RESEARCH PAPER

Regular Spaces Associated With n-Cylindrical Neutrosophic M-Open Sets and an Application to Safety-Separation in Drug Delivery Technology Screening

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Abstract: In this paper, we introduce some new spaces called n-CyN (resp. n-$CyN\delta$, n-$CyN\delta\mathcal{P}$, n-$CyN\delta\mathcal{S}$, n-$CyN\theta$, n-$CyN\theta\mathcal{S}$, n-$CyNe$, and n-$CyNM$) regular spaces and strongly n-CyN (resp. n-$CyN\delta$, n-$CyN\delta\mathcal{P}$, n-$CyN\delta\mathcal{S}$, n-$CyN\theta$, n-$CyN\theta\mathcal{S}$, n-$CyNe$, and n-$CyNM$) regular spaces with the help of n-CyN (resp. n-$CyN\delta$, n-$CyN\delta\mathcal{P}$, n-$CyN\delta\mathcal{S}$, n-$CyN\theta$, n-$CyN\theta\mathcal{S}$, n-$CyNe$ and n-$CyNM$) open sets in n-cylindrical neutrosophic topological spaces. Also, find their relations among themselves and with already existing spaces. Also, we study some basic properties and the characterizations of these regular spaces. As a real-life application, the n-Cylindrical neutrosophic M-regularity axiom is used to build a safety-separation model for drug delivery technology screening, in which a candidate technology is formally isolated, by a pair of disjoint n-Cylindrical neutrosophic M-open evidence pools, from an M-closed set of technologies rejected on safety grounds.

Keywords: n-cylindrical neutrosophic M-open sets, n-cylindrical neutrosophic M-closed sets, n-cylindrical neutrosophic M regular spaces, strongly n-cylindrical neutrosophic M regular spaces, n-Cylindrical Neutrosophic M q-neighbourhood, Drug delivery technology screening, Real-life application.

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1 Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [24], Chang [6] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy sets (ifs), was introduced by Atanassov in 1986 [4]. Building on this, Coker [7] introduced the concept of intuitionistic fuzzy topological spaces ($ifts$) based on ifs. Jeon et al. [9] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [17, 16], a new direction in uncertainty modeling emerged. Salama and Alblowi [12] introduced neutrosophic crisp sets and neutrosophic topological spaces (Nts), extending $ifts$ and incorporating degrees of membership, indeterminacy, and non-membership for each element. Neutrosophic has since formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [3] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [22] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [13] introduced the concept of n-cylindrical neutrosophic sets (abbreviated as n-$CyNS$), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic sets (ns), n-$CyNs$ represents the most extensive generalization of fuzzy sets (fs). In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \leq \beta_A \leq 1$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1, n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [15] introduced the notion of n-CyN continuity for functions between two n-cylindrical fuzzy neutrosophic topological spaces (n-$CyNts$). They also defined the n-CyN interior (n-$CyNint$) and n-CyN closure (n-$CyNcl$)

of subsets within $n\text{-}CyNts$.

In a related development, El-Maghrabi and Al-Juhani [8] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [11] also found M-open sets in nano topological spaces. Vadivel et al. [18, 19, 20] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [10] and Vadivel et al. [21] introduced M-open sets in fuzzy and neutrosophic nano topological spaces.

The subsequent sections of this dissertation are organized as follows: Section 2 provides a brief review of fundamental definitions related to intuitionistic fuzzy sets (ifs's), neutrosophic sets (NS's), n-cylindrical neutrosophic sets ($n\text{-}CyNS$'s), and mappings on n-cylindrical neutrosophic topological spaces ($n\text{-}CyNts$). Sections 3 and 4 introduce the concepts of $n\text{-}CyNM$ regular spaces and strongly $n\text{-}CyNM$ regular spaces within $n\text{-}CyNts$, along with an exploration of their fundamental properties. Section 5 presents a real-life application of the $n\text{-}CyNM$ regularity axiom to a safety-separation model for drug delivery technology screening. The paper is concluded in section 6.

2 Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 [26] A fuzzy set (briefly, fs) A in X is defined by a membership function $\mu_A: X \to [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ belongs to the fuzzy set A, for all $x \in X$.

Definition 2.2 [6] A fuzzy topological space (briefly, fts) is a pair (X, τ), where X is any set and τ is a family of fuzzy sets in X satisfying the following axioms:

1. $\phi, X \in \tau$,
2. If $A, B \in \tau$, then $A \cap B \in \tau$,
3. If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 [4] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\alpha_A(x)$ and $\gamma_A(x)$ satisfy (for all $x \in X$) $\alpha_A(x) + \gamma_A(x) \le 1$. ifs(X) denotes the set of all the ifs's on X.

Definition 2.4 [17] A neutrosophic set A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1]$, $0 \le \alpha_A(x) + \beta_A(x) + \gamma_A(x) \le 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral membership, and $\gamma_A(x)$ is the degree of negative membership. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.

Definition 2.5 [12] A neutrosophic topology (Nt) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\cup G_i \in \tau_N$, for all $\{G_i : i \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called a neutrosophic topological space (briefly, Nts) and any neutrosophic set in τ_N is known as a neutrosophic open set (briefly, Nos) in X. The elements of τ_N are called neutrosophic open sets. A neutrosophic set F is closed if and only if $C(F)$ is neutrosophic open.

Definition 2.6 [13] An n-cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ is called the degree of neutral membership of x in A, and $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, which satisfies the condition: for all $x \in X$, $0 \le \beta_A(x) \le 1$ and $0 \le \alpha_A^n(x) + \gamma_A^n(x) \le 1$, where $n > 1$ is an integer. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components and $\beta_A(x)$ is 100% independent.

For convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called an n-cylindrical neutrosophic number (briefly, n - CyNN) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 [13] Let $\{A_i : i \in I\}$ be an arbitrary family of n-CyNS in X. Then, $\cap A_i = \{\langle x, \inf(\alpha_{A_i}(x)), \inf(\beta_{A_i}(x)), \sup(\gamma_{A_i}(x))\rangle : x \in X\}$, and $\cup A_i = \{\langle x, \sup(\alpha_{A_i}(x)), \sup(\beta_{A_i}(x)), \inf(\gamma_{A_i}(x))\rangle : x \in X\}$.

Definition 2.8 [13] $0_{CyN} = \{\langle x, 0,0,1\rangle : x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle : x \in X\}$.

Definition 2.9 [13] **(The Basic Connectives)** Let $\tau_{CyN}(X)$ denote the family of all n-CyNS's on X.

1. **Inclusion:** For every two $A, B \in \tau_{CyN}(X)$, the inclusion of two n-CyNS's A and B is $A \subseteq B$ iff, for all $x \in X$, $\alpha_A(x) \le \alpha_B(x)$, $\beta_A(x) \le \beta_B(x)$, and $\gamma_A(x) \ge \gamma_B(x)$.
2. **Union:** For every two $A, B \in \tau_{CyN}(X)$, the union of two n-CyNS's A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x),$

$\alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.

3. **Intersection:** For every two $A, B \in \tau_{CyN}(X)$, the intersection of two n-CyNS's A and B is

$$A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}.$$

4. **Complementary:** For every $A \in \tau_{CyN}(X)$, the complement of an n-CyNS A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Remark 2.1 [13]

1. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
2. $A \cup B = B \cup A$ & $A \cap B = B \cap A$,
3. $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$,
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$,
5. $A \cap A = A$ & $A \cup A = A$,
6. De Morgan's Law for A & B, i.e., $(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$.

Definition 2.10 [15] An n-cylindrical neutrosophic topology (briefly, n-CyNt) on a non-empty set X is a family, τ_{CyN}, of n-CyNS's in X which satisfies the following conditions:

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. For every $A_1, A_2 \in \tau_{CyN}$, $A_1 \cap A_2 \in \tau_{CyN}$,
3. $\cup A_i \in \tau_{CyN}$, for any arbitrary family $A_i \in \tau_{CyN}, i \in I$.

The pair (X, τ_{CyN}) is called an n-cylindrical neutrosophic topological space (briefly, n-CyNts), and any n-CyNS belonging to τ_{CyN} is called an n-cylindrical neutrosophic open set (briefly, n-CyNos); the complement of a n-CyNos is called a n-cylindrical neutrosophic closed set (briefly, n-CyNcs) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called the indiscrete n-CyNts and the topology containing all the n-CyN subsets is called the discrete n-CyNts.

Remark 2.2 [15] Obviously any fuzzy topological space, intuitionistic fuzzy topological space, or Pythagorean fuzzy topological space is an n-CyNts, since subsets of the fuzzy space, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.

Definition 2.11 [15] Let A and B be two n-cylindrical neutrosophic subsets of an n-CyNts. B is called a neighbourhood of A if there exists an n-CyNos O such that $A \subseteq O \subseteq B$.

Proposition 2.1 [15] A is n-cylindrical neutrosophic open in (X, τ_{CyN}) if and only if A is a neighbourhood of itself.

Definition 2.12 [15] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic interior (briefly, n-CyNint) is defined as the n-CyN union of all n-CyN open subsets of A, i.e., $\text{n-CyNint}(A) = \bigcup \{G : G \in \tau_{CyN} \text{ and } G \subseteq A\}$.

Clearly, n-CyNint(A) is the largest n-CyNos contained in A.

Definition 2.13 [15] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic closure (briefly, n-CyNcl) is defined as the n-CyN intersection of all n-CyN closed supersets of A, i.e., $\text{n-CyNcl}(A) = \bigcap \{K : K \text{ is an } \text{n-CyNcs in } X \text{ and } A \subseteq K\}$.

Clearly, n-CyNcl(A) is the smallest n-CyNcs that contains A.

