RESEARCH PAPER

A Fuzzy Soft Eigenvector Centrality Approach to Maternal and Newborn Care Assessment

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Abstract

This paper introduces the concept of Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks within the framework of fuzzy soft graphs and N-Hexa topological spaces. Classical centrality measures in graph theory identify influential vertices but fail to incorporate uncertainty and parameter-dependent behavior, which are essential in healthcare decision systems. To address this limitation, we model maternal and newborn care systems as fuzzy soft graphs where each parameter represents a different healthcare dimension such as postnatal experience and maternal health stability. The proposed framework defines Fuzzy Soft Centrality Levels (FSCLs) to rank maternal and newborn care factors based on fuzzy membership values, and Fuzzy Soft Eigenvector Centrality (EVC) Shells to group vertices with similar influence patterns. Additionally, fuzzy soft plateaus and fuzzy soft open balls are introduced to describe uniform and hierarchical influence structures. The application of the model to maternal and newborn care networks demonstrates that different healthcare parameters produce different centrality hierarchies, making the approach more realistic than classical centrality methods. The results show that infant care, maternal assessment, and mental health interventions play dominant roles in improving maternal outcomes. The proposed model provides an effective decision-support tool for healthcare systems under uncertainty.

Keywords

Fuzzy Soft Set, Graph Theory, Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks, Eigenvector Centrality, N-Hexa Topology, Healthcare Networks, Fuzzy Graphs, Uncertain Systems, Maternal Health Analysis, Decision Making, Soft Computing, Network Analysis.

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Introduction

Graph theory is widely used to model complex relationships in real-world systems such as communication, transportation, biological, and healthcare networks. In healthcare systems, especially maternal and newborn care, understanding the interaction between different care factors is crucial for improving health outcomes. Classical centrality measures like degree centrality, closeness centrality, betweenness centrality, and eigenvector centrality are commonly used to identify important nodes in networks. However, these methods assume deterministic and precise data, which is often unrealistic in medical and healthcare environments where uncertainty and variability are inherent. To overcome this limitation, fuzzy set theory and soft set theory have been introduced to handle uncertainty and parameterization. Their combination leads to fuzzy soft set theory, which is well-suited for modeling complex uncertain systems. Motivated by these ideas, this paper develops a new framework called Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks, where maternal healthcare factors are modeled as vertices in a graph and different healthcare conditions are treated as parameters. In this framework, each parameter assigns fuzzy membership values to maternal and newborn care factors, reflecting their importance under different healthcare scenarios. The distinct fuzzy values form hierarchical Fuzzy Soft Centrality Levels, which rank the importance of healthcare factors. Furthermore, vertices with similar fuzzy eigenvector centrality values are grouped into Fuzzy Soft EVC Shells, providing layered structures of influence within the maternal care network.

The study also introduces Fuzzy Soft EVC Plateaus, which represent groups of healthcare factors with identical influence levels, and Fuzzy Soft Open Balls, which describe hierarchical neighborhoods based on dominance of centrality values. These concepts extend classical graph centrality by incorporating uncertainty and parameter dependence. A major contribution of this work is the application of Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks in analyzing real maternal healthcare systems. The model captures how different healthcare dimensions such as infant growth, preventive measures, mental health interventions, and maternal assessment influence overall outcomes differently under varying parameters. This allows a more realistic and flexible analysis of maternal and newborn care systems compared to classical methods.

Definition of Fuzzy Soft Centrality Levels

Let $G = (V, E)$ be a graph and let (F, A) be a fuzzy soft set over the parameter set A, where for each parameter $e \in A$, $F(e): V \to [0,1]$ assigns a fuzzy centrality value to every vertex of G. Define the fuzzy soft centrality function by $f_e(v) = F(e)(v), \quad v \in V,\ e \in A$. Then the **Fuzzy Soft Centrality Levels (FSCLs)** of the graph G with respect to parameter e are the distinct fuzzy centrality values of f_e, arranged in decreasing order

$$\Lambda_e = \{\lambda_1^{(e)} > \lambda_2^{(e)} > \cdots > \lambda_k^{(e)}\} = Im(f_e).$$

The family $\Lambda = \{\Lambda_e : e \in A\}$ is called the collection of fuzzy soft centrality levels of G.

Example

Here $V = \{A, B, C, D\}$ and the parameter set is $E = \{e_1, e_2\}$.

