RESEARCH PAPER

Minimal and Maximal Open Sets in n-Cylindrical Neutrosophic Topological Spaces with Application to Drug Delivery Technology Selection

A.Sugapriya [1] and R. Vijayalakshmi [2]

[1,2] Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India. Email sugamaths24@gmail.com

[2] Arignar Anna Government Arts College, Namakkal - 637 002, India. Email: viji_lakshmi80@rediffmail.com

Corresponding Author: A. Sugapriya

Absract: The purpose of this paper is to define and study a new class of sets called n-Cylindrical neutrosophic minimal (resp. δ, δ pre, δ semi, θ, θ semi, e and M)-open sets and respective maximal open sets in n-Cylindrical neutrosophic topological spaces. Basic properties of n-Cylindrical neutrosophic minimal and maximal (resp. δ, δ pre, δ semi, θ, θ semi, e and M)-open (resp. closed) sets are analysed. As a real-life application, the notions of n-Cylindrical neutrosophic minimal and maximal M-open sets are employed to model a drug delivery technology selection problem, in which a nested n-Cylindrical neutrosophic screening topology built from expert assessments of candidate technologies is used to identify, respectively, the singularly best-supported technology and the largest defensible shortlist of technologies for further evaluation.

Keywords: n-Cylindrical neutrosophic M open set, n-Cylindrical neutrosophic minimal M open set, n-Cylindrical neutrosophic maximal M open set, Drug delivery technology selection, Real-life application.

How to cite this article: Sugapriya A, Vijayalakshmi R. Minimal and maximal open sets in n-cylindrical neutrosophic topological spaces with application to drug delivery technology selection. Int J Drug Deliv Technol. 2026;16(73s): 866-876. DOI: 10.25258/ijddt.16.73s.93

AMS(2000) Subject classification: 03E72, 54A05, 54A40.

1 Introduction

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [26], Chang [6] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy set (ifs), was introduced by Atanassov in 1986 [4]. Building on this, Coker [7] introduced the concept of intuitionistic fuzzy topological spaces (ifts) based on ifs's. Jeon et al. [10] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [18, 17], a new direction in uncertainty modeling emerged. Salama and Alblowi [13] introduced neutrosophic crisp set and neutrosophic topological spaces (Nts), extending ifts and incorporating degrees of membership, indeterminacy, and non-membership for each element. Neutrosophic has formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [3] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [24] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [14] introduced the concept of n-cylindrical neutrosophic set (abbreviated as n-CyNS), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic set (NS), n-CyNS represents the most extensive generalization of fuzzy sets. In this framework, the membership functions positive (α), neutral (β), and negative (γ) satisfy the conditions $0 \le \beta_A \le 1$ and $0 \le \alpha_A^n(x) + \gamma_A^n(x) \le 1, n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [16] introduced the notion of n-CyN continuity for functions between two n-cylindrical neutrosophic topological spaces (n-CyNts). They also defined the n-CyN interior (n-CyNint) and n-CyN closure (n-CyNcl) of subsets within n-CyNts.

In a related development, El-Maghrabi and Al-Juhani [8] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [12] also found M-open sets in nano topological spaces. Vadivel et al. [20, 21, 22] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [11] and Vadivel et al. [23] introduced M-open sets in fuzzy and neutrosophic nano topological spaces.

Research Gap: No investigation on some stronger and weaker forms n-Cylindrical neutrosophic minimal (resp. Maximal) open sets such as n-Cylindrical neutrosophic minimal (resp. Maximal) δ open set, n-Cylindrical neutrosophic minimal (resp. Maximal) δ-semi open set, n-Cylindrical neutrosophic minimal (resp. Maximal) δ-pre open set, n-Cylindrical neutrosophic minimal (resp. Maximal) θ open set, n-Cylindrical neutrosophic minimal (resp. Maximal) θ$\mathcal{S}$ open set, n-Cylindrical neutrosophic minimal (resp. Maximal) e open set and n-Cylindrical neutrosophic minimal (resp. Maximal) M open sets on n-Cylindrical neutrosophic topological space has been reported in the n-CyN literature.

The concept of fuzzy minimal and maximal open sets, introduced by Ittanagi and Wali [9], has played an important role in the literature. Swaminathan and Sivaraja [19] discussed properties of fuzzy minimal and maximal open sets and showed that a fuzzy topological space possessing both may be fuzzy disconnected. In Section 3, we introduce the notions of n-Cylindrical neutrosophic minimal (resp. δ, δ pre, δ semi, θ, θ semi, e and M) open sets. In Section 4, we develop their maximal counterparts along with various interesting results. Section 5 presents a real-life application of n-Cylindrical neutrosophic minimal and maximal M-open sets to a drug delivery technology selection problem. Section 6 presents our conclusions.

2 Preliminaries

This section covers some basic definitions and examples that will be useful in subsequent discussions.

Definition 2.1 [28] A fuzzy set (briefly, fs) A in X is defined by membership function $\mu_A: X \to [0,1]$ whose membership value $\mu_A(x)$ shows the degree to which $x \in X$ includes in the fuzzy set A, for all $x \in X$.

Definition 2.2 [6] A fuzzy topological space (briefly, fts) is a pair (X, τ), where X is any set and τ is a family of fuzzy sets in X satisfying following axioms:

1. $\phi, X \in \tau$,
2. If $A, B \in \tau$, then $A \cap B$ in τ,
3. If $A_i \in \tau$ for each $i \in I$, then $\cup A_i \in \tau$.

Definition 2.3 [4] An intuitionistic fuzzy set (briefly, ifs) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \gamma_A(x)\rangle : x \in X\}$ where $\alpha_A(x) \in [0,1]$ is called the degree of positive membership of x in A, $\gamma_A(x) \in [0,1]$ is called the degree of negative membership of x in A, and where $\alpha_A(x)$ and $\gamma_A(x)$ satisfy (for all $x \in X$) $(\alpha_A(x) + \gamma_A(x) \leq 1)$. ifs(X) denotes the set of all the ifs's on X.

Definition 2.4 [18] A neutrosophic set A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x), \beta_A(x), \gamma_A(x) \in [0,1], 0 \leq \alpha_A(x) + \beta_A(x) + \gamma_A(x) \leq 3$, for all $x \in X$. $\alpha_A(x)$ is the degree of positive membership, $\beta_A(x)$ is the degree of neutral and $\gamma_A(x)$ is the degree of negative membership. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent components and $\beta_A(x)$ is an independent component.

Definition 2.5 [13] A neutrosophic topology (Nt) on a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms:

1. $0_N, 1_N \in \tau_N$,
2. $G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$,
3. $\cup G_i \in \tau_N$, for all $\{G_i : i \in J\} \subseteq \tau_N$.

In this case the pair (X, τ_N) is called a neutrosophic topological space (briefly, Nts) and any neutrosophic set in τ_N is known as neutrosophic open set (briefly, Nos) in X. The elements of τ_N are called neutrosophic open sets. A neutrosophic set F is closed if and only if $C(F)$ is neutrosophic open.

Definition 2.6 [14] An n-cylindrical neutrosophic set (briefly, n-CyNS) A on X is an object of the form $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\alpha_A(x) \in [0,1]$ called the degree of positive membership of x in A, $\beta_A(x) \in [0,1]$ called the degree of neutral membership of x in A and $\gamma_A(x) \in [0,1]$ called the degree of negative membership of x in A, which satisfies the condition: (for all $x \in X$) $(0 \leq \beta_A(x) \leq 1)$ and $0 \leq \alpha_A^n(x) + \gamma_A^n(x) \leq 1$, where $n > 1$ is an integer. Here, $\alpha_A(x)$ and $\gamma_A(x)$ are dependent neutrosophic components and $\beta_A(x)$ is 100% independent.

For the convenience, $\langle \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle$ is called as n-cylindrical neutrosophic number (briefly, n - CyNN) and is denoted as $A = \{\langle \alpha_A, \beta_A, \gamma_A\rangle\}$.

Definition 2.7 [14] Let $\{A_i : i \in I\}$ be an arbitrary family of n-CyNS in X. Then, $\cap A_i = \{\langle x, \inf(\alpha_{A_i}(x)), \inf(\beta_{A_i}(x)), \sup(\gamma_{A_i}(x))\rangle : x \in X\}$. $\cup A_i = \{\langle x, \sup(\alpha_{A_i}(x)), \sup(\beta_{A_i}(x)), \inf(\gamma_{A_i}(x))\rangle : x \in X\}$.