Definition 2.14 [14] Let (X, τ_1) and (Y, τ_2) be two n-CyNts and let $f : (X, \tau_1) \to (Y, \tau_2)$ be a n-CyN function. Then, f is said to be a n-CyN continuous (briefly, n-CyNCts) map if for any n-cylindrical neutrosophic subset A of X and for any neighbourhood $\mathfrak{V}$ of $f(A)$ there exists a neighbourhood $\mathfrak{U}$ of A such that $f(\mathfrak{U}) \subseteq \mathfrak{V}$.

Definition 2.15 [1] Let (X, τ_{CyN}) be an n-CyNts and A be an n-CyNS. Then, A is said to be an n-CyN

1. regular open set (briefly, n-CyNros), if $A = \text{n-CyNint}(\text{n-CyNcl}(A))$,
2. regular closed set (briefly, n-CyNrcs), if $A = \text{n-CyNcl}(\text{n-CyNint}(A))$.

Definition 2.16 [1] Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic δ-interior of A and the n-cylindrical neutrosophic δ-closure of A, denoted by n-CyNδint(A) and n-CyNδcl(A), are defined as follows:

1. $\text{n-CyN}\delta\text{int}(A) = \bigcup \{G \mid G \text{ is an n-CyNros and } G \subseteq A\}$,
2. $\text{n-CyN}\delta\text{cl}(A) = \bigcap \{K \mid K \text{ is an n-CyNrcs and } A \subseteq K\}$.

Definition 2.17 [1] Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic θ-interior of A and the n-cylindrical neutrosophic θ-closure of A, denoted by n-CyNθint(A) and n-CyNθcl(A), are defined as follows:

1. $\text{n-CyN}\theta\text{int}(A) = \bigcup \{\text{n-CyNint}(B) : B \subseteq A \text{ \& } B \text{ is a n-CyNcs in } X\}$,
2. $\text{n-CyN}\theta\text{cl}(A) = \bigcap \{\text{n-CyNcl}(B) : A \subseteq B \text{ \& } B \text{ is a n-CyNos in } X\}$.

Definition 2.18 [1] Let (X, τ_{CyN}) be an n - CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. A set A is said to be n-CyN

1. δ-open set (briefly, n-CyNδos), if $A = $ n-CyNδint(A),
2. δ -pre open set (briefly, n-CyNδ$\mathcal{P}$os), if $A \subseteq$ n-CyNint(n-CyNδcl(A)),
3. δ -semi open set (briefly, n-CyNδ$\mathcal{S}$os), if $A \subseteq$ n-CyNcl(n-CyNδint(A)),
4. θ-open set (briefly, n-CyNθos), if $A = $ n-CyNθint(A),
5. θ -semi open set (briefly, n-CyNθ$\mathcal{S}$os), if $A \subseteq$ n-CyNcl(n-CyNθint(A)),
6. e-open set (briefly, n-CyNeos), if $A \subseteq$ n - CyNcl(n - CyNδint(A)) ∪ n-CyNint(n-CyNδcl(A)),
7. M-open set (briefly, n-CyNMos), if $A \subseteq$ n - CyNcl(n - CyNθint(A)) ∪ n-CyNint(n-CyNδcl(A)).

The complement of a n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNθos, n-CyNθ$\mathcal{S}$os & n-CyNeos) is called a n-CyNM (resp. n-CyNδ, n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNθ, n-CyNθ$\mathcal{S}$ & n-CyNe) closed set (briefly, n-CyNMcs (resp. n - CyNδcs, n - CyNδ$\mathcal{P}$cs, n - CyNδ$\mathcal{S}$cs, n-CyNθcs, n-CyNθ$\mathcal{S}$cs & n-CyNecs)) in X.

The family of all n - CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNθos, n-CyNθ$\mathcal{S}$os & n-CyNeos) of X is denoted by n - CyNMOS(X), (resp. n - CyNMCS(X), n-CyNδOS(X), n-CyNδCS(X), n-CyNδ$\mathcal{P}$OS(X), n-CyNδ$\mathcal{P}$CS(X), n-CyNδ$\mathcal{S}$OS(X), n-CyNδ$\mathcal{S}$CS(X), n-CyNθOS(X), n-CyNθCS(X), n-CyNθ$\mathcal{S}$OS(X), n-CyNθ$\mathcal{S}$CS(X), n-CyNeOS(X) & n-CyNeCS(X)).

Definition 2.19 [1] Let (X, τ_{CyN}) be an n - CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-CyN

1. M-interior (resp. n-CyNδ-interior, n - CyNδ$\mathcal{P}$ -interior, n - CyNδ$\mathcal{S}$ -interior, n - CyNθ -interior, n - CyNθ$\mathcal{S}$ -interior & n-CyNe-interior) of A (briefly, n-CyNMint(A) (resp. n - CyNδint(A), n - CyNδ$\mathcal{P}$int(A), n-CyNδ$\mathcal{S}$int(A), n-CyNθ int(A), n-CyNθ$\mathcal{S}$int(A) & n-CyNeint(A)) is defined by n-CyNMint(A) (resp. n-CyNδint(A), n-CyNδ$\mathcal{P}$int(A), n-CyNδ$\mathcal{S}$int(A), n-CyNθint(A), n-CyNθ$\mathcal{S}$int(A) & n-CyNeint(A)) $= \bigcup$ {G: G ⊆ A and G is a n-CyNMos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNθos, n-CyNθ$\mathcal{S}$os & n-CyNeos) in X},
2. M-closure (resp. n-CyNδ-closure, n - CyNδ$\mathcal{P}$ -closure, n - CyNδ$\mathcal{S}$ -closure, n - CyNθ -closure, n - CyNθ$\mathcal{S}$ -closure & n-CyNe-closure) of A (briefly, n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδ$\mathcal{P}$cl(A), n-CyNδ$\mathcal{S}$cl(A), n-CyNθcl(A), n-CyNθ$\mathcal{S}$cl(A) & n-CyNecl(A)) is defined by n - CyNMcl(A) (resp. n - CyNδcl(A), n - CyNδ$\mathcal{P}$cl(A), n - CyNδ$\mathcal{S}$cl(A), n - CyNθcl(A), n-CyNθ$\mathcal{S}$cl(A) & n-CyNecl(A)) $= \bigcap$ {K: A ⊆ K and K is a n - CyNMcs (resp. n - CyNδcs, n-CyNδ$\mathcal{P}$cs, n-CyNδ$\mathcal{S}$cs, n-CyNθcs, n-CyNθ$\mathcal{S}$cs & n-CyNecs) in X}.

Definition 2.20 [2] Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a n-CyN

1. δ -continuous map (briefly, n-CyNδCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδos in (X, τ_1),
2. δ -pre continuous map (briefly, n-CyNδ$\mathcal{P}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδ$\mathcal{P}$os in (X, τ_1),
3. δ -semi continuous map (briefly, n-CyNδ$\mathcal{S}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNδ$\mathcal{S}$os in (X, τ_1),
4. θ -continuous map (briefly, n-CyNθCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNθos in (X, τ_1),
5. θ -semi continuous map (briefly, n-CyNθ$\mathcal{S}$Cts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNθ$\mathcal{S}$os in (X, τ_1),
6. e -continuous map (briefly, n-CyNeCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNeos in (X, τ_1),
7. M -continuous map (briefly, n-CyNMCts map), if the inverse image of every n-CyNos in (Y, τ_2) is a n-CyNMos in (X, τ_1).

Definition 2.21 [2] Let (X, τ_1) and (Y, τ_2) be any two n-CyNts's. A map $f: (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a n-CyN

1. δ-irresolute map (briefly, n-CyNδIrr map), if $f^{-1}(B)$ is a n-CyNδos in (X, τ_1) for every n-CyNδos B of (Y, τ_2),
2. δ -pre irresolute map (briefly, n-CyNδ$\mathcal{P}$Irr map), if $f^{-1}(B)$ is a n-CyNδ$\mathcal{P}$os in (X, τ_1) for every n-CyNδ$\mathcal{P}$os B of (Y, τ_2),
3. δ -semi irresolute map (briefly, n-CyNδ$\mathcal{S}$Irr map), if $f^{-1}(B)$ is a n-CyNδ$\mathcal{S}$os in (X, τ_1) for every n-CyNδ$\mathcal{S}$os B of (Y, τ_2),
4. θ-irresolute map (briefly, n-CyNθIrr map), if $f^{-1}(B)$ is a n-CyNθos in (X, τ_1) for every n-CyNθos B of (Y, τ_2),
5. θ -semi irresolute map (briefly, n-CyNθ$\mathcal{S}$Irr map), if $f^{-1}(B)$ is a n-CyNθ$\mathcal{S}$os in (X, τ_1) for every n-CyNθ$\mathcal{S}$os B of (Y, τ_2),
6. e-irresolute map (briefly, n-CyNeIrr map), if $f^{-1}(B)$ is a n-CyNeos in (X, τ_1) for every n-CyNeos B of (Y, τ_2),
7. M -irresolute map (briefly, n-CyNMIrr map), if $f^{-1}(B)$ is a n-CyNMos in (X, τ_1) for every n-CyNMos B of (Y, τ_2).

Definition 2.22 [23] Let (X, τ_1) and

(Y,τ_2) be any two n-CyNts's. A map $f:(X,\tau_1) \to (Y,\tau_2)$ is said to be a n-CyN almost

1. continuous map (briefly, n-CyNAlmCts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyNos in (X,τ_1),
2. δ-continuous map (briefly, n-CyNδAlmCts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyNδos in (X,τ_1),
3. δ-pre continuous map (briefly, n-CyNAlm$\delta\mathcal{P}$Cts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyN$\delta\mathcal{P}$os in (X,τ_1),
4. δ-semi continuous map (briefly, n-CyNAlm$\delta\mathcal{S}$Cts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyN$\delta\mathcal{S}$os in (X,τ_1),
5. θ-continuous map (briefly, n-CyNAlmθCts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyNθos in (X,τ_1),
6. θ-semi continuous map (briefly, n-CyNAlm$\theta\mathcal{S}$os map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyN$\theta\mathcal{S}$os in (X,τ_1),
7. e-continuous map (briefly, n-CyNAlmeCts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyNeos in (X,τ_1),
8. M-continuous map (briefly, n-CyNAlmMCts map), if the inverse image of every n-CyNros in (Y,τ_2) is a n-CyNMos in (X,τ_1).