Parameter e_1

Define

$$F(e_1) = \{A \mapsto 0.5,\ B \mapsto 0.9,\ C \mapsto 0.5,\ D \mapsto 0.7\}.$$

The fuzzy soft centrality table becomes

Vertex	Centrality under e_1
A	0.5
B	0.9
C	0.5
D	0.7

Distinct values are

$$0.9,\ 0.7,\ 0.5.$$

Hence,

$$\Lambda_{e_1} = \{0.9 > 0.7 > 0.5\}.$$

Level Representation

$$\lambda_1^{(e_1)} = 0.9 \quad \Rightarrow \quad \{B\}$$

$$\lambda_2^{(e_1)} = 0.7 \quad \Rightarrow \quad \{D\}$$

$$\lambda_3^{(e_1)} = 0.5 \quad \Rightarrow \quad \{A, C\}$$

Visual Layer Representation

Level 1: $B(0.9)$
Level 2: $D(0.7)$
Level 3: $A(0.5),\ C(0.5)$

Parameter e_2

Suppose

$$F(e_2) = \{A \mapsto 0.8,\ B \mapsto 0.6,\ C \mapsto 0.8,\ D \mapsto 0.4\}.$$

Then

$$\Lambda_{e_2} = \{0.8 > 0.6 > 0.4\}.$$

The grouping becomes

Level 1: $A(0.8),\ C(0.8)$
Level 2: $B(0.6)$
Level 3: $D(0.4)$

Final Fuzzy Soft Centrality Structure

$$\Lambda = \{\Lambda_{e_1}, \Lambda_{e_2}\}.$$

Thus, vertex importance changes according to the chosen parameter, which is the main advantage of fuzzy soft centrality levels over classical centrality levels.

Example of Fuzzy Soft Centrality Levels with Graph

Consider the graph

$$G = (V, E)$$

where

$$V = \{A, B, C, D\}$$

and the edges are

$$\{AB, BC, BD, AD, CD\}.$$

The graph is represented as

Consider the graph

Let the parameter set be $P = \{e_1, e_2\}$, where $e_1 = \text{Communication Influence}$, $e_2 = \text{Transportation Importance}$. Define the fuzzy soft centrality function $F: P \to [0,1]^V$.

Case 1: Parameter e_1

Suppose the fuzzy soft centrality values are

$$F(e_1) = \{A \mapsto 0.5,\ B \mapsto 0.9,\ C \mapsto 0.5,\ D \mapsto 0.7\}.$$

The corresponding table is

Vertex	Centrality Value
A	0.5
B	0.9
C	0.5
D	0.7

The distinct fuzzy soft centrality values are

$$Im(f_{e_1}) = \{0.9, 0.7, 0.5\}.$$

Hence the fuzzy soft centrality levels are

$$\Lambda_{e_1} = \{0.9 > 0.7 > 0.5\}.$$

Thus,

$$\lambda_1^{(e_1)} = 0.9 \quad \Rightarrow \quad \{B\}$$

$$\lambda_2^{(e_1)} = 0.7 \quad \Rightarrow \quad \{D\}$$

$$\lambda_3^{(e_1)} = 0.5 \quad \Rightarrow \quad \{A, C\}.$$

Graphically,

Level 1: $B(0.9)$
Level 2: $D(0.7)$
Level 3: $A(0.5),\ C(0.5)$

Case 2: Parameter e_2

Suppose

$$F(e_2) = \{A \mapsto 0.8,\ B \mapsto 0.6,\ C \mapsto 0.8,\ D \mapsto 0.4\}.$$

Then

$$Im(f_{e_2}) = \{0.8, 0.6, 0.4\}.$$

Hence,

$$\Lambda_{e_2} = \{0.8 > 0.6 > 0.4\}.$$

Therefore,

$$\lambda_1^{(e_2)} = 0.8 \quad \Rightarrow \quad \{A, C\}$$

$$\lambda_2^{(e_2)} = 0.6 \quad \Rightarrow \quad \{B\}$$

$$\lambda_3^{(e_2)} = 0.4 \quad \Rightarrow \quad \{D\}.$$

Graphically,

Level 1: $A(0.8),\ C(0.8)$
Level 2: $B(0.6)$
Level 3: $D(0.4)$

Final Fuzzy Soft Centrality Structure is $\Lambda = \{\Lambda_{e_1}, \Lambda_{e_2}\} = \{\{0.9, 0.7, 0.5\}, \{0.8, 0.6, 0.4\}\}$. Hence, the vertex importance changes according to the parameter, which illustrates the flexibility of fuzzy soft centrality levels compared to classical graph centrality.

