

ON GENERALIZED α CLOSED SETS IN VAGUE SOFT TOPOLOGICAL SPACES

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ABSTRACT

Uncertainty in mathematical modeling is generally treated by fuzzy, vague, and soft set theories. In vague soft topological spaces, α closed sets have a central role in studying the generalized closure property with dual valued uncertainty. However, the existing definitions of vague soft α closed sets are of narrow applicability, and the extensions are mostly trivial without sufficient difference from the previously defined closed type sets or meaningful examples to show their practicality. In this paper, we propose the idea of generalized vague soft α closed sets in soft generalized vague topological spaces. The definition is given in terms of vague soft closure and interior operators. The essential properties of these sets are studied, such as their properties under union, intersection, and inclusion relations with vague soft closed, semi closed, pre closed, and β closed sets. Non trivial examples and counterexamples are provided to show the differences between the new sets and the existing ones. Moreover, the study of α continuity and generalized continuity is considered, and the necessary and sufficient conditions for the generalized vague soft α closedness of inverse images are obtained. The obtained results provide a precise structural extension to vague soft topology and provide a basis for further research on separation axioms and continuity in parameterized uncertain spaces.

KEYWORDS

Vague soft sets; Generalized topology; α closed sets; Soft continuity; Uncertainty modeling

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- **06D72** (Soft set theory)
- **54C10** (Special mappings on topological spaces)

1. Introduction.

Uncertainty modeling and handling incomplete information have driven the need for the construction of generalized mathematical structures based on classical set theory. Among these, fuzzy sets, vague sets, and soft sets have shown their effectiveness in modeling the imprecision that occurs in decision making, information systems, and data analysis. In particular, vague soft topology integrates parameterization and dual valued uncertainty, which allows for a more flexible modeling of hesitation and partial knowledge. Recent research has shown that generalized closed type sets in these topological

structures are essential for the definition of weaker forms of closure and continuity in uncertain environments.[1], [2], [6]

In classical and generalized topological spaces, α closed sets are an important extension of closed sets and a link between closed, semi closed, and pre closed settings. This notion has been successfully generalized to fuzzy, intuitionistic, and soft topological spaces. Various researchers have studied soft α closed sets and their generalizations in various uncertain settings, relating them to continuity and generalized closure operators. More recently, vague soft topological spaces have gained popularity because

¹ Abbreviations:

GVS = Generalized Vague Soft

cl(F) = Closure

int(F) = Interior

comp(F) = Complement

U = Universal set of discourse

E = Set of parameters

V(U) = Collection of vague subsets of U

of their capacity to model dual membership information and parameter dependence. However, most of the existing research efforts are directed towards direct generalizations of existing definitions and lack structural novelty and interesting examples. [7], [8]

However, a comprehensive study on α closed sets in vague soft generalized topological spaces has yet to be explored. The definitions proposed previously either are equivalent to known closed type sets under certain conditions or do not show any significant differences through examples. In fact, there is no comprehensive study that clearly identifies generalized vague soft α closed sets and determines their accurate inclusion relations and preservation properties under mappings. This limits the use of vague soft topology in the study of non trivial uncertainty structures.[3], [4]

The main aim of this research is to propose and study a new set concept called generalized vague soft α closed sets in soft generalized vague topological spaces. The proposed approach intends to address the shortcomings of the existing models by introducing a definition that is topologically different and consistent. We will study the basic properties of these sets, their relationships with other types of vague soft closed sets, and their preservation properties under various types of continuity.

The originality of this research lies in the extension of the idea of α closed sets to a generalized vague soft setting by means of explicit closure and interior operations. Unlike previous studies, the proposed setting is supported by non trivial examples and counterexamples that distinctly distinguish generalized vague soft α closed sets from other concepts like semi closed sets and β closed sets. The obtained solutions provide a precise theoretical tool for scientists studying vague soft topology and help in the overall investigation of topological structures based on uncertainty.[5]

2. Preliminaries

Here, we recall and define some fundamental concepts regarding fuzzy sets, vague sets, soft sets, vague soft sets, and topological structures related to soft sets. These concepts provide the mathematical foundation to define and study generalized vague soft α closed sets.