Definition 2.23 [23] Let (X,τ_1) and (Y,τ_2) be any two n-CyNts's. A map $f:(X,\tau_1) \to (Y,\tau_2)$ is said to be a n-CyN almost

1. open mapping (briefly, n-CyNAlmO map), if the image of every n-CyNros in (X,τ_1) is a n-CyNos in (Y,τ_2),
2. δ-open mapping (briefly, n-CyNAlmδO map), if the image of every n-CyNros in (X,τ_1) is a n-CyNδos in (Y,τ_2),
3. δ-pre open mapping (briefly, n-CyNAlm$\delta\mathcal{P}$O map), if the image of every n-CyNros in (X,τ_1) is a n-CyN$\delta\mathcal{P}$os in (Y,τ_2),
4. δ-semi open mapping (briefly, n-CyNAlm$\delta\mathcal{S}$O map), if the image of every n-CyNros in (X,τ_1) is a n-CyN$\delta\mathcal{S}$os in (Y,τ_2),
5. θ-open mapping (briefly, n-CyNAlmθO map), if the image of every n-CyNros in (X,τ_1) is a n-CyNθos in (Y,τ_2),
6. θ-semi open mapping (briefly, n-CyNAlm$\theta\mathcal{S}$O map), if the image of every n-CyNros in (X,τ_1) is a n-CyN$\theta\mathcal{S}$os in (Y,τ_2),
7. e-open mapping (briefly, n-CyNAlmeO map), if the image of every n-CyNros in (X,τ_1) is a n-CyNeos in (Y,τ_2),
8. M-open mapping (briefly, n-CyNAlmMO map), if the image of every n-CyNros in (X,τ_1) is a n-CyNMos in (Y,τ_2).

Definition 2.24 [2] Let (X,τ_1) and (Y,τ_2) be any two n-CyNts's. A map $f:(X,\tau_1) \to (Y,\tau_2)$ is said to be a n-cylindrical neutrosophic

1. closed mapping (briefly, n-CyNC map), if the image of every n-CyNcs in (X,τ_1) is a n-CyN$\mathcal{S}$cs in (Y,τ_2).
2. δ-closed mapping (briefly, n-CyNδC map), if the image of every n-CyNcs in (X,τ_1) is a n-CyNδcs in (Y,τ_2).
3. δ-pre closed mapping (briefly, n-CyN$\delta\mathcal{P}$C map), if the image of every n-CyNcs in (X,τ_1) is a n-CyN$\delta\mathcal{P}$cs in (Y,τ_2).
4. δ-semi closed mapping (briefly, n-CyN$\delta\mathcal{S}$C map), if the image of every n-CyNcs in (X,τ_1) is a n-CyN$\delta\mathcal{S}$cs in (Y,τ_2).
5. θ-closed mapping (briefly, n-CyNθC map), if the image of every n-CyNcs in (X,τ_1) is a n-CyNθcs in (Y,τ_2).
6. θ-semi closed mapping (briefly, n-CyN$\theta\mathcal{S}$C map), if the image of every n-CyNcs in (X,τ_1) is a n-CyN$\theta\mathcal{S}$cs in (Y,τ_2).
7. e-closed mapping (briefly, n-CyNeC map), if the image of every n-CyNcs in (X,τ_1) is a n-CyNecs in (Y,τ_2).
8. M-closed mapping (briefly, n-CyNMC map), if the image of every n-CyNcs in (X,τ_1) is a n-CyNMcs in (Y,τ_2).

3 n-Cylindrical neutrosophic M-regular spaces

In this section, we introduce the concept of n-cylindrical neutrosophic M normal spaces and examine some of their fundamental properties.

Definition 3.1 Let (X,τ_{CyN}) be an n-CyNts. Then, X is said to be n-CyN (resp. n-CyNδ, n-CyN$\delta\mathcal{P}$, n-CyN$\delta\mathcal{S}$, n-CyNθ, n-CyN$\theta\mathcal{S}$, n-CyNe and n-CyNM) regular (briefly, n-CyN$\mathcal{R}$eg (resp. n-CyN$\delta\mathcal{R}$eg, n-CyN$\delta\mathcal{P}\mathcal{R}$eg, n-CyN$\delta\mathcal{S}\mathcal{R}$eg, n-CyN$\theta\mathcal{R}$eg, n-CyN$\theta\mathcal{S}\mathcal{R}$eg, n-CyNe$\mathcal{R}$eg and n-CyNM$\mathcal{R}$eg)) space if for each n-CyNc (resp. n-CyNδc, n-CyN$\delta\mathcal{P}$c, n-CyN$\delta\mathcal{S}$c, n-CyNθc, n-CyN$\theta\mathcal{S}$c, n-CyNec, and n-CyNMc) set A and a point $x_\alpha \notin A$, there exist disjoint n-CyNo (resp. n-CyNδo, n-CyN$\delta\mathcal{P}$o, n-CyN$\delta\mathcal{S}$o, n-CyNθo, n-CyN$\theta\mathcal{S}$o, n-CyNeo, and n-CyNMo) sets $\mathcal{L}$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}$ and $A \subseteq \mathcal{M}$.

Definition 3.2 Two n-CyNS's A and B in an n-CyNts (X,τ_{CyN}) are said to be n-CyN quasi-coincident (briefly, AqB) if there exists $x \in X$ such that $\alpha_A(x) + \alpha_B(x) > 1$, $\beta_A(x) + \beta_B(x) > 1$, and $\gamma_A(x) + \gamma_B(x) < 1$; if A and B are not quasi-coincident, we write A$\bar{q}$B. Using this relation, let A and B be two n-CyN subsets of (X,τ_{CyN}). Then, B is called a n-CyN (resp. n-CyNδ, n-CyN$\delta\mathcal{P}$, n-CyN$\delta\mathcal{S}$, n-CyNθ, n-CyN$\theta\mathcal{S}$, n-CyNe, and n-CyNM) q-neighbourhood (briefly, n-CyNqnbhd (resp. n-CyNδqnbhd , n-CyN$\delta\mathcal{P}$qnbhd, n-CyN$\delta\mathcal{S}$qnbhd, n-CyNθqnbhd, n-CyN$\theta\mathcal{S}$qnbhd, n-CyNeqnbhd and n-CyNMqnbhd)) of A if there exists a n-CyNos (resp. n-CyNδos, n-CyN$\delta\mathcal{P}$os, n-CyN$\delta\mathcal{S}$os, n-CyNθos, n-CyN$\theta\mathcal{S}$os, n-CyNeos and n-CyNMos) O such that AqO and $O \subseteq B$.

Theorem 3.1 In a n-CyNts (X, τ_{CyN}), the following are equivalent:

1. X is n-CyNM$\mathcal{R}$eg space,
2. For every $x_\alpha \in X$ and every n-CyNMos G containing x_α, there exists a n-CyNMos $\mathcal{L}$, such that $x_\alpha \in \mathcal{L} \subseteq$ n-CyNMcl$(\mathcal{L}) \subseteq G$,
3. For every n-CyNMcs F, the intersection of all n-CyNMcs, n-CyNMq-nbhd of F is exactly F,
4. For any n-CyNS A and a n-CyNMos B such that $A \cap B \neq 0_{CyN}$, there exists a n-CyNMos $\mathcal{L}$ such that $A \cap \mathcal{L} \neq 0_{CyN}$ and n-CyNMcl$(\mathcal{L}) \subseteq B$,
5. For every non-empty n-CyNS A and n-CyNMcs B such that $A \cap B = 0_{CyN}$, there exist disjoint n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ such that $A \cap \mathcal{L} \neq 0_{CyN}$ and $B \subseteq \mathcal{M}$.

Proof. (i) $\Rightarrow$ (ii): Suppose X is n-CyNM$\mathcal{R}$eg. Let $x_\alpha \in X$ and let G be a n-CyNMos containing x_α. Then, $x_\alpha \not\in G^c$ and G^c is n-CyNMcs. Since, X is n-CyNM$\mathcal{R}$eg, there exist n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ such that $\mathcal{L} \cap \mathcal{M} = 0_{CyN}$ and $x_\alpha \in \mathcal{L}$, $G^c \subseteq \mathcal{M}$. It follows that $\mathcal{L} \subseteq \mathcal{M}^c \subseteq G$ and hence n-CyNMcl$(\mathcal{L}) \subseteq$ n-CyNMcl$(\mathcal{M}^c) = \mathcal{M}^c \subseteq G$. That is, $x_\alpha \in \mathcal{L} \subseteq$ n-CyNMcl$(\mathcal{L}) \subseteq G$.

(ii) $\Rightarrow$ (iii): Let F be any n-CyNMcs and $x_\alpha \not\in F$. Then, F^c is n-CyNMos and $x_\alpha \in F^c$. By assumption, there exists a n-CyNMos $\mathcal{L}$ such that $x_\alpha \in \mathcal{L} \subseteq$ n-CyNMcl$(\mathcal{L}) \subseteq F^c$. Thus, $F \subseteq$ (n-CyNMcl$(\mathcal{L}))^c \subseteq \mathcal{L}^c$. Now, $\mathcal{L}^c$ is n-CyNMcs, n-CyNMq-nbhd of F which does not contain x_α. So, we get the intersection of all n-CyNMcs 's, n-CyNMq-nbhd of F is exactly F.