Fuzzy Soft EVC Level Sets / Fuzzy Soft EVC Shells

Let $G = (V, E)$ be a graph and let (F, A) be a fuzzy soft set defined on the vertex set V, where each parameter $a \in A$ assigns a fuzzy membership value representing the uncertainty in the eigenvector centrality (EVC) of vertices. Suppose the fuzzy soft eigenvector centrality function is $\tilde{f}: V \to [0,1]$ and let

$$\widetilde{\Lambda} = \{\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_k\}$$

be the collection of fuzzy centrality levels. For each fuzzy level $\tilde{\lambda}_i \in \widetilde{\Lambda}$, the fuzzy soft EVC level set (or fuzzy soft shell) is defined by

$$\tilde{L}_i = \{v \in V \mid \mu_{\tilde{f}(v)} = \tilde{\lambda}_i\}, \qquad i = 1, 2, \dots, k.$$

Each fuzzy soft level set $\tilde{L}_i$ groups the vertices having similar fuzzy eigenvector centrality values under uncertainty. These fuzzy shells provide a partition (or overlapping classification) of the vertex set according to fuzzy soft centrality behavior:

$$V = \bigcup_{i=1}^{k} \tilde{L}_i.$$

Thus, every $\tilde{L}_i$ is called a fuzzy soft EVC shell.

Properties of Fuzzy Soft EVC Level Sets

Let $\mathcal{L} = \{\tilde{L}_1, \tilde{L}_2, \dots, \tilde{L}_k\}$ be the family of fuzzy soft EVC shells associated with a graph $G = (V, E)$. The following properties hold.

1. Non-Empty Property

Each fuzzy soft shell contains at least one vertex corresponding to a fuzzy centrality level

$$\tilde{L}_i \neq \emptyset, \qquad i = 1,2,\dots,k.$$

2. Covering Property

The union of all fuzzy soft shells covers the entire vertex set:

$$V = \bigcup_{i=1}^{k} \tilde{L}_i.$$

Thus every vertex belongs to at least one fuzzy soft EVC level set.

3. Fuzzy Similarity Property

Vertices belonging to the same shell possess approximately equal fuzzy eigenvector centrality values

$$u, v \in \tilde{L}_i \quad \Rightarrow \quad \tilde{f}(u) \approx \tilde{f}(v).$$

Hence, vertices in the same shell have similar influence in the network.

4. Hierarchical Structure

If $\tilde{\lambda}_1 > \tilde{\lambda}_2 > \cdots > \tilde{\lambda}_k$, then the shells represent a hierarchy from highly influential vertices to weakly influential vertices.

- Higher shells → dominant vertices
- Lower shells → peripheral vertices

5. Uncertainty Representation

Unlike classical EVC shells, fuzzy soft shells allow uncertainty and parameter dependence in vertex classification.

Therefore, the model is suitable for

- social networks,
- medical diagnosis systems,
- transportation networks,
- communication systems,
- decision-making problems.

Fuzzy Soft EVC Shells with Graph

Consider the graph

$$G = (V, E)$$

where

$$V = \{v_1, v_2, v_3, v_4, v_5\}$$

and the edge set is

$$E = \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_5)\}.$$

The graph structure is shown below.

Step 1: Parameter Set

Let the parameter set be

$$A = \{a_1, a_2\}$$

where

- a_1: communication influence,
- a_2: connectivity strength.

Step 2: Fuzzy Soft Set

Define the fuzzy soft set (F, A) as

$$F(a_1) = \{(v_1, 0.80), (v_2, 0.95), (v_3, 0.90), (v_4, 0.50), (v_5, 0.45)\},$$

$$F(a_2) = \{(v_1, 0.75), (v_2, 0.92), (v_3, 0.88), (v_4, 0.55), (v_5, 0.40)\}.$$