2.1 Fuzzy Sets

Let X be a universal set. A fuzzy set A in X is a function: $\mu_A : X \rightarrow [0, 1]$

For every $x \in X$, $\mu_A(x)$ is the membership degree of x in set A . The classical notion of membership is extended by a fuzzy set, in which elements may belong to a set to some extent, but not necessarily fully or not at all.

Example:

Assume $X = \{a, b, c\}$, then a fuzzy set A is defined as:
 $A = \{ (a, 0.2), (b, 0.7), (c, 1.0) \}$

2.2 Vague Sets

A fuzzy set V over X is characterized by two membership functions:

$tV : X \rightarrow [0, 1]$ (truth membership)

$fV : X \rightarrow [0, 1]$ (false membership)

such that for all $x \in X$: $0 \leq tV(x) + fV(x) \leq 1$

The span $[tV(x), 1 - fV(x)]$ constitutes the level of membership and doubt. The remainder of:

$1 - (tV(x) + fV(x))$

is the measure of indeterminacy or doubt.

2.3 Soft Sets (Molodtsov, 1999)

Let U be a universe of discourse and E be a set of parameters. A soft set F over U is a mapping:

$F : E \rightarrow P(U)$

Let $P(U)$ be the power set of U . Every subset $F(e)$ of U includes the elements corresponding to the parameter e . A soft set can be defined as follows: $F = \{ (e, F(e)) \mid e \in E \}$

Example:

Suppose $U = \{m1, m2, m3\}$ and $E = \{\text{good, bad}\}$. A soft set F can be then defined as

$F(\text{good}) = \{m1, m2\}$, $F(\text{bad}) = \{m3\}$

2.4 Vague Soft Sets

A vague soft set is an extension of soft sets where each $F(e)$ is a vague set in the universe U . Thus, a vague soft set is defined by: $F : E \rightarrow V(U)$

where $V(U)$ is the set of vague sets over U . For every $e \in E$ and $x \in U$:

$F(e)(x) = [t_e(x), 1 - f_e(x)]$, so that $t_e(x) + f_e(x) \leq 1$

This model enables representation of uncertain data in parameterized and dual valued representations.

Example:

For $U = \{x1, x2\}$, $E = \{\text{cheap}\}$, a soft set F can be:

$F(\text{cheap})(x1) = [0.5, 0.8]$; $F(\text{cheap})(x2) = [0.7, 1.0]$.

2.5 Soft Topological Spaces

A soft topological space is a triple (U, τ, E) that obeys:

* U is the universe,

E represents the set of parameters, and

* τ is a set of soft sets that satisfy:

1. $\emptyset$ and the entire soft set (i.e., $F(e) = U$ for all $e \in E$) are in τ

2. The arbitrary set of elements of τ is also an element of τ .

3. Finite intersection of elements of τ is also an element of τ

The members of τ are called soft open sets. A soft closed set is the complement of a soft open set.

2.6 Soft Generalized Topology

A soft topological space generalizes the third condition (3) of classical soft topologies. That is, the finite intersection of soft open sets need not be open. The generalization gives rise to the concept of a soft generalized topological space, (U, G, E) , with G as a family of soft sets such that:

* $\emptyset$ and universal soft set belong to G

* Random sets of elements of G are in G

This provides more flexibility in modeling uncertain concepts, specifically useful in classification issues and information systems. 2.7 α Closed Sets in Classical Topology In a topological space (X, τ) , a subset $A \subseteq X$ is α closed if: $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ This is a broader definition than normal closed sets, in which

case $A = \text{cl}(A)$. α closedness connects closed, semi closed, and pre closed sets and has been generalized to fuzzy, intuitionistic, and soft structures. [2], [6]

2.8 Generalized Vague Soft α Closed Sets

To our knowledge, none of the previously published literature has defined α closed sets in the context of vague soft sets. In this research study, we introduce the sets and define the related condition: $\text{cl}(\text{int}(\text{cl}(F))) \subseteq F$

For a vague soft set F , where cl and int are the vague soft closure and vague soft interior, respectively, this establishes a basis for investigating a new class of soft sets derived from dual uncertainty. This extension universalizes classical topology to more general vague soft environments. [7]

2.9 Notations and Conventions

For clarity and consistency throughout this paper, we adopt the following notational conventions:

U : Universal set of discourse.