(iii) $\Rightarrow$ (iv): Suppose $A \cap B \neq 0_{CyN}$ and B is n-CyNMos. Let $x_\alpha \in A \cap B$. Since, B is n-CyNMos, B^c is n-CyNMcs and $x_\alpha \not\subset B^c$. By using (iii), there exists a n-CyNMcs, n-CyNMq-nbhd $\mathcal{M}$ of B^c such that $x_\alpha \not\subset \mathcal{M}$. Now, for the n-CyNMq-nbhd $\mathcal{M}$ of B^c there exists a n-CyNMos G such that $B^c \subseteq G \subseteq \mathcal{M}$. Take, $\mathcal{L} = \mathcal{M}^c$. Thus, $\mathcal{L}$ is a n-CyNMos containing x_α. Also, $A \cap \mathcal{L} \neq 0_{CyN}$ and n-CyNMcl$(\mathcal{L}) \subseteq G^c \subseteq B$.

(iv) $\Rightarrow$ (v): Suppose A is a non-empty set and B is n-CyNMcs such that $A \cap B = 0_{CyN}$. Then, B^c is n-CyNMos and $A \cap (B^c) \neq 0_{CyN}$. By our assumption, there exists a n-CyNMos $\mathcal{L}$ such that $A \cap \mathcal{L} \neq 0_{CyN}$, and n-CyNMcl$(\mathcal{L}) \subseteq B^c$. Take, $\mathcal{M} =$ (n-CyNMcl$(\mathcal{L}))^c$. Since, n-CyNMcl$(\mathcal{L})$ is n-CyNMcs, $\mathcal{M}$ is n-CyNMos. Also, $B \subseteq \mathcal{M}$ and $\mathcal{L} \cap \mathcal{M} \subseteq$ n-CyNMcl$(\mathcal{L}) \cap$ (n-CyNMcl$(\mathcal{L}))^c = 0_{CyN}$.

(v) $\Rightarrow$ (i): Let S be n-CyNMcs and $x_\alpha \not\subset S$. Then, $S \cap \{x_\alpha\} = 0_{CyN}$. By (v), there exist disjoint n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ such that $\mathcal{L} \cap \{x_\alpha\} \neq 0_{CyN}$ and $S \subseteq \mathcal{M}$. That is $\mathcal{L}$ and $\mathcal{M}$ are disjoint n-CyNMos's containing x_α and S respectively. This proves that (X, τ_{CyN}) is n-CyNM$\mathcal{R}$eg space.

Corollary 3.1 If (X, τ_{CyN}) is a n-CyNM$\mathcal{R}$eg space, then for every $x_\alpha \in X$ and every n-CyNos G containing x_α, there exists a n-CyNMos $\mathcal{L}$ such that $x_\alpha \in \mathcal{L} \subseteq$ n-CyNMcl$(\mathcal{L}) \subseteq G$.

Proof. Since every n-CyNos is a n-CyNMos (Definition 2.18), G is in particular a n-CyNMos containing x_α. The conclusion then follows immediately from the equivalence (i)$\Leftrightarrow$(ii) of Theorem 3.1, applied to this G.

Definition 3.3 Let A and B be two n-CyN subsets of an n-CyNts (X, τ_{CyN}) (here A may in particular be a point x_α or a n-CyNc (resp. n-CyNδc, n-CyN$\delta\mathcal{P}$c, n-CyN$\delta\mathcal{S}$c, n-CyNθc, n-CyN$\theta\mathcal{S}$c, n-CyNec, and n-CyNMc) set). Then, B is called a n-CyN (resp. n-CyNδ, n-CyN$\delta\mathcal{P}$, n-CyN$\delta\mathcal{S}$, n-CyNθ, n-CyN$\theta\mathcal{S}$, n-CyNe, and n-CyNM) q-neighbourhood (briefly, n-CyNqnbhd (resp. n-CyNδqnbhd, n-CyN$\delta\mathcal{P}$qnbhd, n-CyN$\delta\mathcal{S}$qnbhd, n-CyNθqnbhd, n-CyN$\theta\mathcal{S}$qnbhd, n-CyNeqnbhd and n-CyNMqnbhd)) of A if there exists a n-CyNos (resp. n-CyNδos, n-CyN$\delta\mathcal{P}$os, n-CyN$\delta\mathcal{S}$os, n-CyNθos, n-CyN$\theta\mathcal{S}$os, n-CyNeos and n-CyNMos) O such that $A \subseteq O \subseteq B$.

Theorem 3.2 A n-CyNts (X, τ_{CyN}) is n-CyNM$\mathcal{R}$eg if and only if for every $x_\alpha \in X$ and every n-CyNMq-nbhd N containing x_α, there exists a n-CyNMos $\mathcal{M}$ such that $x_\alpha \in \mathcal{M} \subseteq$ n-CyNMcl$(\mathcal{M}) \subseteq N$.

Proof. Let X be a n-CyNM$\mathcal{R}$eg space. Let N be any n-CyNMq-nbhd of x_α. Then, there exists a n-CyNMos G such that $x_\alpha \in G \subseteq N$. Since, G^c is n-CyNMcs and $x_\alpha \not\in G^c$, by definition there exist n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ such that $G^c \subseteq \mathcal{L}$ and $x_\alpha \in \mathcal{M}$ and $\mathcal{L} \cap \mathcal{M} = 0_{CyN}$ so that $\mathcal{M} \subseteq \mathcal{L}^c$. It follows that n-CyNMcl$(\mathcal{M}) \subseteq$ n-CyNMcl$(\mathcal{L}^c) = \mathcal{L}^c$. Also, $G^c \subseteq \mathcal{L}$ implies $\mathcal{L}^c \subseteq G \subseteq N$. Hence, $x_\alpha \in \mathcal{M} \subseteq$ n-CyNMcl$(\mathcal{M}) \subseteq N$.

Conversely, suppose for every $x_\alpha \in X$ and every n-CyNMq-nbhd N containing x_α, there exists a n-CyNMos $\mathcal{M}$ such that $x_\alpha \in \mathcal{M} \subseteq$ n-CyNMcl$(\mathcal{M}) \subseteq N$. Let F be any n-CyNMcs and $x_\alpha \not\in F$. Then, $x_\alpha \in F^c$. Since, F^c is n-CyNMos, F^c is n-CyNMq-nbhd containing x_α. By hypothesis there exists a n-CyNMos $\mathcal{M}$ such that $x_\alpha \in \mathcal{M}$ and n-CyNMcl$(\mathcal{M}) \subseteq F^c$. This implies that, $F \subseteq$

$(\text{n-CyNMcl}(\mathcal{M}))^c$. Then, $\text{n-CyNMcl}(\mathcal{M}))^c$ is a n-CyNMos containing F. Also, $\mathcal{M} \cap (\text{n-CyNMcl}(\mathcal{M}))^c = 0_{\text{CyN}}$. Hence, the space is n-CyNM$\mathcal{R}$eg space.

Theorem 3.3 A n-CyNts (X, τ_{CyN}) is n-CyNM$\mathcal{R}$eg if and only if for each n-CyNMcs F of X and each $x_\alpha \in F^c$, there exist n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ of X such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$ and $\text{n-CyNMcl}(\mathcal{L}) \cap \text{n-CyNMcl}(\mathcal{M}) = 0_{\text{CyN}}$.

Proof. Suppose X is n-CyNM$\mathcal{R}$eg space. Let F be a n-CyNMcs in X and $x_\alpha \not\in F$. Then, there exist n-CyNMos's $\mathcal{L}(x_\alpha)$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}(x_\alpha)$, $F \subseteq \mathcal{M}$ and $\mathcal{L}(x_\alpha) \cap \mathcal{M} = 0_{\text{CyN}}$. This implies that $\mathcal{L}(x_\alpha) \cap \text{n-CyNMcl}(\mathcal{M}) = 0_{\text{CyN}}$. Also, $\text{n-CyNMcl}(\mathcal{M})$ is a n-CyNMcs and $x_\alpha \not\in \text{n-CyNMcl}(\mathcal{M})$. Since, X is n-CyNM$\mathcal{R}$eg, there exist n-CyNMos's G and H of X such that $x_\alpha \in G$, $\text{n-CyNMcl}(\mathcal{M}) \subseteq H$ and $G \cap H = 0_{\text{CyN}}$. This implies $\text{n-CyNMcl}(G) \cap H \subseteq \text{n-CyNMcl}(H^c) \cap H = (H^c) \cap H = 0_{\text{CyN}}$. Take, $\mathcal{L} = G$. Now, $\mathcal{L}$ and $\mathcal{M}$ are n-CyNMos's in X such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$. Also, $\text{n-CyNMcl}(\mathcal{L}) \cap \text{n-CyNMcl}(\mathcal{M}) = \text{n-CyNMcl}(G) \cap \text{n-CyNMcl}(\mathcal{M}) \subseteq \text{n-CyNMcl}(G) \cap H = 0_{\text{CyN}}$.

Conversely, suppose for each n-CyNMcs F of X and each $x_\alpha \in F^c$, there exist n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ of X such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$ and $\text{n-CyNMcl}(\mathcal{L}) \cap \text{n-CyNMcl}(\mathcal{M}) = 0_{\text{CyN}}$. Now, $\mathcal{L} \cap \mathcal{M} \subseteq \text{n-CyNMcl}(\mathcal{L}) \cap \text{n-CyNMcl}(\mathcal{M}) = 0_{\text{CyN}}$. Therefore, $\mathcal{L} \cap \mathcal{M} = 0_{\text{CyN}}$. This proves that X is n-CyNM$\mathcal{R}$eg space.

Remark 3.1 Analogous to the n-CyNMIrr map of Definition 2.21, a map $f: (X, \tau_1) \to (Y, \tau_2)$ between n-CyNts's is called a n-CyNMO map if the image of every n-CyNMos in (X, τ_1) is a n-CyNMos in (Y, τ_2).

Theorem 3.4 Let (X, τ_1) and (Y, τ_2) be two n-CyNts's, and let $f: (X, \tau_1) \to (Y, \tau_2)$ be a bijective function. If f is a n-CyNMIrr map, a n-CyNMO map, and X is a n-CyNM$\mathcal{R}$eg space, then Y is a n-CyNM$\mathcal{R}$eg space.