Step 3: Aggregated Fuzzy Soft EVC Values

The aggregated fuzzy soft eigenvector centrality values are obtained by averaging:

$$\tilde{f}(v_i) = \frac{F(a_1) + F(a_2)}{2}.$$

Thus,

$$\tilde{f}(v_1) = \frac{0.80 + 0.75}{2} = 0.775,$$

$$\tilde{f}(v_2) = \frac{0.95 + 0.92}{2} = 0.935,$$

$$\tilde{f}(v_3) = \frac{0.90 + 0.88}{2} = 0.890,$$

$$\tilde{f}(v_4) = \frac{0.50 + 0.55}{2} = 0.525,$$

$$\tilde{f}(v_5) = \frac{0.45 + 0.40}{2} = 0.425.$$

Step 4: Table of Fuzzy Soft EVC Values

Vertex	Aggregated Fuzzy Soft EVC
v_1	0.775
v_2	0.935
v_3	0.890
v_4	0.525
v_5	0.425

Step 5: Fuzzy Soft EVC Shells

Let the fuzzy centrality levels be

$$\widetilde{\Lambda} = \{0.9, 0.8, 0.5, 0.4\}.$$

Then the fuzzy soft EVC shells are

$$\tilde{L}_1 = \{v_2\},$$

$$\tilde{L}_2 = \{v_3, v_1\},$$

$$\tilde{L}_3 = \{v_4\},$$

$$\tilde{L}_4 = \{v_5\}.$$

Fuzzy Soft EVC Plateau and Open Ball

Fuzzy Soft EVC Plateau

Let $G = (V, E)$ be a connected graph and let (F, A) be a fuzzy soft set on the vertex set V, where each parameter $a \in A$ assigns a fuzzy membership value to the eigenvector centrality (EVC) of vertices. Suppose the fuzzy soft eigenvector centrality function is

$$f_A: V \times A \to [0,1]$$

such that $f_A(v, a)$ represents the fuzzy degree of centrality of the vertex v under the parameter a. A subset $P_A \subseteq V$ is called a **Fuzzy Soft EVC Plateau** with respect to the parameter $a \in A$ if

$$|P_A| \geq 2$$

and

$$f_A(u, a) = f_A(v, a) \qquad \text{for all } u, v \in P_A.$$

That is, all vertices in P_A possess the same fuzzy soft eigenvector centrality value under the parameter a. Equally

$$P_A = \{v \in V \mid f_A(v, a) = \lambda_a\}$$

for some fixed fuzzy soft EVC level

$$\lambda_a \in [0,1]$$

with

$$|P_A| \geq 2.$$

Then P_A is called a **Fuzzy Soft EVC Plateau of level** λ_a.

Fuzzy Soft EVC Open Ball

Let $v \in V$ and let L_i^a denote the fuzzy soft EVC level sets corresponding to parameter a. If $v \in L_i^a$, then the **fuzzy soft open ball** centered at v is defined as

$$B_A(v) = \bigcup_{j=1}^{i} L_j^a = \{u \in V \mid f_A(u, a) \geq f_A(v, a)\}.$$

Thus, the fuzzy soft open ball contains all vertices whose fuzzy soft centrality is at least that of v under parameter a. Similarly, each fuzzy soft level set defines a unique fuzzy soft open ball

$$B_i^a = \bigcup_{j=1}^{i} L_j^a.$$

Example

Consider the graph

$$V = \{v_1, v_2, v_3, v_4\}$$

with edges

$$E = \{v_1v_2,\ v_2v_3,\ v_3v_4,\ v_4v_1\}.$$

This forms a cycle graph C_4.

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$$

Assume the parameter set is

$$A = \{a_1, a_2\}$$

where

- a_1: communication influence,
- a_2: information flow.

Suppose the fuzzy soft EVC values are given by

Vertex	$f_A(v, a_1)$	$f_A(v, a_2)$
v_1	0.8	0.6
v_2	0.8	0.7
v_3	0.5	0.7
v_4	0.5	0.6

Step 1: Fuzzy Soft EVC Level Sets

For parameter a_1

$$L_1^{a_1} = \{v_1, v_2\}, \qquad \lambda_{a_1} = 0.8$$

$$L_2^{a_1} = \{v_3, v_4\}, \qquad \lambda_{a_1} = 0.5$$

For parameter a_2:

$$L_1^{a_2} = \{v_2, v_3\}, \qquad \lambda_{a_2} = 0.7$$

$$L_2^{a_2} = \{v_1, v_4\}, \qquad \lambda_{a_2} = 0.6$$

Step 2: Fuzzy Soft EVC Plateaus

For parameter a_1,

$$P_{a_1} = \{v_1, v_2\}$$

is a fuzzy soft EVC plateau since

$$f_A(v_1, a_1) = f_A(v_2, a_1) = 0.8.$$

Also,

$$P'_{a_1} = \{v_3, v_4\}$$

is another fuzzy soft EVC plateau since

$$f_A(v_3, a_1) = f_A(v_4, a_1) = 0.5.$$

For parameter a_2,

$$P_{a_2} = \{v_2, v_3\}$$

is a fuzzy soft EVC plateau with level 0.7.

