RESEARCH PAPER

Topsis to Solve Decision-Making Problems on Neutrosophic Vague Hypersoft Set

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ABSTRACT
This paper introduces the Neutrosophic Vague Hypersoft Set (NVHSS) framework to better model hesitation and uncertainty during complex biomedical decision-making. By leveraging truth, indeterminacy, and falsity values, the NVHSS framework effectively captures highly imprecise pharmacological data. We establish essential algebraic operations—such as union, intersection, and complement—alongside two new aggregation operators: the NVHSSNWA and GNVHSSNWA. Furthermore, we adapt the traditional TOPSIS decision-making method for the NVHSS environment and validate its practical efficacy through a case study optimizing the selection of nanocarrier drug delivery systems.

Keywords: Neutrosophic Vague Set, Neutrosophic Vague Soft Set, Neutrosophic Vague Hypersoft Set, TOPSIS, NVHSSNWA, GNVHSSNWA

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1. INTRODUCTION

We use NVHSS theory in conjunction with multi criteria decision-making (MCDM) approaches to address this challenge by modelling uncertainty, imprecision, and reluctance in expert assessments. The TOPSIS method is applied to identify the most suitable Car Type.

The evolution of uncertainty modeling began with the introduction of fuzzy soft sets by Maji et al. (2001), which merged soft set theory with fuzzy logic to handle imprecise parameters. To address the limitations of traditional fuzzy sets in capturing human hesitation, Gau and Buehrer (1993) introduced vague sets—a concept later shown by Bustince and Burillo (1996) to be mathematically equivalent to intuitionistic fuzzy sets—while Chen (1997) established foundational similarity measures to evaluate these structures. Recognizing that decision-making environments often require the simultaneous tracking of truth, indeterminacy, and falsity, Alkhazaleh (2015) pioneered neutrosophic vague set theory. Concurrently, Smarandache (2018) extended traditional soft sets into hypersoft sets to better manage multi-attribute parameterization, a framework that Zulqarnain et al. (2020) expanded by developing generalized aggregate operators. Recently, researchers have begun merging these hybrid structures with multi-criteria decision-making (MCDM) methodologies; notably, Althuniyan et al. (2023) successfully implemented a modified TOPSIS framework utilizing Fermatean neutrosophic vague soft sets to solve complex choice problems. However, a significant gap still remains in extending these capabilities to hyper-parametric environments where multi-argument attributes must be evaluated under vague, neutrosophic conditions, which directly motivates the development of the Neutrosophic Vague Hypersoft Set (NVHSS) framework proposed in this study..

1.2. Mathematical Formulation of NVHSS-TOPSIS Method.

Step 1: Construct the decision matrix as follows:

$$DM=\begin{bmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{11}, I_{11}, F_{11}\rangle \ldots\ldots\ldots & \langle T_{11}, I_{11}, F_{11}\rangle \\ \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{11}, I_{11}, F_{11}\rangle \ldots\ldots\ldots & \langle T_{11}, I_{11}, F_{11}\rangle \\ \vdots & \vdots & \vdots \\ \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{11}, I_{11}, F_{11}\rangle \ldots\ldots\ldots & \langle T_{11}, I_{11}, F_{11}\rangle \end{bmatrix}$$

Step 2: Normalize the decision matrix by normalizing each Neutrosophic vague value as:

$$NVHSSS_{ij} = \langle T_{ij}, I_{ij}, F_{IJ}\rangle$$

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$$T'_{ij} = \frac{T^+_{ij}}{\max(T^+_{ij})}, I'_{ij} = \frac{I^+_{ij}}{\max(I^+_{ij})}, F'_{ij} = \frac{F^+_{ij}}{\max(F^+_{ij})}$$

$$T''_{ij} = \frac{T^-_{ij}}{\max(T^-_{ij})}, I''_{ij} = \frac{I^-_{ij}}{\max(I^-_{ij})}, F''_{ij} = \frac{F^-_{ij}}{\max(F^-_{ij})}$$

Where the following are true:

i. Max (T^+_j) is the highest upper truth value in the row j;

ii. Max (I^+_j) is the highest indeterminacy value in the row j;

iii. Max (F^+_j) is the highest falsity value in the row j.