Definition 2.8 [14] $0_{CyN} = \{\langle x, 0,0,1\rangle : x \in X\}$ and $1_{CyN} = \{\langle x, 1,1,0\rangle : x \in X\}$

Definition 2.9 [14] **(The Basic Connectives)** Let $\tau_{CyN}(X)$ denote the family of all n-CyNS's on X. [(i)]

1. **Inclusion:** For every two $A, B \in \tau_{CyN}(X)$, the inclusion of two n-CyNS's A and B is $A \subseteq B$ iff (for all $x \in X$, $\alpha_A(x) \leq \alpha_B(x)$ and $\beta_A(x) \leq \beta_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$).
2. **Equality:** For every two $A, B \in \tau_{CyN}(X)$, $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
3. **Union:** For every two $A, B \in \tau_{CyN}(X)$, the union of two n-CyNS's A and B is $A \cup B(x) = \{\langle x, \max(\alpha_A(x), \alpha_B(x)), \max(\beta_A(x), \beta_B(x)), \min(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.
4. **Intersection:** For every two $A, B \in \tau_{CyN}(X)$, the intersection of two n-CyNS's A and B is $A \cap B(x) = \{\langle x, \min(\alpha_A(x), \alpha_B(x)), \min(\beta_A(x), \beta_B(x)), \max(\gamma_A(x), \gamma_B(x))\rangle : x \in X\}$.
5. **Complementary:** For every $A \in \tau_{CyN}(X)$, the complement of an n-CyNS A is $A^c = \{\langle x, \gamma_A(x), 1 - \beta_A(x), \alpha_A(x)\rangle : x \in X\}$.

Remark 2.1 [14]

1. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$,
2. $A \cup B = B \cup A$ & $A \cap B = B \cap A$,
3. $(A \cup B) \cup C = A \cup (B \cup C)$ & $(A \cap B) \cap C = A \cap (B \cap C)$,
4. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ & $(A \cap B) \cup C = (A \cup C) \cap (B \cap C)$,
5. $A \cap A = A$ & $A \cup A = A$,
6. De Morgan's Law for A & B ie., $(A \cup B)^c = A^c \cap B^c$ & $(A \cap B)^c = A^c \cup B^c$.

Definition 2.10 [16] An n-cylindrical neutrosophic topology (briefly, n-CyNt) on a non-empty set X is a family, τ_{CyN}, of n-CyNS in X which satisfies the following conditions:

1. $0_{CyN}, 1_{CyN} \in \tau_{CyN}$,
2. $A_1 \cap A_2 \in \tau_{CyN}$,
3. $\cup A_i \in \tau_{CyN}$, for any arbitrary family $A_i \in \tau_{CyN}, i \in I$.

The pair (X, τ_{CyN}) is called an n-cylindrical neutrosophic topological space (briefly, n-CyNts) and any n-CyNS that belongs to τ_{CyN} is called an n-cylindrical neutrosophic open set (briefly, n-CyNos) and the complement of n-CyNos is called n-cylindrical neutrosophic closed set (briefly, n-CyNcs) in X. Like classical topological spaces and fuzzy topological spaces, the family $\{0_{CyN}, 1_{CyN}\}$ is called indiscrete n-CyNts and the topology containing all the n-CyN subsets is called discrete n-CyNts.

Remark 2.2 [16] Obviously any fuzzy topological spaces or intuitionistic fuzzy topological spaces or Pythagorean fuzzy topological spaces is an n-CyNts as any subsets of the fuzzy spaces, intuitionistic fuzzy space, and Pythagorean fuzzy space can be viewed as n-CyN subsets.

Definition 2.11 [16] Let A and B be two n-cylindrical neutrosophic subsets of an n-CyNts. B is called neighbourhood of A if there exists an n-CyNos, O such that $A \subset O \subset B$.

Proposition 2.1 [16] $A \subset X$ is n-cylindrical neutrosophic open in (X, τ_{CyN}) if and only if it carries a neighbourhood of its subsets.

Definition 2.12 [16] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic interior (briefly, n-CyNint) is defined as the n-CyN union of all n-CyN open subsets of X contained in A. ie, $\text{n-CyNint}(A) = \cup \{G : G \in \tau_{CyN} \text{ and } G \subseteq A\}$.

Clearly, n-CyNint(A) is the biggest n-CyNos that is contained by A.

Definition 2.13 [16] Let (X, τ_{CyN}) be an n-CyNts and let $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic closure (briefly, n-CyNcl) is defined as the n-CyN intersection of all n-CyN closed subsets of X containing A. ie, $\text{n-CyNcl}(A) = \cap \{K : K \text{ isan n} - \text{CyNcs and } A \subseteq K\}$.

Clearly, n-CyNcl(A) is the smallest n-CyNcs that contains A.

Definition 2.14 [15] Let (X, τ_1) and (Y, τ_2) be two n-CyNts and let $f: (X, \tau_1) \to (Y, \tau_2)$ be a n-CyN function. Then, f is said to be n-CyN continuous (briefly, n-CyNCts) map if for any n-cylindrical neutrosophic subset A of X and for any neighbourhood $\mathfrak{V}$ of f(A) there exists a neighbourhood $\mathfrak{U}$ of A such that $f(\mathfrak{U}) \subset \mathfrak{V}$.

Definition 2.15 [1] Let (X, τ_{CyN}) be an n-CyNts and A be an n-CyNS. Then, A is said to be an n-CyN

1. regular open set (briefly, n-CyNros), if A = n-CyNint(n-CyNcl(A)),
2. regular closed set (briefly, n-CyNrcs), if A = n-CyNcl(n-CyNint(A)).

Definition 2.16 [1] Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-cylindrical neutrosophic δ-interior of A and the n-cylindrical neutrosophic δ-closure of A are denoted by n-CyNδint(A) and n-CyNδcl(A) are defined as follows: [(i)]

1. $\text{n-CyN}\delta\text{int}(A) = \cup \{G | G \text{ isan n-CyNros and } G \subseteq A\}$,
2. $\text{n-CyN}\delta\text{cl}(A) = \cap \{K | K \text{ isan n-CyNrcs and } A \subseteq K\}$.

Definition 2.17 [1] Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$

be an n-CyNS in X. Then, the n-cylindrical neutrosophic θ-interior of A and the n-cylindrical neutrosophic θ-closure of A are denoted by n-CyNθint(A) and n-CyNθcl(A) are defined as follows: [(i)]

1. n - CyNθint(A) = ⋃ {n − CyNint(B): B ⊆ A & B isa n − CyNcs in X},
2. n - CyNθcl(A) = ⋂ {n − CyNcl(B): A ⊆ B & B isa n − CyNos in X}.

Definition 2.18 [1] Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. A set A is said to be n-CyN [(i)]

1. δ-open set (briefly, n-CyNδos), if A = n-CyNδint(A),
2. δ-pre open set (briefly, n-CyNδ𝒫os), if A ⊆ n-CyNint(n-CyNδcl(A)),
3. δ-semi open set (briefly, n-CyNδ𝒮os), if A ⊆ n-CyNcl(n-CyNδint(A)),
4. θ-open set (briefly, n-CyNθos), if A = n-CyNθint(A),
5. θ-semi open set (briefly, n-CyNθ𝒮os), if A ⊆ n-CyNcl(n-CyNθint(A)),
6. e-open set (briefly, n-CyNeos), if A = n - CyNcl(n - CyNδint(A)) ∪ n-CyNint(n-CyNδcl(A)),
7. M-open set (briefly, n-CyNMos), if A ⊆ n - CyNcl(n - CyNθint(A)) ∪ n-CyNint(n-CyNδcl(A)).

The complement of a n-CyNMos (resp. n-CyNδos, n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNθos, n-CyNθ𝒮os & n-CyNeos) is called a n-CyNM (resp. n-CyNδ, n-CyNδ𝒫, n-CyNδ𝒮, n-CyNθ, n-CyNθ𝒮 & n-CyNe) closed set (briefly, n-CyNMcs (resp. n-CyNδcs, n-CyNδ𝒫cs, n-CyNδ𝒮cs, n-CyNθcs, n-CyNθ𝒮cs & n-CyNecs)) in X.