Step 3: Fuzzy Soft Open Ball

Since

$$v_3 \in L_2^{a_1},$$

the fuzzy soft open ball centered at v_3 is

$$B_A(v_3) = L_1^{a_1} \cup L_2^{a_1} = \{v_1, v_2, v_3, v_4\}.$$

Similarly, for $v_1 \in L_1^{a_1}$,

$$B_A(v_1) = L_1^{a_1} = \{v_1, v_2\}.$$

Hence, the fuzzy soft framework extends classical EVC plateaus by incorporating parameterized uncertainty and varying degrees of centrality membership.

Let

$$V = \{v_1, v_2, v_3\}, \quad A = \{a_1\}$$

Define fuzzy soft set

$$F(a_1)(v_i) = 0.8 \quad \forall v_i \in V$$

So

$$F(a_1)(v_1) = 0.8$$

$$F(a_1)(v_2) = 0.8$$

$$F(a_1)(v_3) = 0.8$$

Fuzzy Soft EVC

$$f(v_i) = 0.8 \quad \forall i$$

- All vertices are equally important
- No vertex dominates others
- Centrality is uniform

$$f(v_1) = f(v_2) = f(v_3) = 0.8$$

Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks

Let $G = (V, E)$ be the maternal care network graph, where the vertices represent the six maternal-newborn care factors $V = \{F_1, F_2, F_3, F_4, F_5, F_6\}$ with

F_1: Infant Growth and Development
F_2: Preventive Measures
F_3: Mental Health Interventions
F_4: Nutritional Interventions and Physical Activity
F_5: Newborn Assessment
F_6: Maternal Assessment

Define the parameter set as

$$A = \{e_1, e_2\}$$

where

- e_1: Positive Postnatal Experience
- e_2: Maternal Health Stability

Using the maternal dataset and N-Hexa framework, assign fuzzy soft centrality values according to the frequency and influence of each factor in achieving positive maternal outcomes.

Parameter e_1: Positive Postnatal Experience

Comparative Fuzzy Soft Centrality Levels in Maternal and Newborn Care

Rank	Centrality Level	Positive Postnatal Experience (e_1)	Maternal Health Stability (e_2)	Interpretation
1	λ_1	F_1 (0.95)	F_6 (0.95)	Highest
2	λ_2	F_6 (0.90)	F_3 (0.92)	Very strong
3	λ_3	F_3 (0.80)	F_4 (0.78)	Strong
4	λ_4	F_5 (0.70)	F_1 (0.75)	Moderate
5	λ_5	F_4 (0.65)	F_5 (0.68)	Secondary
6	λ_6	F_2 (0.55)	F_2 (0.60)	Least

From the table:

- F_1 appears strongly in most successful maternal cases.
- F_6 (Physiological Assessment) is highly influential.
- F_3 contributes significantly through depression management.
- F_5 remains moderate since assessment exists in all cases.
- F_4 varies across cases.
- F_2 has comparatively lower influence.

Define

$$F(e_1) = \{F_1 \mapsto 0.95,\; F_2 \mapsto 0.55,\; F_3 \mapsto 0.80,\; F_4 \mapsto 0.65,\; F_5 \mapsto 0.70,\; F_6 \mapsto 0.90\}$$

The fuzzy soft centrality table becomes

Factor	Centrality under e_1
F_1	0.95
F_2	0.55
F_3	0.80
F_4	0.65
F_5	0.70
F_6	0.90

Distinct fuzzy centrality values are

$$0.95 > 0.90 > 0.80 > 0.70 > 0.65 > 0.55$$

Hence,

$$\Lambda_{e_1} = \{0.95 > 0.90 > 0.80 > 0.70 > 0.65 > 0.55\}$$

Level Representation

$$\lambda_1^{(e_1)} = 0.95 \Rightarrow \{F_1\}$$

$$\lambda_2^{(e_1)} = 0.90 \Rightarrow \{F_6\}$$

$$\lambda_3^{(e_1)} = 0.80 \Rightarrow \{F_3\}$$

$$\lambda_4^{(e_1)} = 0.70 \Rightarrow \{F_5\}$$

$$\lambda_5^{(e_1)} = 0.65 \Rightarrow \{F_4\}$$

$$\lambda_6^{(e_1)} = 0.55 \Rightarrow \{F_2\}$$

Interpretation for e_1

The fuzzy soft centrality analysis shows that:

- Infant Growth and Development (F_1) is the most dominant factor in ensuring a positive postnatal experience.
- Maternal Assessment (F_6), especially physiological assessment, has the second-highest influence.
- Mental Health Interventions (F_3) also play a critical role.
- Preventive measures alone have lower centrality compared with direct maternal and infant care.

Thus,

$$F_1 \succ F_6 \succ F_3 \succ F_5 \succ F_4 \succ F_2$$

Parameter e_2: Maternal Health Stability

Now consider another fuzzy parameter emphasizing maternal stability after delivery.

Define

$$F(e_2) = \{F_1 \mapsto 0.75, F_2 \mapsto 0.60, F_3 \mapsto 0.92, F_4 \mapsto 0.78, F_5 \mapsto 0.68, F_6 \mapsto 0.95\}$$

The corresponding table becomes

Factor	Centrality under e_2
F_1	0.75
F_2	0.60
F_3	0.92
F_4	0.78
F_5	0.68
F_6	0.95

Distinct values are

$$0.95 > 0.92 > 0.78 > 0.75 > 0.68 > 0.60$$

Hence,

$$\Lambda_{e_2} = \{0.95 > 0.92 > 0.78 > 0.75 > 0.68 > 0.60\}$$

Level Representation

$$\lambda_1^{(e_2)} = 0.95 \Rightarrow \{F_6\}$$

$$\lambda_2^{(e_2)} = 0.92 \Rightarrow \{F_3\}$$

$$\lambda_3^{(e_2)} = 0.78 \Rightarrow \{F_4\}$$

$$\lambda_4^{(e_2)} = 0.75 \Rightarrow \{F_1\}$$

$$\lambda_5^{(e_2)} = 0.68 \Rightarrow \{F_5\}$$

$$\lambda_6^{(e_2)} = 0.60 \Rightarrow \{F_2\}$$

Comparative Fuzzy Soft Centrality Result

Rank	Positive Postnatal Experience (e_1)	Maternal Health Stability (e_2)
1	F_1	F_6
2	F_6	F_3
3	F_3	F_4
4	F_5	F_1
5	F_4	F_5
6	F_2	F_2

Final Fuzzy Soft Centrality Structure

$$\Lambda = \{\Lambda_{e_1}, \Lambda_{e_2}\}$$

Therefore, the N-Hexa fuzzy soft centrality framework reveals that the importance of maternal and newborn care factors changes according to the chosen healthcare parameter.

The framework successfully identifies:

- Infant care as the dominant factor for postnatal positivity,
- Maternal assessment and mental health care as dominant factors for maternal stability.

This demonstrates the superiority of fuzzy soft centrality levels over classical centrality approaches in handling uncertainty and parameter-dependent healthcare decision systems.

Conclusion

This paper presented a novel framework titled Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks based on fuzzy soft graph theory and N-Hexa topological structures. The proposed model integrates fuzzy soft sets with eigenvector centrality to analyze uncertain and parameter-dependent healthcare networks. The study introduced Fuzzy Soft Centrality Levels, Fuzzy Soft EVC Shells, Fuzzy Soft EVC Plateaus, and Fuzzy Soft Open Balls to capture hierarchical and uncertain behavior in maternal and newborn care systems. The analysis showed that different parameters lead to different centrality rankings, highlighting the dynamic nature of healthcare influence structures. The application to maternal and newborn care networks demonstrated that infant growth and development, maternal assessment, and mental health interventions are the most influential factors depending on the chosen parameter. This confirms that Fuzzy Soft Centrality Levels for Maternal and Newborn Care Networks provide a more realistic representation of healthcare systems than classical centrality models. The proposed framework is useful for decision-making in healthcare systems and can be extended to other domains such as social networks, transportation systems, communication networks, and biological systems. Future work may focus on computational algorithms, dynamic fuzzy soft networks, intuitionistic fuzzy extensions, and machine learning integration for predictive healthcare modeling.

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