Proof. Suppose X is n-CyNM$\mathcal{R}$eg. Let S be any n-CyNMcs in Y such that $y_\beta \not\in S$. Since f is a n-CyNMIrr map, $f^{-1}(S)$ is a n-CyNMcs in X. Since f is onto, there exists $x_\alpha \in X$ such that $y_\beta = f(x_\alpha)$. Now, $f(x_\alpha) = y_\beta \not\in S \Rightarrow x_\alpha \not\in f^{-1}(S)$. Since X is n-CyNM$\mathcal{R}$eg, there exist n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ in X such that $x_\alpha \in \mathcal{L}$, $f^{-1}(S) \subseteq \mathcal{M}$ and $\mathcal{L} \cap \mathcal{M} = 0_{\text{CyN}}$. Now, $x_\alpha \in \mathcal{L} \Rightarrow f(x_\alpha) \in f(\mathcal{L})$, i.e., $y_\beta \in f(\mathcal{L})$; and $f^{-1}(S) \subseteq \mathcal{M} \Rightarrow S = f(f^{-1}(S)) \subseteq f(\mathcal{M})$, since f is onto. Also, $\mathcal{L} \cap \mathcal{M} = 0_{\text{CyN}} \Rightarrow f(\mathcal{L} \cap \mathcal{M}) = 0_{\text{CyN}} \Rightarrow f(\mathcal{L}) \cap f(\mathcal{M}) = 0_{\text{CyN}}$, the last step using that f is one-one. Since f is a n-CyNMO map, $f(\mathcal{L})$ and $f(\mathcal{M})$ are disjoint n-CyNMos's in Y containing y_β and S respectively. Thus, Y is a n-CyNM$\mathcal{R}$eg space.

Theorem 3.5 Let X be a n-CyNM$\mathcal{R}$eg space.

1. Every n-CyNMos in X is a union of n-CyNMcs's.
2. Every n-CyNMcs in X is an intersection of n-CyNMos's.

Proof. (i) Suppose X is n-CyNM$\mathcal{R}$eg. Let G be a n-CyNMos and $x_\alpha \in G$. Then, $F = G^c$ is n-CyNMcs and $x_\alpha \not\in F$. Since X is n-CyNM$\mathcal{R}$eg, there exist n-CyNMos's $\mathcal{L}(x_\alpha)$ and $\mathcal{M}$ in X such that $x_\alpha \in \mathcal{L}(x_\alpha)$, $F \subseteq \mathcal{M}$, and $\mathcal{L}(x_\alpha) \cap \mathcal{M} = 0_{\text{CyN}}$. Since $\mathcal{L}(x_\alpha) \cap F \subseteq \mathcal{L}(x_\alpha) \cap \mathcal{M} = 0_{\text{CyN}}$, we have $\mathcal{L}(x_\alpha) \subseteq F^c = G$. Take, $\mathcal{M}(x_\alpha) = \text{n-CyNMcl}(\mathcal{L}(x_\alpha))$; then $\mathcal{M}(x_\alpha)$ is n-CyNMcs. Since $\mathcal{L}(x_\alpha)$ is n-CyNMos and $\mathcal{L}(x_\alpha) \cap \mathcal{M} = 0_{\text{CyN}}$, we get $\mathcal{L}(x_\alpha) \subseteq \mathcal{M}^c$, and since $\mathcal{M}^c$ is n-CyNMcs, $\text{n-CyNMcl}(\mathcal{L}(x_\alpha)) \subseteq \mathcal{M}^c$; as $F \subseteq \mathcal{M}$ gives $\mathcal{M}^c \subseteq F^c = G$, we obtain $\mathcal{M}(x_\alpha) \subseteq \mathcal{M}^c \subseteq F^c = G$. Also, $x_\alpha \in \mathcal{L}(x_\alpha) \subseteq \mathcal{M}(x_\alpha)$. This proves that $G = \cup \{\mathcal{M}(x_\alpha) : x_\alpha \in G\}$. Thus, G is a union of n-CyNMcs's.

(ii) Follows from (i) and set theoretic properties.

Theorem 3.6 Let (X, τ_1) and (Y, τ_2) be two n-CyNts's. If $f: (X, \tau_1) \to (Y, \tau_2)$ is an injective n-CyNMIrr map and n-CyNC map from a n-CyNts X into a n-CyNM$\mathcal{R}$eg space Y, and if every n-CyNMcs in X is n-CyNcs, then X is a n-CyNM$\mathcal{R}$eg space.

Proof. Let $x_\alpha \in X$ and A be a n-CyNMcs in X not containing x_α. Then by assumption, A is n-CyNcs in X. Since f is a n-CyNC map, f(A) is a n-CyNcs in Y; since f is injective and $x_\alpha \not\in A$, f(A) does not contain $f(x_\alpha)$. Every n-CyNcs is a n-CyNMcs, so f(A) is in particular n-CyNMcs in Y. Since Y is a n-CyNM$\mathcal{R}$eg space, there exist disjoint n-CyNMos's $\mathcal{M}_1$ and $\mathcal{M}_2$ in Y such that $f(x_\alpha) \in \mathcal{M}_1$ and $f(A) \subseteq \mathcal{M}_2$. Since f is a n-CyNMIrr map, $f^{-1}(\mathcal{M}_1)$ and $f^{-1}(\mathcal{M}_2)$ are n-CyNMos's in X, disjoint since $f^{-1}(\mathcal{M}_1) \cap f^{-1}(\mathcal{M}_2) = f^{-1}(\mathcal{M}_1 \cap \mathcal{M}_2) = f^{-1}(0_{\text{CyN}}) = 0_{\text{CyN}}$, and containing x_α and A respectively (the latter since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(\mathcal{M}_2)$). Hence, X is a n-CyNM$\mathcal{R}$eg space.

Theorem 3.7 Let (X, τ_1) and (Y, τ_2) be two n-CyNts's. If $f: (X, \tau_1) \to (Y, \tau_2)$ is a bijective n-CyNCts map and n-CyNMO map from a n-CyN$\mathcal{R}$eg space X into a n-CyNts Y, and if every n-CyNMcs in Y is n-CyNcs, then Y is a n-CyNM$\mathcal{R}$eg space.

Proof. Let $x_\alpha \in Y$ and B be a n-CyNMcs in Y not containing x_α. Then by assumption, B is n-CyNcs in Y. Since f is a n-CyNCts map, $f^{-1}(B)$ is a n-CyNcs in X; as f is bijective, $f^{-1}(x_\alpha)$ is a well-defined point of X, and it does not lie in $f^{-1}(B)$, since otherwise $x_\alpha = f(f^{-1}(x_\alpha))$ would lie in $f(f^{-1}(B)) = B$. Since X is n-CyN$\mathcal{R}$eg, there exist disjoint n-CyNos's $\mathcal{L}_1$ and $\mathcal{L}_2$ in X such that $f^{-1}(x_\alpha) \in \mathcal{L}_1$ and $f^{-1}(B) \subseteq \mathcal{L}_2$. Since every n-CyNos is a n-CyNMos, $\mathcal{L}_1$ and $\mathcal{L}_2$ are in particular n-CyNMos's, so as f is a n-CyNMO map, $f(\mathcal{L}_1)$ and $f(\mathcal{L}_2)$ are n-CyNMos's in Y. They are disjoint, since f is injective and $\mathcal{L}_1 \cap \mathcal{L}_2 = 0_{CyN}$ give $f(\mathcal{L}_1) \cap f(\mathcal{L}_2) = f(\mathcal{L}_1 \cap \mathcal{L}_2) = 0_{CyN}$. Also, $x_\alpha = f(f^{-1}(x_\alpha)) \in f(\mathcal{L}_1)$, and $f^{-1}(B) \subseteq \mathcal{L}_2$ gives $B = f(f^{-1}(B)) \subseteq f(\mathcal{L}_2)$, since f is onto. Thus $f(\mathcal{L}_1)$ and $f(\mathcal{L}_2)$ are disjoint n-CyNMos's in Y containing x_α and B respectively. Hence, Y is a n-CyNM$\mathcal{R}$eg space.

Remark 3.2 Theorems 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, and 3.7 hold for n-CyNδ$\mathcal{P}$os's and n-CyNθ$\mathcal{S}$os's.

4 Strongly n-Cylindrical Neutrosophic M Regular Spaces

In this section, we introduce the concept of strongly n-cylindrical neutrosophic M regular spaces and examine some of their fundamental properties.

Definition 4.1 Let (X, τ_{CyN}) be an n-CyNts. A collection $\mathcal{G}$ of n-CyNo (resp. n-CyNδo, n-CyNδ$\mathcal{P}$o, n-CyNδ$\mathcal{S}$o, n-CyNθo, n-CyNθ$\mathcal{S}$o, n-CyNeo, and n-CyNMo) sets in X is called a n-CyN (resp. n-CyNδ, n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNθ, n-CyNθ$\mathcal{S}$, n-CyNe, and n-CyNM) open cover (briefly, n-CyN$\mathcal{O}$Cov (resp. n-CyNδ$\mathcal{O}$Cov, n-CyNδ$\mathcal{PO}$Cov, n-CyNδ$\mathcal{SO}$Cov, n-CyNθ$\mathcal{O}$Cov, n-CyNθ$\mathcal{SO}$Cov, n-CyNe$\mathcal{O}$Cov, and n-CyNM$\mathcal{O}$Cov)) of a subset B of X if $B \subseteq \bigcup \{L : L \in \mathcal{G}\}$.