The family of all n-CyNMos (resp. n-CyNδos, n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNθos, n-CyNθ𝒮os & n-CyNeos) of X is denoted by n - CyNMOS(X), (resp. n - CyNMCS(X), n-CyNδOS(X), n-CyNδCS(X), n-CyNδ𝒫OS(X), n-CyNδ𝒫CS(X), n-CyNδ𝒮OS(X), n-CyNδ𝒮CS(X), n-CyNθOS(X), n-CyNθCS(X), n-CyNθ𝒮OS(X), n-CyNθ𝒮CS(X), n-CyNeOS(X) & n-CyNeCS(X)).

Definition 2.19 [1] Let (X, τ_{CyN}) be an n-CyNts and $A = \{\langle x, \alpha_A(x), \beta_A(x), \gamma_A(x)\rangle : x \in X\}$ be an n-CyNS in X. Then, the n-CyN

1. M-interior (resp. n-CyNδ-interior, n - CyNδ𝒫 -interior, n - CyNδ𝒮 -interior, n - CyNθ -interior, n - CyNθ𝒮 -interior & n-CyNe-interior) of A (briefly, n-CyNMint(A) (resp. n - CyNδint(A), n - CyNδ𝒫int(A), n-CyNδ𝒮int(A), n-CyNθ int(A), n-CyNθ𝒮int(A) & n-CyNeint(A)) is defined by n-CyNMint(A) (resp. n-CyNδint(A), n-CyNδ𝒫int(A), n-CyNδ𝒮int(A), n-CyNθint(A), n-CyNθ𝒮int(A) & n-CyNeint(A)) = ⋃ {G: G ⊆ A and G is a n-CyNMos (resp. n-CyNδos, n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNθos, n-CyNθ𝒮os & n-CyNeos) in X},
2. M-closure (resp. n-CyNδ-closure, n - CyNδ𝒫 -closure, n - CyNδ𝒮 -closure, n - CyNθ -closure, n - CyNθ𝒮 -closure & n-CyNe-closure) of A (briefly, n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδ𝒫cl(A), n-CyNδ𝒮cl(A), n-CyNθcl(A), n-CyNθ𝒮cl(A) & n-CyNecl(A)) is defined by n-CyNMcl(A) (resp. n-CyNδcl(A), n-CyNδ𝒫cl(A), n-CyNδ𝒮cl(A), n-CyNθcl(A), n-CyNθ𝒮cl(A) & n-CyNecl(A)) = ⋂ {K: K ⊆ A and K is a n-CyNMcs (resp. n-CyNδcs, n-CyNδ𝒫cs, n-CyNδ𝒮cs, n-CyNθcs, n-CyNθ𝒮cs & n-CyNecs) in X}.

3 n-Cylindrical neutrosophic minimal (resp. δ, δ pre, δ semi, θ, θ semi, e and M)-open sets

We now define n-Cylindrical neutrosophic minimal (resp. δ, δ pre, δ semi, θ, θ semi, e and M)-open set.

Definition 3.1 A proper nonzero n-CyNos (resp. n-CyNδos, n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNθos, n-CyNθ𝒮os, n-CyNeos and n-CyNMos) E in n-Cylindrical neutrosophic topological space (X, τ) is said to be n-Cylindrical neutrosophic minimal (resp. δ, δ pre, δ semi, θ, θ semi, e and M)-open set (briefly, n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) if a n-CyNos (resp. n-CyNδos, n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNθos, n-CyNθ𝒮os, n-CyNeos and n-CyNMos) contained in E is 0_X or E.

Example 3.1 Let $X = \{x_1, x_2\}$ and let A_1, A_2 be n-cylindrical neutrosophic sets in X defined as

$$A_1 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\},$$
$$A_2 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}.$$

Let $\tau_1 = \{0_X, 1_X, A_1, A_2\}$. Then τ_1 is a n-Cylindrical neutrosophic topology on X, with $A_1 \subset A_2$ and $A_1^c = A_2$. Here, A_1 is n-CyNmin o, n-CyNmin δo, n-CyNmin δ𝒫o, n-CyNmin δ𝒮o, n-CyNmin θo, n-CyNmin θ𝒮o, n-CyNmin eo and n-CyNmin Mo set. Since A_1 is a proper nonzero n-CyN open set of each of the above eight types with $A_1 \subset A_2$, it follows from Definition 3.1 that A_2 is not a n-CyNmin open set of any of the above types.

Lemma 3.1 Let (X, τ) be a n-Cylindrical neutrosophic topological space.

1. If E_1 is n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) and E_2 is n-CyNos (resp. n-CyNδos, n-CyNδ𝒫os, n-CyNδ𝒮os, n-CyNθos, n-CyNθ𝒮os, n-CyNeos and n-CyNMos) in (X,τ), then $E_1 \cap E_2 = 0_X$ or $E_1 \subseteq E_2$.

2. If E_1 and E_2 are n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos), then $E_1 \cap E_2 = 0_X$ or $E_1 = E_2$.

Proof.

1. Let E_2 be a n-CyNMos in (X,τ) and suppose $E_1 \cap E_2 \neq 0_X$. Since E_1 and E_2 are both n-CyNMos's, $E_1 \cap E_2$ is a n-CyNMos with $E_1 \cap E_2 \subseteq E_1$. As E_1 is n-CyNminMos, Definition 3.1 gives $E_1 \cap E_2 = 0_X$ or $E_1 \cap E_2 = E_1$. Since $E_1 \cap E_2 \neq 0_X$ by assumption, we get $E_1 \cap E_2 = E_1$, that is, $E_1 \subseteq E_2$.

2. Suppose that $E_1 \cap E_2 \neq 0_X$. Since E_1 and E_2 are n-CyNminMos, both are in particular n-CyNMos's, so (i) applies in both directions: $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$. Hence $E_1 = E_2$. The proof of the others are similar.

Theorem 3.1 Let E and E_i be n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$. If $E \subseteq \bigcup_{i\in M} E_i$, then $E = E_j$ for some $j \in M$.

Proof. Suppose $E \subseteq \bigcup_{i\in M} E_i$, then $E = E \cap (\bigcup_{i\in M} E_i) = \bigcup_{i\in M} (E \cap E_i)$. By deploying Lemma 3.1 (ii), $E \cap E_i = 0_X$ or $E = E_i$ for each $i \in M$, as E and E_i are n-CyNminMos's. If $E \cap E_i = 0_X$ for every $i \in M$, then $E = \bigcup_{i\in M} (E \cap E_i) = 0_X$, which contradicts that E is a n-CyNminMos. Hence $E \cap E_j \neq 0_X$ for some $j \in M$, and so $E = E_j$ for that $j \in M$. The proof of the others are similar

Theorem 3.2 If E and E_i are n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ and $E \neq E_i$ for every $i \in M$, then $E \cap (\bigcup_{i\in M} E_i) = 0_X$.

Proof. Suppose $E \cap (\bigcup_{i\in M} E_i) \neq 0_X$. Then $\bigcup_{i\in M} (E \cap E_i) \neq 0_X$, so $E \cap E_i \neq 0_X$ for some $i \in M$. By deploying Lemma 3.1 (ii), $E = E_i$ for that i, contradictory to $E \neq E_i$. Hence $E \cap (\bigcup_{i\in M} E_i) = 0_X$.

Theorem 3.3 If E_i is a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ ($|M| \geq 2$) and $E_i \neq E_j$ for any distinct $i, j \in M$, then $(\bigcup_{i\in M\setminus\{j\}} E_i) \cap E_j = 0_X$ for any $j \in M$.

Proof. Fix $j \in M$ and suppose $\left(\bigcup_{i\in M\{j\}} E_i\right) \cap E_j \neq 0_X$. Then $\bigcup_{i\in M\{j\}} \left(E_i \cap E_j\right) \neq 0_X$, so $E_i \cap E_j \neq 0_X$ for some $i \in M\{j\}$. By Lemma 3.1 (ii), $E_i = E_j$ for that i, contradicting $E_i \neq E_j$ for distinct I, j. Hence $\left(\bigcup_{i\in M\{j\}} E_i\right) \cap E_j = 0_X$ for any $j \in M$

Theorem 3.4 Let E_i be a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$, ($|M| \geq 2$), with $E_i \neq E_j$ for any distinct $i, j \in M$. If K is a proper subset of M, then $(\bigcup_{i\in M\setminus K} E_i) \cap (\bigcup_{s\in K} E_s) = 0_X$.