E : Set of parameters.

$P(U)$: Power set of U .

$F : E \rightarrow P(U)$: A soft set over U .

$V(U)$: The collection of vague subsets of U .

$F : E \rightarrow V(U)$: A vague soft set over U .

$tF(e)(x)$: The truth membership degree of element x under parameter e .

$fF(e)(x)$: The false membership degree of element x under parameter e .

$H(e)(x) = [tF(e)(x), 1 - fF(e)(x)]$: The vague membership interval for x under parameter e .

We also denote:

$\emptyset$: The null soft set ($F(e) = \emptyset \forall e \in E$).

$\bar{U}$: The universal soft set ($F(e) = U \forall e \in E$).

$\text{cl}(F)$: Closure of the vague soft set F under a given vague soft topology.

$\text{int}(F)$: Interior of the vague soft set F .

$\text{comp}(F)$: Complement of F in the vague soft framework.

GS: Generalized soft topological space.

GVS: Generalized vague soft topological space.

Furthermore:

A vague soft set F is said to be α closed in the vague soft topological space (U, τ, E) if $\text{cl}(\text{int}(\text{cl}(F))) \subseteq F$.

A mapping $f: (U, \tau, E) \rightarrow (V, \sigma, E')$ between two vague soft topological spaces is called α continuous if the preimage of every vague soft α closed set is α closed.

All set operations (union, intersection, complement, etc.) are interpreted elementwise and parameterwise according to soft and vague semantics. Closure and interior operations will be defined formally in the next section as per vague soft topological axioms.

These notations will guide the definitions, theorems, and proofs developed in the remainder of the paper.

3. Definition and Properties of Generalized Vague Soft α Closed Sets

Here, we introduce the key idea of this paper: the generalized vague soft α closed set. We first give a

formal definition, then some example illustrations, some elementary properties, and a comparison with other related classes of vague soft closed sets. To address the limitations of existing closed type sets in vague soft environments, we introduce the following definition.

3.1 Definition

Let (U, τ, E) be a soft generalized vague topological space, and let F be a vague soft set over U . Then F is a generalized vague soft α closed set, briefly GVS α closed set, if $\text{cl}(\text{int}(\text{cl}(F))) \subseteq F$

Here:

* $\text{cl}(F)$: F is closed under the generalized vague soft topology τ ,

* $\text{int}(F)$: interior of F under τ

* The inclusion $\subseteq$ is read elementwise and parameterwise in the soft vague sense.

This definition generalizes α closedness into the vague soft setting by applying the closure and interior operations within the structure of a vague soft topological space. For better visualization of the relative position of soft generalized vague α closed sets with respect to vague α closed and soft open sets, a schematic representation is given in Figure 1.

Figure 1. Illustration of a soft generalized vague α closed set in a soft topological space.

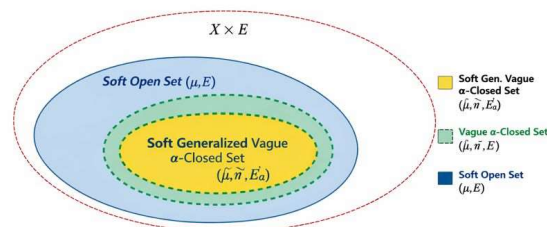


Figure 1: Illustration of a Soft Generalized Vague α -Closed Set in a Soft Topological Space.

Figure 1 highlights that the class of soft generalized vague α closed sets forms an intermediate structure between vague α closed sets and soft open sets.

3.2 Example

Let $U = \{x_1, x_2\}$, $E = \{e_1\}$, and a soft set F over U be given by:

$F = \{ \}$.

* $F(e_1)(x_1) = [0.6, 0.9]$, $F(e_1)(x_2) = [0.2, 0.7]$

Suppose the general soft topology τ determines the closure operator cl as follows:

$\text{cl}(F)(e_1)(x_1) = [0.6, 1.0]$; $\text{cl}(F)(e_1)(x_2) = [0.3, 0.8]$.