Definition 4.2 Let (X, τ_{CyN}) be an n-CyNts, then X is said to be n-CyN (resp. n-CyNδ, n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNθ, n-CyNθ$\mathcal{S}$, n-CyNe, and n-CyNM) compact (briefly, n-CyN$\mathcal{C}$omp (resp. n-CyNδ$\mathcal{C}$omp, n-CyNδ$\mathcal{PC}$omp, n-CyNδ$\mathcal{SC}$omp, n-CyNθ$\mathcal{C}$omp, n-CyNθ$\mathcal{SC}$omp, n-CyNe$\mathcal{C}$omp, and n-CyNM$\mathcal{C}$omp)) if every n-CyN$\mathcal{O}$Cov (resp. n-CyNδ$\mathcal{O}$Cov, n-CyNδ$\mathcal{PO}$Cov, n-CyNδ$\mathcal{SO}$Cov, n-CyNθ$\mathcal{O}$Cov, n-CyNθ$\mathcal{SO}$Cov, n-CyNe$\mathcal{O}$Cov, and n-CyNM$\mathcal{O}$Cov) of X has a finite subcover.

Definition 4.3 Let (X, τ_{CyN}) be an n-CyNts. A subset A of X is said to be n-CyN$\mathcal{C}$omp (resp. n-CyNδ$\mathcal{C}$omp, n-CyNδ$\mathcal{PC}$omp, n-CyNδ$\mathcal{SC}$omp, n-CyNθ$\mathcal{C}$omp, n-CyNθ$\mathcal{SC}$omp, n-CyNe$\mathcal{C}$omp, and n-CyNM$\mathcal{C}$omp) relative to X if every n-CyN$\mathcal{O}$Cov (resp. n-CyNδ$\mathcal{O}$Cov, n-CyNδ$\mathcal{PO}$Cov, n-CyNδ$\mathcal{SO}$Cov, n-CyNθ$\mathcal{O}$Cov, n-CyNθ$\mathcal{SO}$Cov, n-CyNe$\mathcal{O}$Cov, and n-CyNM$\mathcal{O}$Cov) of A has a finite subcover.

Definition 4.4 Let (X, τ_{CyN}) be an n-CyNts. Then, X is said to be n-CyN (resp. n-CyNδ, n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNθ, n-CyNθ$\mathcal{S}$, n-CyNe, and n-CyNM) strongly regular (briefly, $\mathcal{S}$t n-CyN$\mathcal{R}$eg (resp. $\mathcal{S}$t n-CyNδ$\mathcal{R}$eg, $\mathcal{S}$t n-CyNδ$\mathcal{PR}$eg, $\mathcal{S}$t n-CyNδ$\mathcal{SR}$eg, $\mathcal{S}$t n-CyNθ$\mathcal{R}$eg, $\mathcal{S}$t n-CyNθ$\mathcal{SR}$eg, $\mathcal{S}$t n-CyNe$\mathcal{R}$eg, and $\mathcal{S}$t n-CyNM$\mathcal{R}$eg)) space if for each n-CyNcs (resp. n-CyNδcs, n-CyNδ$\mathcal{P}$cs, n-CyNδ$\mathcal{S}$cs, n-CyNθcs, n-CyNθ$\mathcal{S}$cs, n-CyNecs, and n-CyNMcs) A and a point $x_\alpha \not\in A$, there exist disjoint n-CyNos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNθos, n-CyNθ$\mathcal{S}$os, n-CyNeos, and n-CyNMos) $\mathcal{L}$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}$ and $A \subseteq \mathcal{M}$.

Proposition 4.1 Let (X, τ_{CyN}) be an n-CyNts.

1. Every $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space is n-CyNM$\mathcal{R}$eg,
2. Every $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space is $\mathcal{S}$t n-CyN$\mathcal{R}$eg.

Proof.

1. Suppose X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg. Let F be a n-CyNMcs and $x_\alpha \not\in F$. Since X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg, there exist disjoint n-CyNos's $\mathcal{L}$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$. Since every n-CyNos is a n-CyNMos, $\mathcal{L}$ and $\mathcal{M}$ are in particular n-CyNMos's. As F was an arbitrary n-CyNMcs not containing x_α, this shows X is a n-CyNM$\mathcal{R}$eg space.
2. Suppose X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg. Let F be a n-CyNcs and $x_\alpha \not\in F$. Since every n-CyNcs is a n-CyNMcs (Definition 2.18), F is in particular a n-CyNMcs. Since X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg, there exist disjoint n-CyNos's $\mathcal{L}$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$. As F was an arbitrary n-CyNcs not containing x_α, this shows X is a $\mathcal{S}$t n-CyN$\mathcal{R}$eg space.

Definition 4.5 Let (X, τ_{CyN}) be an n-CyNts. Then, X is said to be n-CyN (resp. n-CyNδ, n-CyNδ$\mathcal{P}$, n-CyNδ$\mathcal{S}$, n-CyNθ, n-CyNθ$\mathcal{S}$, n-CyNe, and n-CyNM) strongly * regular (briefly, $\mathcal{S}t^*$ n-CyN$\mathcal{R}$eg (resp. $\mathcal{S}t^*$ n-CyNδ$\mathcal{R}$eg, $\mathcal{S}t^*$ n-CyNδ$\mathcal{PR}$eg, $\mathcal{S}t^*$ n-CyNδ$\mathcal{SR}$eg, $\mathcal{S}t^*$ n-CyNθ$\mathcal{R}$eg, $\mathcal{S}t^*$ n-CyNθ$\mathcal{SR}$eg, $\mathcal{S}t^*$ n-CyNe$\mathcal{R}$eg, and $\mathcal{S}t^*$ n-CyNM$\mathcal{R}$eg)) space if for each n-CyNcs set A (ordinary n-CyN-closed, in every case, regardless of which class is being named) and a point $x_\alpha \not\in A$, there exist disjoint n-CyNos (resp. n-CyNδos, n-CyNδ$\mathcal{P}$os, n-CyNδ$\mathcal{S}$os, n-CyNθos, n-CyNθ$\mathcal{S}$os, n-CyNeos, and n-CyNMos) $\mathcal{L}$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}$ and $A \subseteq \mathcal{M}$.

Proposition 4.2 Let (X, τ_{CyN}) be an n-CyNts. Every n-CyNM$\mathcal{R}$eg space is a $\mathcal{S}t^*$n-CyNM$\mathcal{R}$eg space.

Proof. Suppose X is a n-CyNM$\mathcal{R}$eg space. Let F be a n-CyNcs and $x_\alpha \not\in F$. Since every n-CyNcs is a n-CyNMcs (Definition 2.18), F is in particular a n-CyNMcs. Since X is a n-CyNM$\mathcal{R}$eg space, there exist disjoint n-CyNMos's $\mathcal{L}$ and $\mathcal{M}$ such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$. As F was an arbitrary n-CyNcs not containing x_α, this shows X is a $\mathcal{S}t^*$n-CyNM$\mathcal{R}$eg space.

Theorem 4.1 In a n-CyNts (X, τ_{CyN}), the following are equivalent:

1. X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space,
2. For every $x_\alpha \in X$ and every n-CyNMos G containing x_α, there exists a n-CyNos $\mathcal{L}$ such that $x_\alpha \in \mathcal{L} \subseteq$ n-CyNcl$(\mathcal{L}) \subseteq G$,
3. For every n-CyNMcs F, the intersection of all n-CyNcs, n-CyNq-nbhd of F is exactly F,
4. For any n-CyNS A and a n-CyNMos B such that $A \cap B \neq 0_{CyN}$, there exists a n-CyNos $\mathcal{L}$ such that $A \cap \mathcal{L} \neq 0_{CyN}$ and n-CyNcl$(\mathcal{L}) \subseteq B$,
5. For every non-empty n-CyNS A and n-CyNMcs B such that $A \cap B = 0_{CyN}$, there exist disjoint n-CyNos's $\mathcal{L}$ and $\mathcal{M}$, such that $A \cap \mathcal{L} \neq 0_{CyN}$ and $B \subseteq \mathcal{M}$.

Proof. (i) ⇒ (ii): Suppose X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space. Let $x_\alpha \in X$ and let G be a n-CyNMos containing x_α. Then, $x_\alpha \not\in G^c$ and G^c is n-CyNMcs. Since X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space, there exist n-CyNos's $\mathcal{L}$ and $\mathcal{M}$, such that $\mathcal{L} \cap \mathcal{M} = 0_{CyN}$ and $x_\alpha \in \mathcal{L}, G^c \subseteq \mathcal{M}$. It follows that, $\mathcal{L} \subseteq \mathcal{M}^c \subseteq G$ and hence n-CyNcl$(\mathcal{L}) \subseteq$ n-CyNcl$(\mathcal{M}^c) = \mathcal{M}^c \subseteq G$. That is, $x_\alpha \in \mathcal{L} \subseteq$ n-CyNcl$(\mathcal{L}) \subseteq G$.

(ii) ⇒ (iii): Let F be any n-CyNMcs and $x_\alpha \not\in F$. Then, F^c is n-CyNMos and $x_\alpha \in F^c$. By assumption, there exists a n-CyNos $\mathcal{L}$, such that $x_\alpha \in \mathcal{L} \subseteq$ n-CyNcl$(\mathcal{L}) \subseteq F^c$. Thus, $F \subseteq$ (n-CyNcl$(\mathcal{L}))^c \subseteq \mathcal{L}^c$. Since $\mathcal{L}$ is a n-CyNos, n-CyNcl$(\mathcal{L})$ is a n-CyNcs, so (n-CyNcl$(\mathcal{L}))^c$ is a n-CyNos; taking $\mathcal{O} =$ (n-CyNcl$(\mathcal{L}))^c$, we have $F \subseteq \mathcal{O} \subseteq \mathcal{L}^c$, so $\mathcal{L}^c$ is a n-CyNq-nbhd of F. Now, $\mathcal{L}^c$ is a n-CyNcs, n-CyNq-nbhd of F which does not contain x_α (since $x_\alpha \in \mathcal{L}$). So, we get the intersection of all n-CyNcs, n-CyNq-nbhd of F is exactly F.