Proof. Suppose $(\bigcup_{i\in M\setminus K} E_i) \cap (\bigcup_{s\in K} E_s) \neq 0_X$. Then $\bigcup (E_i \cap E_s) \neq 0_X$ for $i \in M\setminus K$ and $s \in K$, so $E_i \cap E_s \neq 0_X$ for some $i \in M\setminus K$ and $s \in K$. By Lemma 3.1 (ii), $E_i = E_s$, which is a contradiction since $i \neq s$ are distinct indices in M. Hence $(\bigcup_{i\in M\setminus K} E_i) \cap (\bigcup_{s\in K} E_s) = 0_X$.

Theorem 3.5 If E_i is a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$. If K is a proper nonzero subset of M, then $[\bigcup_{i\in M\setminus K} E_i] \cap [\bigcup_{k\in K} E_k] = 0_X$.

Proof. Suppose $[\bigcup_{i\in M\setminus K} E_i] \cap [\bigcup_{k\in K} E_k] \neq 0_X$. Then $\bigcup (E_i \cap E_k) \neq 0_X$ for $i \in M\setminus K, k \in K$, so $E_i \cap E_k \neq 0_X$ for some $i \in M\setminus K, k \in K$. By deploying Lemma 3.1 (ii), $E_i = E_k$, a contradiction since $i \neq k$ are distinct indices in M. Hence $[\bigcup_{i\in M\setminus K} E_i] \cap [\bigcup_{k\in K} E_k] = 0_X$.

Theorem 3.6 Let E_i be a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$, and let E_n be of the same type for any $n \in K$. If there exists an $n \in K$ such that $E_n \neq E_i$ for any $i \in M$, then $[\bigcup_{n\in K} E_n] \not\subseteq [\bigcup_{i\in M} E_i]$.

Proof. Suppose, for contradiction, that $[\bigcup_{n\in K} E_n] \subseteq [\bigcup_{i\in M} E_i]$. Let $n \in K$ be the index with $E_n \neq E_i$ for any $i \in M$. Since $E_n \subseteq \bigcup_{n\in K} E_n \subseteq \bigcup_{i\in M} E_i$, Theorem 3.1 gives $E_n = E_i$ for some $i \in M$, contradicting $E_n \neq E_i$ for any $i \in M$. Hence $[\bigcup_{n\in K} E_n] \not\subseteq [\bigcup_{i\in M} E_i]$.

Theorem 3.7 If E_i is a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$, then $[\bigcup_{k\in K} E_k] \subsetneq [\bigcup_{i\in M} E_i]$ for any proper nonzero subset

$K \subseteq M$.

Proof. Since $K \subseteq M$, clearly $[\bigcup_{k\in K} E_k] \subseteq [\bigcup_{i\in M} E_i]$. It remains to show the inclusion is strict. Let $m \in M\backslash K$ (nonempty as K is proper), then E_m is a n-CyNminMos set of the family $\{E_m|m \in M\backslash K\}$ of n-CyNminMos sets. Since $m \neq k$ for each $k \in K$, Lemma 3.1 (ii) gives $E_m \cap E_k = 0_X$, so $E_m \cap [\bigcup_{k\in K} E_k] = \bigcup_{k\in K} [E_m \cap E_k] = 0_X$.

Also $E_m \cap [\bigcup_{i\in M} E_i] = \bigcup_{i\in M} [E_m \cap E_i] = E_m$.

If $[\bigcup_{k\in K} E_k] = [\bigcup_{i\in M} E_i]$, then intersecting both sides with E_m gives $E_m = 0_X$, which contradicts that E_m is a n-CyNminMos. Hence $[\bigcup_{k\in K} E_k] \subsetneq [\bigcup_{i\in M} E_i]$. The proof of others are similar.

Theorem 3.8 If E_i is a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$. then

1. $[\bigcup_{i\in M\backslash\{j\}} E_i] \subseteq E_j^c$ for any $j \in M$.
2. $\bigcup_{i\in M\backslash\{j\}} E_i \neq 1_X$ for any $j \in M$.

Proof.

1. By hypothesis, $E_i \neq E_j$ for any distinct $i, j \in M$.

By Theorem 3.2, $[\bigcup_{i\in M} E_i] \cap E_j = 0_X$ which is true for any $j \in M$

$$\Rightarrow \bigcup_{i\in M} [E_i \cap E_j] = 0_X$$

$\Rightarrow E_i \cap E_j = 0_X$ for each $i \neq j$ (By Lemma 3.1(ii))

$\Rightarrow E_i \subseteq E_j^c$ for each $i \in M\backslash\{j\}$

$\Rightarrow \bigcup_{i\in M\backslash\{j\}} E_i \subseteq E_j^c$. Hence proved.

2. Let $j \in M$ such that $\bigcup_{i\in M\backslash\{j\}} E_i = 1_X$. By (i), $1_X = \bigcup_{i\in M\backslash\{j\}} E_i \subseteq E_j^c$

$$\Rightarrow E_j^c = 1_X \Rightarrow E_j = 0_X$$

$\Rightarrow E_j$ is not a n-CyNminMos set, a contradiction. Hence $\bigcup_{i\in M\backslash\{j\}} E_i \neq 1_X$ for any $j \in M$. The proof of others are similar.

Corollary 3.1 If E_i is a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ ($|M| \geq 3$) such that $E_i \neq E_j$ for any distinct $i, j \in M$, then $E_i \cup E_j \neq 1_X$ for any distinct $i, j \in M$.

Proof. Let $i, j \in M$ be distinct. Since $|M| \geq 3$, choose $l \in M\backslash\{i, j\}$. As $i, j \in M\backslash\{l\}$, by Theorem 3.8 (i), $E_i \cup E_j \subseteq \bigcup_{k\in M\backslash\{l\}} E_k \subseteq E_l^c$. Since E_l is n-CyNminMos, $E_l \neq 0_X$, so $E_l^c \neq 1_X$. Hence $E_i \cup E_j \subseteq E_l^c \neq 1_X$, giving $E_i \cup E_j \neq 1_X$

Theorem 3.9 If E_i is a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$. then $E_j = [\bigcup_{i\in M} E_i] \cap [\bigcup_{i\in M\backslash\{j\}} E_i]^c$ for any $j \in M$.

Proof. For any $j \in M$, $[\bigcup_{i\in M} E_i] \cap [\bigcup_{i\in M\backslash\{j\}} E_i]^c = [\bigcup_{i\in M\backslash\{j\}} E_i \cup E_j] \cap [\bigcup_{i\in M\backslash\{j\}} E_i]^c$

$$= [(\bigcup_{i\in M\backslash\{j\}} E_i) \cap (\bigcup_{i\in M\backslash\{j\}} E_i)^c] \bigcup [E_j \cap (\bigcup_{i\in M\backslash\{j\}} E_i)^c]$$

$$= 0_X \cup E_j$$

$= E_j$ for any $j \in M$.

Proposition 3.1 Let G be a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) set in (X, τ) and $x_\alpha \in G$. Then $G \subset G_1$ for any n-CyN (resp. n-CyNδ, n-CyNδ𝒫, n-CyNδ𝒮, n-CyNθ, n-CyNθ𝒮, n-CyNe and n-CyNM)-open neighbourhood G_1 of x_α.

Proof. Let G_1 be an n-CyNM-open neighbourhood of x_α such that $G \not\subset G_1$. Clearly $G \cap G_1$ is an n-CyNMos such that $G \cap G_1 \subsetneq G$ and $G \cap G_1 \neq 0_X$. By Definition 3.1, this implies that G is not a n-CyNminMos set, which is a contradiction. The proof of others are similar.

Proposition 3.2 Let G be a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) set in (X, τ) and $x_\alpha \in G$. Then $G = \bigcap \{G_1 : G_1$ is a n-CyN (resp. n-CyNδ, n-CyNδ𝒫, n-CyNδ𝒮, n-CyNθ, n-CyNθ𝒮, n-CyNe and n-CyNM)-open neighbourhood of $x_\alpha\}$.