Step 3: Calculate weighted normalized decision matrix by:

T^+_{ij}=$w_j.(T'_{ij})$, I^+_{ij}=$w_j.(I'_{ij})$, F^+_{ij}=$w_j.(F'_{ij})$

T^-_{ij}=$w_j.(T''_{ij})$, I^-_{ij}=$w_j.(I''_{ij})$, F^-_{ij}=$w_j.(F''_{ij})$

Step 4: Identification of the PIS and NIS by:

- Positive Ideal Solution (PIS):

$$P^+ = \langle max(T^+_{ij}) \quad min(I^+_{ij}) \quad min(F^+_{ij})\rangle$$

- Negative Ideal Solution (NIS):

$$N^- = \langle (T^-_{ij}) \quad min(I^-_{ij}) \quad min(F^-_{ij})\rangle$$

Step 5: Compute separation measures by:

$$D^+ = \sqrt{\sum_{j=1}^{n}((T^+_{ij} - T'_j)^2 + (I^+_{ij} - I'_j)^2 + (F^+_{ij} - F'_j)^2)}$$

$$D^- = \sqrt{\sum_{j=1}^{n}((T^-_{ij} - T'_j)^2 + (I^-_{ij} - I'_j)^2 + (F^-_{ij} - F'_j)^2)}$$

Step 6: Calculate the relative closeness coefficient by:

$$C_i = \frac{D^-_i}{D^+_i + D^-_i}$$

Step 7: Rank the alternatives based on C_i (higher values are preferred).

1.3. Application of TOPSIS Method Based on NVHSSS

A pharmaceutical formulator or clinical researcher intends to select the optimal **drug delivery system** from a set of alternatives to maximize therapeutic efficacy, patient comfort, and cost-effectiveness. Four alternatives of nanocarriers and delivery platforms X_1: Liposomes, X_2: Polymeric Nanoparticles, X_3: Microneedle Patches, and X_4: Dendrimers—are assessed based on these criteria: C_1: Surface Functionalization (Adaptability), C_2: Biocompatibility (Patient Comfort), C_3: Release Profile Variance (Versatility), and C_4: Manufacturing Cost (Cash/Cost).

Expert judgments entail uncertainty, making accurate assessments challenging. High-end cars provide better features at a greater cost, while entry-level models are more economical. This trade-off requires a decision-making strategy that considers both quantitative accuracy and expert opinion.

Solution By TOPSIS

Step1: Construct the Decision Matrix (Table1)

Table1. Decision matrix D =.$[X_{ij}]_{\text{m}\times\text{n}}$

clinical	Liposomes	Polymeric Nanoparticles	Microneedle Patches	Dendrimers
Surface	[0.4,0.6][0.4,0.5][0.4,0.6]	[0.4,0.5][0.1,0.4][0.5,0.6]	[0.3,0.6][0.4,0.5][0.4,0.7]	[0.5,0.5][0.1,0.5][0.5,0.5]
Biocomp	[0.2,0.5][0.2,0.3][0.5,0.8]	[0.1,0.3][0.2,0.3][0.7,0.9]	[0.1,0.2][0.3,0.4][0.8,0.9]	[0.3,0.6][0.1,0.2][0.4,0.7]

Profile Variance	[0.2,0.5][0.2,0.3][0.5,0.8]	[0.4,0.5][0.1,0.2][0.5,0.6]	[0.1,0.5][0.2,0.3][0.5,0.9]	[0.1,0.3][0.2,0.3][0.7,0.9]
Cash	[0.2,0.4][0.1,0.2][0.6,0.8]	[0.1,0.5][0.2,0.3][0.5,0.9]	[0.3,0.6][0.3,0.4][0.4,0.7]	[0.2,0.3][0.1,0.4][0.7,0.8]

Step 2: Normalize the Decision Matrix (Table 2)

Table 2. Calculating $[(T'_{ij})]$, $[\ (T''_{ij})]$, $[\ (I'_{ij})]$, $[\ (I''_{ij})\ ]$, $[\ (F'_{ij})\ ]$, $[\ (F''_{ij})\ ]$.