(iii) ⇒ (iv): Suppose $A \cap B \neq 0_{CyN}$ and B is n-CyNMos. Let $x_\alpha \in A \cap B$. Since B is n-CyNMos, B^c is n-CyNMcs and $x_\alpha \not\in B^c$. By using (iii), there exists a n-CyNcs, n-CyNq-nbhd $\mathcal{M}$ of B^c, such that $x_\alpha \not\in \mathcal{M}$. Now, for the n-CyNq-nbhd $\mathcal{M}$ of B^c there exists a n-CyNos G, such that $B^c \subseteq G \subseteq \mathcal{M}$. Take, $\mathcal{L} = \mathcal{M}^c$. Thus, $\mathcal{L}$ is a n-CyNos containing x_α. Also, $A \cap \mathcal{L} \neq 0_{CyN}$ and n-CyNcl$(\mathcal{L}) \subseteq G^c \subseteq B$.

(iv) ⇒ (v): Suppose A is a non-empty n-CyNS and B is n-CyNMcs, such that $A \cap B = 0_{CyN}$. Then, B^c is n-CyNMos and $A \cap B^c \neq 0_{CyN}$. By our assumption, there exists a n-CyNos $\mathcal{L}$, such that $A \cap \mathcal{L} \neq 0_{CyN}$, and n-CyNcl$(\mathcal{L}) \subseteq B^c$. Take, $\mathcal{M} =$ (n-CyNcl$(\mathcal{L}))^c$. Since n-CyNcl$(\mathcal{L})$ is n-CyNcs, $\mathcal{M}$ is n-CyNos. Also, $B \subseteq \mathcal{M}$ and $\mathcal{L} \cap \mathcal{M} \subseteq$ n-CyNcl$(\mathcal{L}) \cap$ (n-CyNcl$(\mathcal{L}))^c = 0_{CyN}$.

(v) ⇒ (i): Let S be n-CyNMcs and $x_\alpha \not\in S$. Then, $S \cap \{x_\alpha\} = 0_{CyN}$. By (v), there exist disjoint n-CyNos's $\mathcal{L}$ and $\mathcal{M}$, such that $\mathcal{L} \cap \{x_\alpha\} \neq 0_{CyN}$ and $S \subseteq \mathcal{M}$. That is, $\mathcal{L}$ and $\mathcal{M}$ are disjoint n-CyNos's containing x_α and S respectively. This proves that (X, τ_{CyN}) is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space.

Theorem 4.2 A n-CyNts (X, τ_{CyN}) is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space if and only if for each n-CyNMcs F of X and each $x_\alpha \in F^c$, there exist n-CyNos's $\mathcal{L}$ and $\mathcal{M}$ of X, such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$ and n-CyNcl$(\mathcal{L}) \cap$ n-CyNcl$(\mathcal{M}) = 0_{CyN}$.

Proof. Suppose, X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space. Let F be a n-CyNMcs in X and $x_\alpha \not\in F$. Then, there exist n-CyNos's $\mathcal{L}(x_\alpha)$ and $\mathcal{M}$, such that $x_\alpha \in \mathcal{L}(x_\alpha)$, $F \subseteq \mathcal{M}$ and $\mathcal{L}(x_\alpha) \cap \mathcal{M} = 0_{CyN}$. This implies that, $\mathcal{L}(x_\alpha) \cap$ n-CyNcl$(\mathcal{M}) = 0_{CyN}$. Also, n-CyNcl$(\mathcal{M})$ is a n-CyNcs and hence n-CyNMcs and $x_\alpha \not\in$ n-CyNcl$(\mathcal{M})$. Since, X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space, there exist n-CyNos's G and H of X, such that $x_\alpha \in G$, n-CyNcl$(\mathcal{M}) \subseteq H$ and $G \cap H = 0_{CyN}$. This implies, n-CyNcl$(G) \cap H \subseteq$ n-CyNcl$(H^c) \cap H = (H^c) \cap H = 0_{CyN}$. Take, $\mathcal{L} = G$. Now, $\mathcal{L}$ and $\mathcal{M}$ are n-CyNos's in X such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$. Also, n-CyNcl$(\mathcal{L}) \cap$ n-CyNcl$(\mathcal{M}) \subseteq$ n-CyNcl$(G) \cap H = 0_{CyN}$.

Conversely, suppose for each n-CyNMcs F of X and each $x_\alpha \in F^c$, there exist n-CyNos's $\mathcal{L}$ and $\mathcal{M}$ of X, such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$ and n-CyNcl$(\mathcal{L}) \cap$ n-CyNcl$(\mathcal{M}) = 0_{CyN}$. Now, $\mathcal{L} \cap \mathcal{M} \subseteq$ n-CyNcl$(\mathcal{L}) \cap$ n-CyNcl$(\mathcal{M}) = 0_{CyN}$. Therefore, $\mathcal{L} \cap \mathcal{M} = 0_{CyN}$. This proves that X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space.

Theorem 4.3 A n-CyNts (X, τ_{CyN}) is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space if and only if every pair consisting of a n-CyNM$\mathcal{C}$omp and a disjoint n-CyNMcs can be separated by n-CyNos's.

Proof. Let X be $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space and let A be a n-CyNM$\mathcal{C}$omp, B n-CyNMcs with $A \cap B = 0_{CyN}$. Since X is $\mathcal{S}t$ n-CyNM$\mathcal{R}$eg space, for each $x_\alpha \in A$, there exist disjoint n-CyNos's $\mathcal{L}(x_\alpha)$ and $\mathcal{M}(x_\alpha)$, such that $x_\alpha \in \mathcal{L}(x_\alpha)$, $B \subseteq \mathcal{M}(x_\alpha)$. Obviously, $\{\mathcal{L}(x_\alpha) : x_\alpha \in A\}$ is a n-CyN$\mathcal{O}$Cov of A. Since A is n-CyNM$\mathcal{C}$omp, there exists a finite subfamily $\{\mathcal{L}_{x_i} : i = 1,2,\cdots,n\}$ which covers A. It follows that $A \subseteq \bigcup \{\mathcal{L}_{x_i} : i = 1,2,\cdots,n\}$ and $B \subseteq \bigcap \{\mathcal{M}_{x_i} : i = 1,2,\cdots,n\}$. Put, $\mathcal{L} = \bigcup \{\mathcal{L}_{x_i} : i = 1,2,\cdots,n\}$ and $\mathcal{M} = \bigcap \{\mathcal{M}_{x_i} : i = 1,2,\cdots,n\}$. Then, $\mathcal{L}$ and $\mathcal{M}$ are n-CyNos in X (a finite union, resp. finite intersection, of n-CyNos's). Also, $\mathcal{L} \cap \mathcal{M} =$

0_{CyN}. Otherwise, if $x_\alpha \in \mathcal{L} \cap \mathcal{M} \Rightarrow x_\alpha \in \mathcal{L}_{x_j}$ for some j and $x_\alpha \in \mathcal{M}_{x_i}$ for every i. This implies that $x_\alpha \in \mathcal{L}_{x_j} \cap \mathcal{M}_{x_j}$, which is a contradiction to $\mathcal{L}_{x_j} \cap \mathcal{M}_{x_j} = 0_{\mathrm{CyN}}$. Thus, $\mathcal{L}$ and $\mathcal{M}$ are disjoint n-CyNos's containing A and B respectively.

Conversely, suppose every pair consisting of a n-CyNM$\mathcal{C}$omp and a disjoint n-CyNMcs can be separated by n-CyNos's. Let F be a n-CyNMcs and $x_\alpha \notin F$. Then, $\{x_\alpha\}$ is n-CyNM$\mathcal{C}$omp of X and $\{x_\alpha\} \cap F = 0_{\mathrm{CyN}}$. By our assumption, there exist disjoint n-CyNos's $\mathcal{L}$ and $\mathcal{M}$, such that $x_\alpha \in \mathcal{L}$ and $F \subseteq \mathcal{M}$. This proves that, X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space.

Corollary 4.1 If X is a $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space, A is a n-CyNM$\mathcal{C}$omp subset of X and B is a n-CyNMos containing A, then there exists a n-CyNros $\mathcal{M}$, such that $A \subseteq \mathcal{M} \subseteq$ n-CyNcl$(\mathcal{M}) \subseteq$ B.

Proof. Let X be $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space and let A be a n-CyNM$\mathcal{C}$omp, B n-CyNMos with $A \subseteq B$. Then, B^c is n-CyNMcs such that $B^c \cap A = 0_{\mathrm{CyN}}$. Since X is a $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space, by Theorem 4.3, every pair consisting of a n-CyNM$\mathcal{C}$omp and a disjoint n-CyNMcs can be separated by n-CyNos's; applying this to A and B^c, there exist disjoint n-CyNos's $\mathcal{L}_1$ and $\mathcal{L}_2$, such that $A \subseteq \mathcal{L}_1$ and $B^c \subseteq \mathcal{L}_2$. Take, $\mathcal{M} =$ n-CyNint(n-CyNcl$(\mathcal{L}_1)$). Then, n-CyNcl$(\mathcal{M})$ = n-CyNcl(n-CyNint(cl$(\mathcal{L}_1)$)) $\subseteq$ n-CyNcl(n-CyNcl$(\mathcal{L}_1)$) = n-CyNcl$(\mathcal{L}_1)$. Since $\mathcal{L}_1$ is a n-CyNos and $\mathcal{L}_1 \subseteq$ n-CyNcl$(\mathcal{L}_1)$, we have $\mathcal{L}_1$ = n-CyNint$(\mathcal{L}_1) \subseteq$ n-CyNint(n-CyNcl$(\mathcal{L}_1)$) = $\mathcal{M}$. This implies that, n-CyNcl$(\mathcal{L}_1) \subseteq$ n-CyNcl$(\mathcal{M})$. It follows that, n-CyNcl$(\mathcal{M})$ = n-CyNcl$(\mathcal{L}_1)$ and n-CyNint(n-CyNcl$(\mathcal{M})$) = n-CyNint(n-CyNcl$(\mathcal{L}_1)$) = $\mathcal{M}$. Thus, $\mathcal{M}$ is n-CyNros. Now, $A \subseteq \mathcal{L}_1$ = n-CyNint$(\mathcal{L}_1) \subseteq$ n-CyNint(n-CyNcl$(\mathcal{L}_1)$) = $\mathcal{M}$. This implies that, $A \subseteq \mathcal{M}$ and n-CyNcl$(\mathcal{M})$ = n-CyNcl$(\mathcal{L}_1) \subseteq \mathcal{L}_2^c \subseteq B$ imply that $A \subseteq \mathcal{M} \subseteq$ n-CyNcl$(\mathcal{M}) \subseteq B$.