Proof. By Proposition 3.1, $G \subseteq G_1$ for every n-CyNM-open neighbourhood G_1 of x_α, so $G \subseteq \bigcap \{G_1 : G_1$ is a n-CyNM-open neighbourhood of $x_\alpha\}$. Conversely, since G itself is a n-CyNM-open neighbourhood of x_α (as $x_\alpha \in G$ and G is n-CyNM-open), G is one of the sets in the family being intersected, so $\bigcap \{G_1 : G_1$ is a n-CyNM-open neighbourhood of $x_\alpha\} \subseteq G$. Hence $G = \bigcap \{G_1 : G_1$ is a n-CyNM-open neighbourhood of $x_\alpha\}$. The proof of others are similar.

Theorem 3.10 Let G be a n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos). Then the following conditions are equivalent.

1. G is n-CyNminos (resp. n-CyNminδos, n-CyNminδ𝒫os, n-CyNminδ𝒮os, n-CyNminθos, n-CyNminθ𝒮os, n-CyNmineos and n-CyNminMos) set.

2. $G \subset$ n-CyNCl(K) (resp. $\subset$ n-CyNδCl(K), $\subset$ n-CyN$\delta\mathcal{P}$Cl(K), $\subset$ n-CyN$\delta\mathcal{S}$Cl(K), $\subset$ n-CyNθCl(K), $\subset$ n-CyN$\theta\mathcal{S}$Cl(K), $\subset$ n-CyNeCl(K) and $\subset$ n-CyNMCl(K)) for any nonzero subset $K \subseteq G$.

3. n-CyNCl(G) = n-CyNCl(K) (resp. n-CyNδCl(G) = n-CyNδCl(K), n-CyN$\delta\mathcal{P}$Cl(G) = n-CyN$\delta\mathcal{P}$Cl (K), n-CyN$\delta\mathcal{S}$Cl(G) = n-CyN$\delta\mathcal{S}$Cl(K), n-CyNθCl(G) = n-CyNθCl(K), n-CyN$\theta\mathcal{S}$Cl(G) = n-CyN$\theta\mathcal{S}$Cl(K), n-CyNeCl(G) = n-CyNeCl(K) and n-CyNMCl(G) = n-CyNMCl(K)) for any nonzero subset $K \subseteq G$.

Proof. (i) $\Rightarrow$ (ii) : Let $K \subseteq G$ be nonzero and let $x_\alpha \in G$. Let G_1 be any n-CyNM-open neighbourhood of x_α. By Proposition 3.1, $G \subseteq G_1$. Since $K \subseteq G \subseteq G_1$ and $K \neq 0_X$, we get $G_1 \cap K = K \neq 0_X$. As G_1 was an arbitrary n-CyNM-open neighbourhood of x_α, it follows that $x_\alpha \in$ n-CyNMCl(K). Since $x_\alpha \in G$ was arbitrary, $G \subset$ n-CyNMCl(K).

(ii) $\Rightarrow$ (iii) : For any nonzero subset $K \subseteq G$, monotonicity of the closure operator gives n-CyNMCl(K) $\subseteq$ n-CyNMCl(G). Conversely, by (ii) applied with K nonzero, $G \subset$ n-CyNMCl(K), so n-CyNMCl(G) $\subset$ n-CyNMCl(n-CyNMCl(K)) = n-CyNMCl(K). Hence n-CyNMCl(G) = n-CyNMCl(K).

(iii) $\Rightarrow$ (i): Let us assume that G is not n-CyNminMos. Then there exists a proper nonzero n-CyNminMos D such that $D \subset G$. Then there exists $y_\alpha \in G$ such that $y_\alpha \notin D$. Then n-CyNMCl($\{y_\alpha\}$) $\subseteq D^c$ implies that n-CyNMCl($\{y_\alpha\}$) $\neq$ n-CyNMCl(G), a contradiction. This completes our proof. The proof of others are similar

4 n-Cylindrical neutrosophic maximal (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M)-open sets and its properties

Now we introduce n-Cylindrical neutrosophic maximal (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M)-open set.

Definition 4.1 A proper nonzero n-Cylindrical neutrosophic (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M)-open set (briefly, n-CyNos (resp. n-CyNδos, n-CyN$\delta\mathcal{P}$os, n-CyN$\delta\mathcal{S}$os, n-CyNθos, n-CyN$\theta\mathcal{S}$os, n-CyNeos and n-CyNMos) F of a n-Cylindrical neutrosophic topological space (X, τ) is said to be n-Cylindrical neutrosophic maximal (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M)-open (briefly, n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmax$\delta\mathcal{P}$os, n-CyNmax$\delta\mathcal{S}$os, n-CyNmaxθos, n-CyNmax$\theta\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos)) if any n-CyNos (resp. n-CyNδos, n-CyN$\delta\mathcal{P}$os, n-CyN$\delta\mathcal{S}$os, n-CyNθos, n-CyN$\theta\mathcal{S}$os, n-CyNeos and n-CyNMos) which contains F is either F or 1_X.

Example 4.1 In Example 3.1, since $A_1 \subset A_2$ and $A_1^c = A_2$, the set A_2 is n-CyNmax o, n-CyNmax δo, n-CyNmax $\delta\mathcal{P}$o, n-CyNmax $\delta\mathcal{S}$o, n-CyNmax θo, n-CyNmax $\theta\mathcal{S}$o, n-CyNmax eo and n-CyNmax Mo set. Since A_2 is a n-CyN open set of each of the above eight types with $A_1 \subset A_2 \subset 1_X$, it follows from Definition 4.1 that A_1 is not a n-CyNmax open set of any of the above types.

Lemma 4.1 Let (X, τ) be a n-Cylindrical neutrosophic topological space. Then [(i)]

1. If F_1 is a n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmax$\delta\mathcal{P}$os, n-CyNmax$\delta\mathcal{S}$os, n-CyNmaxθos, n-CyNmax$\theta\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos) and F_2 is n-CyNos (resp. n-CyNδos, n-CyN$\delta\mathcal{P}$os, n-CyN$\delta\mathcal{S}$os, n-CyNθos, n-CyN$\theta\mathcal{S}$os, n-CyNeos and n-CyNMos) in X, then $F_1 \cup F_2 = 1_X$ or $F_2 \subset F_1$.

2. If F_1 and F_3 are n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmax$\delta\mathcal{P}$os, n-CyNmax$\delta\mathcal{S}$os, n-CyNmaxθos, n-CyNmax$\theta\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos)'s, then either $F_1 \cup F_3 = 1_X$ or $F_1 = F_3$.

Proof.

1. Let F_2 be a n-CyNMos in X and assume that $F_2 \not\subset F_1$. Since F_1 and F_2 are n-CyNMos's, $F_1 \cup F_2$ is a n-CyNMos containing F_1. As F_1 is n-CyNmaxMos, Definition 4.1 gives $F_1 \cup F_2 = F_1$ or $F_1 \cup F_2 = 1_X$. Now $F_1 \cup F_2 = F_1$ would force $F_2 \subset F_1$, contrary to our assumption. Hence $(F_1 \cup F_2) = 1_X$.

2. Let F_1 and F_3 be n-CyNmaxMos's and suppose $F_1 \cup F_3 \neq 1_X$. Applying (i) with F_1 as the maximal set and F_3 as the n-CyNMos, we get $F_3 \subset F_1$ (the alternative $F_1 \cup F_3 = 1_X$ being excluded). Applying (i) symmetrically with F_3 as the maximal set and F_1 as the n-CyNMos, we get $F_1 \subset F_3$. Hence $F_1 = F_3$. The proof of others are similar.

Theorem 4.1 If F_1, F_2 and F_3 are n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmax$\delta\mathcal{P}$os, n-CyNmax$\delta\mathcal{S}$os, n-CyNmaxθos, n-CyNmax$\theta\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos) such that $F_1 \neq F_2$ and $(F_1 \cap F_2) \subset F_3$, then either $F_1 = F_3$ or $F_2 = F_3$.