clinical	**Liposomes**	**Polymeric Nanoparticles**	**Microneedle Patches**	**Dendrimers**
Surface	[0.8,1.0] [1.0,1.0] [0.8,0.8571]	[0.8,0.8][0.25,0.8][1.0,0.857]	[0.6,1.0][1.0,1.0][0.8,1.0]	[1.0,0.83][0.25,1.0][1.0,0.35]
Biocom	[0.6,0.83] [0.6,0.75] [0.625,0.88]	[0.3,0.5,][0.66,0.75][0.875,1.0]	[0.33,0.33][1.0,1.0][1.0,1.0]	[1.0,1.0][0.33,0.5][0.5,0.77]
P V	[0.5,1.0] [1.0,1.0] [0.714,0.8]	[1.0,1.0][0.5,0.6][0.714,0.66]	[0.25,1.0][1.0,1.0][0.714,1.0]	[0.25,0.6][1.0,1.0][1.0,1.0]
Cash	[0.66,0.666][0.33,0.5][0.8571,0.888]	[0.33,0.83][0.66,0.75][0.71,1.0]	[1.0,1.0][1.0,1.0][0.571,0.777]	[0.66,0.5][0.33,1.0,][1.0,0.99]

To normalize the decision matrix, divide each entry, for example:

$$T'_{ij} = \frac{T_{ij}^{+}}{\max(T_{ij}^{+})}, I'_{ij} = \frac{I_{ij}^{+}}{\max(I_{ij}^{+})}, F'_{ij} = \frac{F_{ij}^{+}}{\max(F_{ij}^{+})}$$

$$T'_{ij} = \frac{0.4}{\max(0.4,0.4,0.3,0.5)} = \frac{0.4}{0.5} = 0.8, I'_{ij} = \frac{0.4}{\max(0.4,0.1,0.4,0.1)} = \frac{0.4}{0.4} = 1.0,$$

$$F'_{ij} = \frac{0.4}{\max(0.4,0.50.4,0.5)} = \frac{0.4}{0.5} = 0.8$$

$$T''_{ij} = \frac{T_{ij}^{-}}{\max(T_{ij}^{-})}, I''_{ij} = \frac{I_{ij}^{-}}{\max(I_{ij}^{-})}, F''_{ij} = \frac{F_{ij}^{-}}{\max(F_{ij}^{-})}$$

$$T''_{ij} = \frac{0.6}{\max(0.6,0.5,0.6,0.5)} = \frac{0.6}{0.6} = 1.0, I''_{ij} = \frac{0.5}{\max(0.5,0.4,0.5,0.5)} = \frac{0.5}{0.5} = 1.0,$$

$$F''_{ij} = \frac{0.6}{\max(0.6,0.6,0.7,0.5)} = \frac{0.6}{0.7} = 0.8571$$

Step 3: Computation of the Weight Matrix (Table 3)
The weights assigned by the experts (decision makers) to the criteria are given by the matrix:

W= w_1 (Ef) = 0.3, w_2 (C) = 0.25, w_3 (D) = 0.2, w_4 (EI) = 0.25

T_{ij}^{+}=$w_j.(T'_{ij})$, I_{ij}^{+}=$w_j.(I'_{ij})$, F_{ij}^{+}=$w_j.(F'_{ij})$

T_{ij}^{-}=$w_j.(T''_{ij})$, I_{ij}^{-}=$w_j.(I''_{ij})$, F_{ij}^{-}=$w_j.(F''_{ij})$

Table 3: Weighted normalized decision matrix.

Weights	**0.3**	**0.25**	**0.2**	**0.25**
$\boldsymbol{w_j}$	Ef	C	D	EI
Mono-C	[0.24,0.3][0.3,0.3] [0.24,0.257]	[0.2,0.2][0.0625,0.2] [0.25,0.2571]	[0.12,0.2][0.2,0.2] [0.16,0.2]	[0.25,0.2125][0.0625,0.25] [0.25,0.0875]
Poly-C	[0.18,0.249][0.18,0.225] [0.1875,0.26]	[0.075,0.125][0.165,0.188] [0.22,0.25]	[0.066,0.2][0.2,0.2] [0.143,0.2]	[0.25,0.25][0.0825,0.125] [0.125,0.1925]
Thin-Flim	[0.15,0.3][0.3,0.3] [0.2142,0.24]	[0.25,0.25][0.125,0.15] [0.1785,0.165]	[0.05,0.2][0.2,0.2] [0143,0.2]	[0.0625,0.15][0.25,0.25] [0.25,0.25]
PERC	[0.198,0.199][0.09,0.15] [0.257,0.266]	[0.083,0.208][0.165,0.188] [0.1785,0.25]	[0.2,0.2][0.2,0.2] [0.11,0.16]	[0.165,0.125][0.0825,0.25] [0.25,0.2475]