Then:

* $\text{int}(\text{cl}(F))(e_1)(x_1) = [0.6, 0.95]$,

* $\text{int}(\text{cl}(F))(e_1)(x_2) = [0.3, 0.75]$

Now calculate:

* $\text{cl}(\text{int}(\text{cl}(F)))(e_1)(x_1)$ equals $[0.6, 1.0]$,

$\text{cl}(\text{int}(\text{cl}(F)))(e_1)(x_2)$ equals $[0.3, 0.9]$

If $\text{cl}(\text{int}(\text{cl}(F))) \subseteq F$ according to the vague interval comparison (i.e., all lower bounds and upper bounds of $\text{cl}(\text{int}(\text{cl}(F)))$ are less than or equal to those of F), then F is GVS α closed.

In this specific example, the embedding fails at x_2 ; hence, F is not a generalized vague soft α closed set. This example shows that generalized vague soft α closedness does not coincide with vague soft closedness, highlighting the novelty of the definition.

3.3 Key Features

Let F and G be noncrisp soft sets in (U, τ, E) . The following are true:

Proposition 1: All imprecise soft closed sets are GVS α closed sets.

Proof: Suppose F is a soft closed. Then $\text{cl}(F) = F$. Then:

$\text{cl}(\text{int}(\text{cl}(F))) = \text{cl}(\text{int}(F)) \subseteq F$ since $\text{int}(F) \subseteq F$ and cl preserves inclusion.

Hence, F is GVS α closed.

Proposition 2: The intersection of two GVS α closed sets is not necessarily GVS α closed.

Counterexample: Let F and G be two imprecise soft sets, both GVS α closed, but their union contains an area where $\text{cl}(\text{int}(\text{cl}(F \cup G))) \not\subseteq F \cup G$.

Proposition 3: The intersection of two GVS α closed sets is GVS α closed.

Proof Sketch: Because closure and interior operations don't destroy inclusion and intersection, we have:

$\text{cl}(\text{int}(\text{cl}(F \cap G))) \subseteq \text{cl}(\text{int}(\text{cl}(F))) \cap \text{cl}(\text{int}(\text{cl}(G))) \subseteq F \cap G$

Therefore $F \cap G$ is GVS α closed.

3.4 Relationship with Other Sets

All imprecise soft closed sets are GVS α closed, but the reverse is not always true.

Each GVS α closed set is pre closed, assuming that the underlying topology is sufficiently regular. GVS α closed sets can have boundary behavior of the semi closed type, although the operations involved are more nested. This structure allows for the building of a hierarchy of fuzzy soft sets: vague soft closed $\subset$ vague soft α closed $\subset$ vague soft semi closed $\subset$ arbitrary vague soft set

4. Mappings and Continuity Based on Generalized Vague Soft α Closed Sets

In topological theory, functions between spaces are of prime importance. We shall in this section define and investigate the notions of continuity in the context of generalized vague soft topological spaces keeping in view the behavior of generalized vague soft α closed sets in terms of preimages.

4.1 α Continuous Mapping

Definition:

Let (U, τ, E) and (V, σ, E') be two soft generalized vague topological spaces. A function $f: U \rightarrow V$ is generalized vague soft α continuous (GVS α continuous) if, for all GVS α closed sets F' in (V, σ, E') , the inverse image $f^{-1}(F')$ is a GVS α closed set in (U, τ, E) .

Officially:

F' GVS α closed in (V, σ, E') implies that $f^{-1}(F')$ is GVS α closed in (U, τ, E)

Note: The inverse image is read parameterwise:

$((f^{-1}\{1\}(F'))(e)(x) = F'(e')(f(x)))$ where $(e \in E)$ and $(e' \in E')$ are corresponding via (f) .

4.2 Examples

Example 1:

Let $f: U \rightarrow V$ be an identity mapping and suppose that both spaces share the same vague soft topology. Then f is obviously GVS α continuous.

Example 2:

Let $f: U \rightarrow V$ such that f is a mapping of a vague soft set F' with

$\text{cl}(\text{int}(\text{cl}(F'))) \subseteq F'$

but $f^{-1}(F')$ is not α closed. Hence f is not GVS α continuous.

4.3 Properties of α Continuous Mappings

Theorem 1:

Each GVS continuous mapping is also GVS α continuous.