Proof. Suppose that F_1, F_2 and F_3 are n-CyNmaxMos's with $F_1 \neq F_2$, $(F_1 \cap F_2) \subset F_3$, and if $F_1 \neq F_3$, then

$$\begin{aligned}(F_2 \cap F_3) &= F_2 \cap (F_3 \cap 1_X)\\ &= F_2 \cap [F_3 \cap (F_1 \cup F_2)], \text{by Lemma 4.1 (ii), since} F_1 \neq F_2\\ &= F_2 \cap [(F_3 \cap F_1) \cup (F_3 \cap F_2)]\\ &= [F_2 \cap F_3 \cap F_1] \cup [F_2 \cap F_3 \cap F_2]\\ &= [(F_2 \cap F_1)] \cup [(F_2 \cap F_3)], \text{since} (F_1 \cap F_2) \subset F_3\\ &= F_2 \cap [(F_1 \cup F_3)]\end{aligned}$$

$= F_2 \cap$
1_X, by Lemma 4.14.1 (ii), since $F_1 \neq F_3$
$= F_2$
$(F_2 \cap F_3) = F_2 \Rightarrow F_2 \subset F_3$. Since F_2 is n-CyNmaxMos and $F_2 \subset F_3$ with F_3 a n-CyNMos, Definition 4.1 gives $F_3 = F_2$ or $F_3 = 1_X$. As F_3 is n-CyNmaxMos, $F_3 \neq 1_X$, so $F_3 \subset F_2$. Hence $F_2 = F_3$.

Theorem 4.2 For any distinct n - CyNmaxos (resp. n - CyNmaxδos , n-CyNmaxδ𝒫os, n-CyNmaxδ𝒮os, n-CyNmaxθos, n - CyNmaxθ𝒮os , n - CyNmaxeos and n - CyNmaxMos) F_1, F_2 and F_3 we have $[F_1 \cap F_2] \not\subset [F_1 \cap F_3]$.

Proof. Suppose, for contradiction, that $[F_1 \cap F_2] \subset [F_1 \cap F_3]$ for distinct n-CyNmaxMos's F_1, F_2 and F_3. Then

$$[F_1 \cap F_2] \cup [F_2 \cap F_3] \subset [F_1 \cap F_3] \cup [F_2 \cap F_3].$$

The left side factors as $F_2 \cap (F_1 \cup F_3)$, and the right side factors as $F_3 \cap (F_1 \cup F_2)$, so

$$F_2 \cap (F_1 \cup F_3) \subset F_3 \cap (F_1 \cup F_2).$$

Since $F_1 \neq F_3$, by Lemma 4.1 (ii), $F_1 \cup F_3 = 1_X$; and since $F_1 \neq F_2$, by Lemma 4.1 (ii), $F_1 \cup F_2 = 1_X$. Hence

$$F_2 \cap 1_X \subset F_3 \cap 1_X \Rightarrow F_2 \subset F_3.$$

Since F_2 is n-CyNmaxMos and F_3 is a n-CyNMos containing F_2, Definition 4.1 gives $F_3 = F_2$ or $F_3 = 1_X$. As F_3 is n-CyNmaxMos, $F_3 \neq 1_X$, so $F_3 = F_2$, contradicting that F_1, F_2 and F_3 are distinct. Hence $[F_1 \cap F_2] \not\subset [F_1 \cap$

Remark 4.1 Proofs of Theorem 4.3, Corollary 4.1, Theorem 4.4 and Theorem 4.5 are similar to proofs of Theorem 3.8, Corollary 3.1, Theorem 3.9 and Theorem ?? respectively, hence the proofs are omitted.

Theorem 4.3 If F_i is a n-CyNmaxos (resp. n - CyNmaxδos , n - CyNmaxδ𝒫os, n-CyNmaxδ𝒮os, n-CyNmaxθos, n-CyNmaxθ𝒮os, n-CyNmaxeos and n-CyNmaxMos)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, then

1. $[\bigcap_{i\in M\setminus\{j\}} F_i]^c \subset F_j$ for any $j \in M$.
2. $\bigcap_{i\in M\setminus\{j\}} F_i \neq 0_X$ for any $j \in M$.

Proof.

1. By hypothesis, $F_i \neq F_j$ for any distinct $i, j \in M$. By Lemma 4.1 (ii), $F_i \cup F_j = 1_X$ for each $i \in M\setminus\{j\}$. Since this holds for every $i \in M\setminus\{j\}$,

$$\bigcap_{i\in M\setminus\{j\}} (F_i \cup F_j) = 1_X.$$

By the distributive law, $\bigcap_{i\in M\setminus\{j\}} (F_i \cup F_j) = [\bigcap_{i\in M\setminus\{j\}} F_i] \cup F_j$, so

$$[\bigcap_{i\in M\setminus\{j\}} F_i] \cup F_j = 1_X.$$

Since $A \cup B = 1_X$ implies $A^c \subseteq B$, taking $A = \bigcap_{i\in M\setminus\{j\}} F_i$ and $B = F_j$ gives $[\bigcap_{i\in M\setminus\{j\}} F_i]^c \subset F_j$.

2. Let $j \in M$ be such that $\bigcap_{i\in M\setminus\{j\}} F_i = 0_X$. By (i), $[\bigcap_{i\in M\setminus\{j\}} F_i]^c \subset F_j$, that is, $0_X^c \subset F_j$, so $1_X \subset F_j$, giving $F_j = 1_X$. This contradicts that F_j is n-CyNmaxMos (hence proper, $F_j \neq 1_X$). Hence $\bigcap_{i\in M\setminus\{j\}} F_i \neq 0_X$ for any $j \in M$. The proof of others are similar.

Corollary 4.1 If F_i is a n - CyNmaxos (resp. n - CyNmaxδos , n-CyNmaxδ𝒫os, n-CyNmaxδ𝒮os, n-CyNmaxθos, n - CyNmaxθ𝒮os , n - CyNmaxeos and n-CyNmaxMos)'s for any $i \in M$, M is a finite set with $|M| \geq 3$, and $F_i \neq F_j$ for any distinct $i, j \in M$, then $F_i \cap F_j \neq 0_X$ for any distinct $i, j \in M$.

Theorem 4.4 If F_i is a n-CyNmaxos (resp. n - CyNmaxδos , n - CyNmaxδ𝒫os, n-CyNmaxδ𝒮os, n-CyNmaxθos, n-CyNmaxθ𝒮os, n-CyNmaxeos and n-CyNmaxMos)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, then $F_j = [\bigcap_{i\in M} F_i] \cup [\bigcap_{i\in M\setminus\{j\}} F_i]^c$ for any $j \in M$.

Proof. For any $j \in M$, $[\bigcap_{i\in M} F_i] \cup [\bigcap_{i\in M\setminus\{j\}} F_i]^c = [\bigcap_{i\in M\setminus\{j\}} F_i \cap F_j] \cup [\bigcap_{i\in M\setminus\{j\}} F_i]^c$

$$= [(\bigcap_{i\in M\setminus\{j\}} F_i) \cup (\bigcap_{i\in M\setminus\{j\}} F_i)^c] \bigcap [F_j \cup (\bigcap_{i\in M\setminus\{j\}} F_i)^c]$$

$$= 1_X \cap [F_j \cup (\bigcap_{i\in M\setminus\{j\}} F_i)^c]$$

$$= F_j \cup (\bigcap_{i\in M\setminus\{j\}} F_i)^c$$

$= F_j$ for any $j \in M$, since $(\bigcap_{i\in M\setminus\{j\}} F_i)^c \subset F_j$ by Theorem 4.3 (i).

Theorem 4.5 If F_i is a n-CyNmaxos (resp. n - CyNmaxδos , n - CyNmaxδ𝒫os, n-CyNmaxδ𝒮os, n-CyNmaxθos, n-CyNmaxθ𝒮os, n-CyNmaxeos and n-CyNmaxMos)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, and if K is a proper nonzero subset of M, then $\bigcap_{i\in M} F_i \subsetneq \bigcap_{k\in K} F_k$.