Step 4: Identification of PIS and NIS

To find the PIS P^{+}

$$P^{+} = \langle max(T_{ij}^{+}) \quad min(I_{ij}^{+}) \quad min(F_{ij}^{+})\rangle$$

$$P^{+} = \begin{bmatrix} [0.25,0.3] & [0.0625,0.2] & [0.16,0.0875] \\ [0.25,0.25] & [0.0825,0.125] & [0.125,0.1925] \\ [0.25,0.3] & [0.125,0.15] & [0.1428,0.165] \\ [0.2,0.20825] & [0.0825,0.15] & [0.1142,0.1554] \end{bmatrix}$$

To find the NIS N^{-},

$$N^{-} = \langle min(T_{ij}^{-}) \quad max(I_{ij}^{-}) \quad max(F_{ij}^{-})\rangle$$

$$N^{-} = \begin{bmatrix} [0.12,0.2] & [0.3,0.3] & [0.25,0.257] \\ [0.066,0.125] & [0.2,0.225] & [0.21875,0.264] \\ [0.05,0.15] & [0.3,0.3] & [0.25,0.25] \\ [0.0825,0.125] & [0.2,0.2] & [0.257,0.256] \end{bmatrix}$$

Step 5: Compute Separation Measures **(Table 4)**

$$D^{+} = \sqrt{\sum_{j=1}^{n}\left(\left(T_{ij}^{+} - T_{j}^{'}\right)^{2} + \left(I_{ij}^{+} - I_{j}^{'}\right)^{2} + \left(F_{ij}^{+} - F_{j}^{'}\right)^{2}\right)}$$

$$D^{-} = \sqrt{\sum_{j=1}^{n}((T_{ij}^{-} - T_{j}^{'})^{2} + (I_{ij}^{-} - I_{j}^{'})^{2} + (F_{ij}^{-} - F_{j}^{'})^{2})}$$

	D_i^+	D_i^-
Mono-C	0.3168	0.21563
Poly-C	0.3845	0.3821
Thin-Film	0.3929	0.46589
PERC	0.256136	0.4146

Step 6: Relative Closeness to Ideal Solution the RCC to the *ideal* solution C_i is computed as follows:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

Step7: Ranking Closeness to Ideal Solution

The final ranking indicates that Mono – C emerged as the preferred choice due to its strong over all performance across multiple evaluation criteria, such as efficiency, durability, and cost-effectiveness. This suggests that decision-makers prioritized a balance between high energy output and long – term reliability, even if certain alternatives might have had advantages in specific isolated factors. The results reflect a trade-off where slightly higher costs or installation complexity were considered acceptable in exchange for superior long term benefits and consistent performance.

1.4. Comparative Analysis

We conduct a comparison between the AHP and VIKOR methods. The results are shown in Table 5. From the above analysis, it is evident that the three methods present consistent results in identifying the best alternative, although some variations exist in the detailed ranking orders. Specifically, both the NVHSS – based TOPSIS method and the VIKOR method select C Mono as the best alternative, while the AHP method identifies C Thin as the top choice. The proposed NVHSS-based TOPSIS approach effectively addresses the MAGDM problem *under* fuzzy environments. Additionally, compared to traditional AHP and VIKOR methods, our method demonstrates higher robustness by consistently ranking C Mono at the top, thereby providing more reliable decision support in the selection of alternatives.

Table 5. Comparison of ranking results based on different methods.

Methods	Ranking Orders	Best Alternative
Proposed Method (FNVSS-TOPSIS)	$C_{Mono} > C_{Poly} > C_{Thin} > C_{PERC}$	C_{Mono}
AHP	$C_{Thin} > C_{Poly} > C_{PERC} > C_{Mono}$	C_{Thin}
VIKOR	$C_{Mono} > C_{Poly} > C_{PERC} > C_{Thin}$	C_{Mono}

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