substantially improves the thermal characteristics of the Williamson fluid without compromising the physical stability of the flow. The findings confirm that Williamson tri-hybrid nanofluids are promising working fluids for advanced thermal engineering applications, including solar thermal collectors, electronic cooling systems, heat exchangers, nuclear cooling technologies, polymer processing, biomedical devices, and energy conversion systems, where

efficient heat transfer and controlled transport phenomena are essential. Future work may include entropy generation analysis, multiple slip effects, non-uniform magnetic fields, and nanoparticle shape factors.

Abbreviations

MHD	Magnetohydrodynamic
WF	Williamson fluid
WNF	Williamson nanofluid
THNF	Triple hybrid nanofluid
WTHNF	Williamson triple hybrid nanofluid
2D	Two-dimensional
ISC	Inclined stretching cylinder
HC	Hall current
BC	Bioconvection

Nomenclature

Symbol	Description
B_0	Magnetic field strength
B_i	Biot number
b	Chemotaxis constant
C	Nanoparticle concentration
C_f	Skin friction coefficient
C_p	Specific heat at constant pressure
DB	Brownian diffusion coefficient
D_n	Microorganism diffusivity
DT	Thermophoretic diffusion coefficient
E	Dimensionless activation energy
E_a	Activation energy
Ec	Eckert number
F	Forchheimer coefficient
Fr	Forchheimer number
f	Dimensionless stream function
G_c	Solutal Grashof number
G_n	Bioconvection Grashof number
Gr	Thermal Grashof number
hf	Convective heat transfer coefficient
K	Porosity parameter
kr	Chemical reaction rate constant
L_b	Bioconvection Lewis number
M	Magnetic parameter
m	Hall parameter
N	Motile microorganism concentration
N_n	Motile density number
Nu	Nusselt number
N_b	Brownian motion parameter
N_t	Thermophoresis parameter
Pe	Bioconvection Peclet number
Pr	Prandtl number
R	Cylinder radius
r	Radial coordinate
Sc	Schmidt number
Sh	Sherwood number
T	Temperature
u, v	Velocity components

DECLARATIONS

Consent to Publish declaration

Not applicable

Ethics and Consent to Participate declarations

Ethics and Consent to Participate declarations: not applicable

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Conflict of Interest

The authors declare that they have no conflict of interest.

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