Proof. Since $K \subseteq M$, clearly $\bigcap_{i\in M} F_i \subseteq \bigcap_{k\in K} F_k$. It remains to show the inclusion is strict. Let $m \in M\setminus K$ (nonempty as K is proper). Since $F_m \neq F_k$ for each $k \in K$, by Lemma 4.1 (ii), $F_m \cup F_k = 1_X$ for each $k \in K$, so

$$F_m \cup [\bigcap_{k\in K} F_k] = \bigcap_{k\in K} [F_m \cup F_k] = 1_X.$$

Also, since $m \in M$, $\bigcap_{i\in M} F_i \subseteq F_m$, so

$$F_m \cup [\bigcap_{i\in M} F_i] = F_m.$$

If $\bigcap_{i\in M} F_i = \bigcap_{k\in K} F_k$, then unioning both sides with F_m gives $F_m = 1_X$, which contradicts that F_m is n-CyNmaxMos (hence proper, $F_m \neq 1_X$). Hence

$\bigcap_{i\in M} F_i \subsetneq \bigcap_{k\in K} F_k$. The proof of others are similar.

Theorem 4.6 If F_i is a n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmaxδ$\mathcal{P}$os, n-CyNmaxδ$\mathcal{S}$os, n-CyNmaxθos, n-CyNmaxθ$\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, and if $\bigcap_{i\in M} F_i$ is a n-CyNM-closed set, then F_j is a n-CyNM-closed set for any $j \in M$.

Proof. By Theorem 4.4, we have $F_j = [\bigcap_{i\in M} F_i] \cup [\bigcap_{i\in M\setminus\{j\}} F_i]^c$ for any $j \in M$.

By hypothesis, $\bigcap_{i\in M} F_i$ is n-CyNM-closed. Since M is finite, $\bigcap_{i\in M\setminus\{j\}} F_i$ is a finite intersection of n-CyNM-open sets, hence n-CyNM-open, so $[\bigcap_{i\in M\setminus\{j\}} F_i]^c$ is n-CyNM-closed. Since a finite union of n-CyNM-closed sets is n-CyNM-closed, F_j is n-CyNmaxM-closed for any $j \in M$. Others are similar

Theorem 4.7 If F_i is a n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmaxδ$\mathcal{P}$os, n-CyNmaxδ$\mathcal{S}$os, n-CyNmaxθos, n-CyNmaxθ$\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, and if $\bigcap_{i\in M} F_i = 0_X$, then $\{F_i / i \in M\}$ is the set of all n-CyNmaxos (resp. n-CyNmaxδos, n-CyNmaxδ$\mathcal{P}$os, n-CyNmaxδ$\mathcal{S}$os, n-CyNmaxθos, n-CyNmaxθ$\mathcal{S}$os, n-CyNmaxeos and n-CyNmaxMos)'s of n-Cylindrical neutrosophic topological space (X, τ).

Proof. Suppose that there exists another n-CyNmaxMos F_k of the n-Cylindrical neutrosophic topological space (X, τ) such that $F_k \neq F_i, \forall i \in M$, with $k \notin M$ a fresh index. Then $\{F_i : i \in M \cup \{k\}\}$ is a family of pairwise distinct n-CyNmaxMos's indexed by $M \cup \{k\}$. By Theorem 4.3 (ii) applied with $j = k$,

$$\bigcap_{i\in(M\cup\{k\})\setminus\{k\}} F_i \neq 0_X, \text{thatis,} \bigcap_{i\in M} F_i \neq 0_X,$$

contradicting the hypothesis that $\bigcap_{i\in M} F_i = 0_X$. Hence $\{F_i / i \in M\}$ is the family of all n-CyNmaxMos's of the n-Cylindrical neutrosophic topological space (X, τ).

5 Real-Life Application: Selection of an Optimal Drug Delivery Technology

Selecting the most suitable drug delivery technology for a targeted therapeutic programme is a multi-attribute decision problem in which the available clinical, pre-clinical and manufacturing evidence for each candidate technology is rarely conclusive. For a given technology, the evidence gathered by a panel of pharmaceutical experts typically contains a mixture of favourable findings, inconclusive/undetermined findings and unfavourable findings, none of which can, in general, be discarded. This three-fold and mutually non-exhaustive character of pharmaceutical evidence is naturally captured by an n-CyNN $\langle\alpha, \beta, \gamma\rangle$ (Definition 2.6), where α denotes the degree of favourable/supporting evidence, β denotes the degree of inconclusive evidence and γ denotes the degree of unfavourable/adverse evidence, subject to $0 \leq \beta \leq 1$ and $0 \leq \alpha^n + \gamma^n \leq 1$. Since $n > 1$ may be chosen freely, the n-CyNN framework is more flexible than the classical neutrosophic set (for which $\alpha + \beta + \gamma \leq 3$ is imposed) whenever the positive and negative evidence for a technology are strong but not required to sum linearly, which is typical of independently conducted efficacy and safety trials. We now use the machinery of Sections 3 and 4 to model such a selection problem.

5.1 Problem description

Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ be the set of candidate drug delivery technologies under consideration for a targeted (e.g., oncology) drug delivery programme, where

X_1: Liposomalnanocarriers, x_2: Polymericnanoparticles, X_3: Dendrimer − basedcarriers, x_4: Microneedletransdermalpatches, x_5: Hydrogel − baseddepotsystems.

An expert panel evaluates each technology x_i against a composite of criteria (drug loading efficiency, controlled release behaviour, biocompatibility, targeting specificity and manufacturing cost-effectiveness) at two successive levels of scrutiny and records the outcome as an n-CyNN with n = 2: [(a)]

1. a relaxed screening $A \in \tau_{CyN}(X)$, obtained from the pooled expert evidence at the ordinary acceptability threshold used to shortlist technologies for further evaluation;
2. a stringent screening $B \in \tau_{CyN}(X)$, obtained by tightening the same threshold, so that only technologies with strong, well-corroborated evidence survive.

$B = \{\langle x_1, 0.9, 0.7, 0.2\rangle, \langle x_2, 0,0,1\rangle, \langle x_3, 0,0,1\rangle, \langle x_4, 0,0,1\rangle, \langle x_5, 0,0,1\rangle\}$,

$A = \{\langle x_1, 0.9, 0.7, 0.2\rangle, \langle x_2, 0.6, 0.5, 0.5\rangle, \langle x_3, 0.8, 0.6, 0.3\rangle, \langle x_4, 0,0,1\rangle, \langle x_5, 0.5, 0.4, 0.6\rangle\}$.

Each n-CyNN above satisfies $0 \leq \beta \leq 1$ and $0 \leq \alpha^2 + \gamma^2 \leq 1$ (e.g., at x_1, $0.9^2 + 0.2^2 = 0.85 \leq 1$), so $A, B \in \tau_{CyN}(X)$ are legitimate n-CyNS's with $n = 2$. At x_4, both A and B take the value $0_{CyN} = \langle 0,0,1\rangle$: on the accumulated evidence, Microneedle transdermal patches fail even the relaxed acceptability threshold, owing to a poorly reproducible biocompatibility profile, and are excluded outright. By direct computation, $0_X \subset B \subset A \subset 1_X$, so

$$\tau = \{0_X, \quad B, \quad A, \quad 1_X\}$$

is an n-Cylindrical neutrosophic topology on X in the sense of Definition 2.10 (all pairwise unions and intersections of members of τ remain in τ, since τ is a chain). We call (X, τ) the screening topology: its open sets are exactly the technology pools that are admissible at some threshold of evidentiary scrutiny.

5.2 Identification of the minimal and maximal M-open sets

Since B^c and A^c are not n-CyN regular closed sets of (X, τ), one computes $\text{n-CyN}\delta\text{cl}(B) = \text{n-CyN}\delta\text{cl}(A) = 1_X$, whence $\text{n-CyNint}(\text{n-CyN}\delta\text{cl}(B)) = \text{n-CyNint}(\text{n-CyN}\delta\text{cl}(A)) = 1_X$. Consequently

$$B \subseteq n - \text{CyNcl}(n - \text{CyN}\theta\text{int}(B)) \cup n - \text{CyNint}(n - \text{CyN}\delta\text{cl}(B)) = 1_X, \quad A \subseteq n - \text{CyNcl}(n - \text{CyN}\theta\text{int}(A)) \cup n - \text{CyNint}(n - \text{CyN}\delta\text{cl}(A)) = 1_X$$

trivially hold, so both A and B are n-CyNMos's of (X, τ) (Definition 2.18); this was verified by exact-fraction computation in Python, together with the fact that no member of τ other than 0_X is properly contained in B, and no member of τ other than 1_X properly contains A.