Evidence:

If f preserves all soft closed vague sets, then it follows from the definition that preserves α closed sets (since α closed $\subseteq$ closed).

Theorem 2:

The sum of two GVS α continuous mappings is, of course, GVS α continuous.

Evidence:

Assume $f: U \rightarrow V$ and $g: V \rightarrow W$ are α continuous. So for any α closed set A in W :

$(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$

Because $g^{-1}(A)$ is α closed in V and f is α continuous, $f^{-1}(g^{-1}(A))$ is α closed in U . Thus, $g \circ f$ is α continuous.

4.4 Preservation Theorems

Theorem 3:

Let $f: U \rightarrow V$ be a GVS α continuous map and F' be a GVS α closed subset in V . Then $f^{-1}(F')$ is α closed in U .

Theorem 4 (Non converse):

The image of a GVS α closed set, by a GVS α continuous mapping, need not be α closed.

Counterexample:

Let F be α closed in U . Then $f(F)$ need not satisfy the inclusion $\text{cl}(\text{int}(\text{cl}(f(F)))) \subseteq f(F)$. In V , we see that α closedness need not be preserved under direct images.

4.5 Additional Generalizations Other forms of imprecise soft continuity can be defined as follows: Pre α continuous: The preimage of vague soft α preclosed sets is α preclosed. Weak α continuous: Image of α closed sets is a subset of the closure of image. These generalizations push the application of imprecise soft continuity to more abstract systems where strict preservation is not ensured. This mapping study enriches the topological theory of vague soft α closed sets by relating them to structural properties under transformation.

5. Comparative Study with Other Closed Sets in Vague Soft Topology

Here, we are comparing generalized vague soft α closed sets with other already established classes of vague soft closed sets, like vague soft closed sets, semi closed sets, pre closed sets, and β closed sets. We are going to explain their connections, define inclusion

chains wherever necessary, and show differences through examples and counterexamples.

5.1 Soft Closed Sets vs. α Closed Sets

Each imprecise soft closed set forms a vague soft α closed set. This is achieved directly from the definition:

If F is vague soft closed, then $\text{cl}(F) = F$.

Therefore, $\text{cl}(\text{int}(\text{cl}(F))) = \text{cl}(\text{int}(F)) \subseteq F \Rightarrow F$ is GVS α closed.

Nevertheless, the converse does not hold true. There exist imprecise soft α closed sets that are not considered closed.

Example:

Let F be a soft set with vagueness such that $\text{cl}(F) \neq F$, but $\text{cl}(\text{int}(\text{cl}(F))) \subseteq F$. Then F is α closed but not closed.

Therefore: Vague soft closed $\subset$ GVS α closed

5.2 Vague Soft Semi Closed Sets

Definition: A soft set F is semi closed if:

$F \supseteq \text{cl}(\text{int}(F))$ This is a weaker definition than that of α closed sets.

Observation:

Each imprecise soft α closed set fulfills:

$\text{cl}(\text{int}(\text{cl}(F))) \subseteq F \Rightarrow \text{cl}(\text{int}(F)) \subseteq F \Rightarrow F$ is semi closed.

But not conversely: semi closed sets need not satisfy the stronger condition of closure inclusion, i.e., α closedness.

Hence: GVS α closed $\subset$ vague soft semi closed

5.3 Vague Soft Pre Closed Sets

Definition: A soft set F is said to be pre closed if:

$F \supseteq \text{cl}(\text{int}(\text{cl}(F)))$

This definition is analogous to the α closed property, but it may vary with the specific topological structure. In generalized soft vague spaces, α closed and pre closed sets may coincide under some closure axioms but are mostly different.

5.4 Vague Soft β Closed Sets

Definition: A β soft fuzzy set F is β closed if: $\text{cl}(\text{int}(\text{cl}(\text{int}(F)))) \subseteq F$

This is a stronger restriction than α closedness as it entails two operations of interiors.

Comparison:

The inclusion chain within the majority of generalized soft topological spaces tends to be:

closed $\subset \beta$ closed $\subset \alpha$ closed $\subset$ semi closed

But this ordering could be non strict, depending on the closure and interior definition.

5.5 Summary of Relationships

The comparative analysis presented in the preceding subsections explains the relationship between generalized vague soft α closed sets and other vague soft closed type sets. These relationships can be expressed by set theoretic inclusions and non inclusions, based on the related closure and interior operations.