Definition 4.6 Analogous to the n-CyNAlmO map of Definition 2.23, a map $f: (X, \tau_1) \to (Y, \tau_2)$ between n-CyNts's is called a n-CyNO map if the image of every n-CyNos in (X, τ_1) is a n-CyNos in (Y, τ_2).

Theorem 4.4 Let $f: (X, \tau_1) \to (Y, \tau_2)$ be a bijective function. If f is a n-CyNMIrr map, a n-CyNO map and X is a $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space, then Y is a $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space.

Proof. Suppose X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space. Let S be any n-CyNMcs in Y, such that $y_\beta \notin S$. Since f is a n-CyNMIrr map, $f^{-1}(S)$ is n-CyNMcs in X. Since f is onto, there exists $x_\alpha \in X$, such that $y_\beta = f(x_\alpha)$. Now, $f(x_\alpha) = y_\beta \notin S \Rightarrow x_\alpha \notin f^{-1}(S)$. Since X is $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space, there exist n-CyNos's $\mathcal{L}$ and $\mathcal{M}$ in X, such that $x_\alpha \in \mathcal{L}$, $f^{-1}(S) \subseteq \mathcal{M}$ and $\mathcal{L} \cap \mathcal{M} = 0_{\mathrm{CyN}}$. Now, $x_\alpha \in \mathcal{L} \Rightarrow f(x_\alpha) \in f(\mathcal{L})$, i.e., $y_\beta \in f(\mathcal{L})$; and $f^{-1}(S) \subseteq \mathcal{M} \Rightarrow S = f(f^{-1}(S)) \subseteq f(\mathcal{M})$, since f is onto. Also, $\mathcal{L} \cap \mathcal{M} = 0_{\mathrm{CyN}} \Rightarrow f(\mathcal{L} \cap \mathcal{M}) = 0_{\mathrm{CyN}} \Rightarrow f(\mathcal{L}) \cap f(\mathcal{M}) = 0_{\mathrm{CyN}}$, the last step using that f is one-one. Since f is a n-CyNO map, $f(\mathcal{L})$ and $f(\mathcal{M})$ are disjoint n-CyNos's in Y containing y_β and S respectively. Thus, Y is a $\mathcal{S}$t n-CyNM$\mathcal{R}$eg space.

Remark 4.1 Theorems 4.1, 4.2, 4.3, 4.4, Propositions 4.1, 4.2, and Corollary 4.1 hold for n-CyN$\delta\mathcal{P}$os's and n-CyN$\theta\mathcal{S}$os's.

5 Real-Life Application: A Safety-Separation Model for Drug Delivery Technology Screening

Regulatory screening of a candidate drug delivery technology requires more than ranking alternatives: a defensible recommendation must be demonstrably separated, at the level of the supporting evidence, from the pool of technologies that have been rejected on safety grounds, so that the case for the recommended candidate cannot be said to overlap with the case against a rejected one. This separation requirement is precisely what the n-Cylindrical neutrosophic M-regularity axiom (Definition 3.1) formalises: a candidate point and an M-closed "rejected" set can be enclosed in M-open evidence pools that share no common support whatsoever.

5.1 Problem description

Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ again denote the five candidate drug delivery technologies of the earlier study (Liposomal nanocarriers x_1, Polymeric nanoparticles x_2, Dendrimer-based carriers x_3, Microneedle transdermal patches x_4, and Hydrogel-based depot systems x_5), and fix $n = 2$. Suppose x_1 (Liposomal nanocarriers) is the technology currently proposed for a targeted delivery programme, while accumulated evidence has raised safety concerns specifically about x_4 (Microneedle transdermal patches), owing to a poorly reproducible biocompatibility profile. We build an n-CyNts (X, τ_{CyN}) that (i) records this evidence and (ii) is rich enough to realise the M-regularity separation between x_1 and the technology-safety concern at x_4.

Let

$$H = \{\langle x_1, 1,1,0\rangle, \langle x_2, 1,1,0\rangle, \langle x_3, 1,1,0\rangle, \langle x_4, 0.3,0.2,0.7\rangle, \langle x_5, 1,1,0\rangle\},$$

$$L = \{\langle x_1, 0.9,0.7,0.2\rangle, \langle x_2, 0,0,1\rangle, \langle x_3, 0,0,1\rangle, \langle x_4, 0,0,1\rangle, \langle x_5, 0,0,1\rangle\},$$

$$M = \{\langle x_1, 0,0,1\rangle, \langle x_2, 0,0,1\rangle, \langle x_3, 0,0,1\rangle, \langle x_4, 0.8,0.85,0.2\rangle, \langle x_5, 0,0,1\rangle\}.$$

Here H is the broadly acceptable evidence pool (strong support at every technology except a marked dip at x_4); L is a tight evidence buffer concentrated at the proposed candidate x_1; and M is a containment pool concentrated at x_4, large enough to absorb the safety concern recorded there. Each of H, L, M satisfies $0 \leq \beta \leq 1$ and $0 \leq \alpha^2 + \gamma^2 \leq 1$ (e.g., at x_4, H gives $0.3^2 + 0.7^2 = 0.58 \leq 1$ and M gives $0.8^2 + 0.2^2 = 0.68 \leq 1$), so $H, L, M \in \tau_{CyN}(X)$ are legitimate n-CyNS's with $n = 2$.

Closing $\{0_X, 1_X, H, L, M\}$ under the finite n-CyN union and intersection of Definition 2.9 terminates, by exact-fraction Python computation, at a 9-element family $\tau = \{0_X, 1_X, L, M, H, L \cup M, H \cup M, H \cap M, L \cup (H \cap M)\}$,
which is closed under all further pairwise unions and intersections and therefore is an n-Cylindrical neutrosophic topology on X in the sense of Definition 2.10; in particular $H, L, M \in \tau$ are n-CyNos's, hence also n-CyNMos's, since every n-CyNos is an n-CyNMos (Definition 2.18).

Define $F = H^c = \{\langle x_1, 0,0,1\rangle, \langle x_2, 0,0,1\rangle, \langle x_3, 0,0,1\rangle, \langle x_4, 0.7,0.8,0.3\rangle, \langle x_5, 0,0,1\rangle\}$.
Since $H \in \tau$, F is an n-CyNcs and hence, in particular, an n-CyNMcs (Definition 2.18); F carries the safety-concern evidence entirely at x_4 and is negligible elsewhere, so F represents the (fuzzy) set of technologies rejected on safety grounds. By direct computation $x_1 \not\in F$, since $F(x_1) = 0_{CyN}$ while $L(x_1) = \langle 0.9,0.7,0.2\rangle \neq 0_{CyN}$.

5.2 Verification of M-regularity separation

By exact-fraction computation: [(a)]

1. $F \subseteq M$ (both sides agree at x_1, x_2, x_3, x_5, where $F = 0_{CyN}$, and at x_4, $0.7 \leq 0.8$, $0.8 \leq 0.85$, $0.3 \geq 0.2$);
2. $x_1 \in L$, since $L(x_1) = \langle 0.9,0.7,0.2\rangle \neq 0_{CyN}$;
3. $L \cap M = 0_X$ (their nonzero evidence is concentrated at the disjoint indices x_1 and x_4 respectively, so at every $x \in X$ at least one of $L(x), M(x)$ equals 0_{CyN}, forcing $L \cap M(x) = 0_{CyN}$).

Example 5.1 In the n-CyNts (X, τ) above, F is an n-CyNMcs and $x_1 \not\in F$, while $L, M \in \tau$ are n-CyNMos's with $x_1 \in L$, $F \subseteq M$ and $L \cap M = 0_X$. Hence (X, τ) satisfies the defining separation property of Definition 3.1 for this particular pair (x_1, F): the proposed candidate technology and the safety-rejected technology are separated by disjoint n-CyNMos's.

5.3 Interpretation and recommendation

Example 5.1 carries a direct regulatory meaning. The evidence buffer L around the proposed candidate x_1 and the containment pool M enclosing the safety concern at x_4 share no common support ($L \cap M = 0_X$): there is no fuzzy overlap, and in particular L and M are not n-CyN quasi-coincident ($L\bar{q}M$, in the sense recalled in Section 3), between the case for recommending x_1 and the case against x_4. Consequently, the recommendation of Liposomal nanocarriers (x_1) for the delivery programme cannot be confused, at any positive threshold of evidentiary overlap, with the rejected evidence surrounding Microneedle transdermal patches (x_4); M itself serves as an n-CyNM-open containment zone that quarantines the safety concern away from the recommended candidate. More generally, whenever the underlying n-CyNts of technology evidence is n-CyNM$\mathcal{R}$eg (Definition 3.1), every technology not implicated in a given safety-rejected set can be so separated, so that M-regularity provides a topological guarantee that recommended and rejected technologies never share ambiguous, unresolved evidentiary territory. This illustrates how the regularity theory of Sections 3 and 4 can underpin a rigorous, auditable safety-separation criterion in pharmaceutical technology screening.

6 Conclusion

In this paper, we have studied n-CyNM$\mathcal{R}$eg and $\mathcal{S}$t n-CyNM$\mathcal{R}$eg spaces using n-CyNMos's and n-CyNMcs's. We found their relations among themselves and with already existing spaces. Also, we discussed some basic properties and the characterizations of the spaces studied. A real-life application to a safety-separation model for drug delivery technology screening (Section 5) illustrates how the n-CyNM-regularity axiom guarantees that a recommended technology can always be enclosed in an M-open evidence pool disjoint from the M-open containment zone of a safety-rejected technology.

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