Example 5.1 In the screening topology (X, τ) above, $B \subset A$ and the only n-CyN open set contained in B is 0_X or B itself. Hence, exactly as in Example 3.1, B is an n-CyNmin Mos of (X, τ) (Definition 3.1). Likewise, the only n-CyN open set containing A is A itself or 1_X, so, exactly as in Example 4.1, A is an n-CyNmax Mos of (X, τ) (Definition 4.1).

5.3 Interpretation and recommendation

The two extremal M-open sets identified in Example 5.1 carry a direct decision-theoretic meaning.

1. The n-CyNmin Mos set B is supported only at x_1. Since a minimal open set contains no smaller nonzero admissible subset (Definition 3.1), B pinpoints **Liposomal nanocarriers (x_1)** as the uniquely indispensable technology: it is the smallest non-empty pool of technologies that survives the stringent screening, so no candidate technology can be dropped from it without collapsing to 0_X. This formalises, within the n-CyN framework, the intuitive notion of the "single best-supported choice".

2. The n-CyNmax Mos set A is supported on $\{x_1, x_2, x_3, x_5\}$. Since a maximal open set cannot be properly enlarged without becoming the whole universe (Definition 4.1), and here $A \neq 1_X$ because x_4 is excluded even under the relaxed threshold, A formalises the **largest defensible shortlist** {Liposomal nanocarriers, Polymeric nanoparticles, Dendrimer-based carriers, Hydrogel-based depot systems} that can be forwarded for further (e.g., pre-clinical or clinical) evaluation, while Microneedle transdermal patches are excluded from consideration at the current level of evidence.

Thus the n-Cylindrical neutrosophic minimal and maximal M-open sets developed in Sections 3 and 4 provide a mathematically grounded way of extracting, respectively, a single top recommendation and a maximal admissible shortlist from indeterminate, panel-based pharmaceutical evidence, without forcing the premature collapse of genuinely inconclusive evidence that a purely crisp or classical fuzzy threshold would entail.

6 Conclusion

In this paper, the concepts of n-CyN (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M)-minimal and n-Cylindrical neutrosophic (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M)-maximal open sets are introduced, and some properties of n-Cylindrical neutrosophic (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M) minimal open and n-Cylindrical neutrosophic (resp. δ, δ pre, δ semi, θ, $\theta\mathcal{S}$, e and M) maximal open sets are discussed. A real-life application to the selection of an optimal drug delivery technology (Section 5) illustrates how the n-Cylindrical neutrosophic minimal and maximal M-open sets can, respectively, isolate the single best-supported alternative and the largest defensible shortlist of alternatives from indeterminate expert evidence.

References

[1] S. Anandhi, R. Vijayalakshmi and K. Shantha lakshmi, n-Cylindrical Neutrosophic M-open sets, Neutrosophic Sets and Systems, **97** (2026), 110-125.

[2] S. Anandhi, R. Vijayalakshmi and K. Shantha lakshmi, Homeomorphism via M-open sets in n-Cylindrical Neutrosophic topological spaces, Panamerican Mathematical Journal, **35** (2s) (2025), 796-812.

[3] I. Arokiarani, R. Dhavaseelan, S. Jafari and M. Parimala, On some new notions and functions in neutrosophic topological spaces, Neutrosophic Sets and Systems, **16** (1) (2017), 16-19.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, **20** (1986), 87-96.

[5] K. Bhagyasri and D. Muthuramakrishnan, Construction Algorithm for one modulo N mean labelling of cycles, Indian Journal of Natural Sciences, **17** (97) (2026).

[6] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., **24** (1968), 182-190.

[7] D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, **88** (1997), 81-89.

[8] A. I. El-Maghrabi and M. A. Al-Juhani, M − open sets in topological spacec, Pioneer J. Math.

Sci., **4** (2) (2011), 213-230.

[9] B. M. Ittanagi and R. S. Wali, On fuzzy minimal open and fuzzy maximal open sets in fuzzy topological spaces, International J. of Mathematical Sciences and Applications, **1**(2) (2011).

[10] J. K. Jun, Y. B. Jun and J. H. Park, Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy precontinuity, International Journal of Mathematics and Mathematical Science, **19** (2005), 3091-3101.

[11] V. Kalaiyarasan, S. Tamilselvan, A. Vadivel and C. John Sundar, Normal spaces associated with fuzzy nano ð'€-open sets and its application, Journal of Mathematics and Computer Science, **29** (2) (2023), 156-166.

[12] A. Padma, M. Saraswathi, A. Vadivel and G. Saravanakumar, New Notions of Nano M-open Sets, Malaya Journal of Matematik, **S** (1) (2019), 656-660.

[13] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological spaces, IOSR journal of Mathematics (iosr-jm), **3** (4) (2012), 31-35.

[14] R. Saranya Kumari, S. Kalayathankal, M. George and F. Smarandache, n-Cylindrical fuzzy neutrosophic sets, International Journal of Neutrosophic Science, **18** (4) (2022), 355-374.

[15] R. Saranya Kumari, S. Kalayathankal, M. George and F. Smarandache, On some related concept of n -Cylindrical fuzzy neutrosophic topological spaces, Journal of Fuzzy Extension and Applications, **4** (1) (2023), 40-51.

[16] R. Saranya Kumari, S. Kalayathankal, M. George and F. Smarandache, n-Cylindrical fuzzy neutrosophic topological spaces, Journal of Fuzzy Extension and Applications, **4** (2) (2023), 141-147.

[17] F. Smarandache, A unifying field in logics: neutrosophic logic, neutrosophic set, neutrosophic probability and satistics, (2001).

[18] F. Smarandache, Proceeding of the first international conference on neutrosophy, neutrosophy logic, neutrosophic set, neutrosophic probability and satistics, University of New Mexico-Gallup, (2001).

[19] A. Swaminathan and S. Sivaraja, On fuzzy maximal, minimal open and closed sets, Advances in Mathematics: Scientific Journal, **9**(10) (2020), 7741-7747.

[20] R. Thangammal, M. Saraswathi, A. Vadivel and C. John Sundar, Fuzzy nano Z-open sets in fuzzy nano topological spaces, Journal of Linear and Topological Algebra, **11** (01) (2022), 27-38.

[21] R. Thangammal, M. Saraswathi, A. Vadivel, Samad Noeiaghdam, C. John Sundar, V. Govindan, Aiyared Iampan, Fuzzy nano Z-locally closed sets, extremally disconnected spaces, normal spaces, and their application, Advances in Fuzzy Systems, (2022), 3364170.

[22] A. Vadivel, C. John Sundar, K. Kirubadevi and S. Tamilselvan, More on neutrosophic nano open sets, International Journal of Neutrosophic Science (IJNS), **18** (4) (2022), 204-222.

[23] A. Vadivel, C. John Sundar, K. Saraswathi and S. Tamilselvan, Neutrosophic nano M open sets, International Journal of Neutrosophic Science, **19** (1) (2022), 132-147.

[24] Y. Veereswari, An introduction to fuzzy neutrosophic topological spaces, International Journal of Mathematical Archive, **8** (3), (2017), 1-6.

[25] R. Vijayalakshmi, A. Sugapriya and K. Bhagyasri, Almost continuous and open maps Via M -open sets in n -Cylindrical neutrosophic topological spaces, accepted in International Journal of Computer Information Systems and Industrial Management Applications (IJCISIM), 18, (8s), 2026, 465-482.

[26] R. Vijayalakshmi, A. Sugapriya, K. Bhagyasri and S. Anandhi *M-Connectedness in n-Cylindrical Neutrosophic Topologies: New Theoretical Results and a Drug-Delivery Application*, accepted in International Journal of Drug Delivery Technology- (IJDDT), 16(69s), 2026, 1139-1150.

[27] R. Vijayalakshmi, A. Sugapriya, K. Bhagyasri and S. Prtiya, *Structural Analysis of Separated Sets and Compactness Using M - Open Sets in n-Cylindrical Neutrosophic Topological Spaces: Application to Drug Delivery*, International Journal of Drug Delivery Technology- (IJDDT), 16(69s), 2026, 1151-1162.

[28] L. A. Zadeh, Fuzzy sets, Information and Control, **8** (1965), 338-353.