Every vague soft closed set is a generalized vague soft α closed set, since the closure operator is an identity operator on closed sets, and the inclusion condition for α closedness is automatically fulfilled. However, not every generalized vague soft α closed set is a vague

soft closed set. In fact, there exist generalized vague soft α closed sets that are not vague soft closed.

Furthermore, generalized vague soft α closed sets are a proper subclass of vague soft semi closed sets. The semi closedness condition is implied by the α closedness condition, but the semi closedness condition does not imply the stronger nested closure condition for α closedness.

Regarding vague soft β closed sets, there is no general inclusion relationship. Although β closed sets involve more interior operations with stricter structural properties, generalized vague soft α closed sets have a different closure and interior pattern. Hence, these two types of sets are incomparable in general.

To avoid repetition, the inclusion relationships between the different vague soft closed type sets introduced in the above subsections are summarized in Table 1. This table emphasizes that generalized vague soft α closed sets are an intermediate and proper class between vague soft closed sets and vague soft semi closed sets in vague soft topological spaces.

Table 1. Inclusion relationships among vague soft closed type sets

Type of Set	Relationship
Vague soft closed sets	$\subset$ Generalized vague soft α closed sets
Generalized vague soft α closed sets	$\subset$ Vague soft semi closed sets
Vague soft β closed sets	In general, incomparable with α closed sets
Semi closed sets	Strictly weaker than α closed sets

The table highlights that generalized vague soft α closed sets form an intermediate class between vague soft closed and semi closed sets, while remaining distinct from β closed sets.

6. Conclusion and Future Work

We define generalized vague soft α closed sets (GVS α closed sets) in soft generalized vague topological spaces. We generalize the classical α closed sets into a parameter based, dual valued framework, illustrating how vague soft topology can be used to generalize uncertainty models.

We introduced GVS α closed sets and their important properties. Examples identify them as a generalization of vague soft closed sets and distinct from other generalized closed sets such as semi closed and β closed sets. Our results proved that vague soft α closed sets are a proper subset of vague soft semi closed sets and a super set of vague soft closed sets with differences in contrast to β closed sets.

We investigated α continuity, a soft vague mapping which is α closed in inverse images. Preservation theorems for closure behavior under composition and continuity mappings were developed. The results generalize vague soft continuity and can be applied in topological modeling for decision systems, artificial intelligence, and data science.

Directions for future research on GVS α closed sets are:

- Current studies on α^* closed and γ closed imprecise soft sets;
- Developing vague soft connectedness and compactness based on α closedness;
- Investigation of separation axioms and boundary definitions of α closed behavior;
- Investigating fuzzy soft hybrid α closed models for multi layer uncertainty analysis;

Applications of the sets are decision making, image processing, and classification of knowledge. The framework facilitates fuzzy soft topology research and improves observation of double uncertainty modeling in mathematical systems.

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Appendix

Appendix A: Summary of Notations

Symbol	Description
U	Universal set of discourse
E	Set of parameters
$P(U)$	Power set of U
$F:E \rightarrow P(U)$	A soft set over U
$V(U)$	Collection of vague subsets of U
$F:E \rightarrow V(U)$	A vague soft set over U
$tF(e)(x)$	Truth membership degree
$fF(e)(x)$	False membership degree
$H(e)(x)$	Vague membership interval
$cl(F)$	Closure of vague soft set F
$int(F)$	Interior of vague soft set F
$comp(F)$	Complement of vague soft set F
$GVS\alpha$ closed	Generalized Vague Soft α Closed Set

Table A.1. Notations and their descriptions used in generalized vague soft topological spaces

Appendix B: Example Membership Interval Calculations

Given $F(e_1)(x_1)=[0.6,0.9]$, and using the closure operator:

- $cl(F)(e_1)(x_1)=[0.6,1.0]$
- $int(cl(F))(e_1)(x_1)=[0.6,0.95]$
- $cl(int(cl(F)))(e_1)(x_1)=[0.6,1.0]$

Comparison with the original set F helps determine if the inclusion condition $cl(int(cl(F))) \subseteq F$ holds.

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