RESEARCH PAPER

PENDANT ECCENTRIC DEGREE DOMINATING FUNCTIONS OF PATH UNION AND CYCLE OF SOME ZERO DIVISOR GRAPHS

[1]R.Geetha, [2]R.Rajeswari
[1]Research scholar (Reg.No.23112012092002), [2]Assistant Professor
[1,2]PG & Research Department of Mathematics, A.P.C. Mahalaxmi College for Women, Thoothukudi.
Affiliated to Manonmaniam Sundaranar University, Tirunelveli-627012, Tamil Nadu, India
Email: geetha2262@gmail.com rajimuthuram@gmail.com

ABSTRACT
A set S the number of vertices in a graph G is said to be a Pendant eccentric Domination if every vertex in $V - S$ is satisfied by pendant domination and eccentric domination. A function $f: V(G) \rightarrow \{0,1,2, \dots \Delta(G) + 2\}$ is called a $PEDDF$ (Pendant Eccentric Degree Dominating Function) if every vertex v of S means PED (Pendant Eccentric Domination) is assigned with $deg(v) + 2$ and other vertices to zero. The weight of $PEDDF$ is denoted by $w_f(G) = \sum_{v \epsilon V} f(v)$. $The\ PEDD$ number denoted by $\gamma_{PEDD}(G)$ is the minimum weight of all possible $PEDDF. In$ this paper $PEDDF$ has been developed for path union and cycle of some ZDG (zero divisor graph). The join graph of some ZDG which has been elucidated as a theorem.

Keywords: ZDG, $PEDDF$, Join graph, Path union graph.
How to cite this article: 1R.Geetha 2R.Rajeswari. PENDANT ECCENTRIC DEGREE DOMINATING FUNCTIONS OF PATH UNION AND CYCLE OF SOME ZERO DIVISOR GRAPHS. Int J Drug Deliv Technol. 2026;16(78s):713-718. DOI: 10.25258/ijddt.16.78s.81.

1.INTRODUCTION

The concept of zero divisor graph appears in two distinct variations. The first was introduced by Beck in 1988 [2].Later in 1999, Anderson and Livingston modified this approach by restricting the vertex set exclusively to the zero divisors of a ring [1]. In graph theory, domination is a fundamental area of study that analyzes how a specific subset of vertices can efficiently influence or control an entire network. A dominating set refers to a collection of vertices chosen such that every vertex in the graph either belongs to this set or is directly connected to a vertex within it. This concept of the minimum dominating set was originally introduced by T.W. Haynes and S.T. Hedetniemi [9]. Building on this, N.C. Demirpolat and E.Kilic introduced the concept of Degree Domination Number [5]. Additionally, several researchers have explored pendant domination and Eccentric domination, as documented in studies [10,12]. Among these variations, Pendant Eccentric Degree Domination (PEDD) is one of the most prominent frameworks. Furthermore, reference [11,14] presents several key developments, including the concept of the line graph of Mycielski graph. Motivated by this we developed $PEDDF$ of path union and cycle of some zero divisor graphs.

2. $PEDDF$ OF CYCLE OF JOIN ZDG:

Definition 2.1[7] A function $f: V(G) \rightarrow \{0,1,2, \dots \Delta(G) + 2\}$ is called a $PEDDF$ (Pendant Eccentric Degree Dominating Function) if every vertex v of S means PED (Pendant Eccentric Domination) is assigned with $deg(v) + 2$ and other vertices to zero. The weight of $PEDDF$ is denoted by $w_f(G) = \sum_{v \epsilon V} f(v)$. $The\ PEDD$ number denoted by $\gamma_{PEDD}(G)$ is the minimum weight of all possible $PEDDF$.

Theorem 2.2 The cycle of join zero divisor graph $C\left(m, \Gamma(z_{2p}) + \Gamma(z_4)\right)$ where $m \geq 3\ and\ p \geq 3$, then $\gamma_{PEDD} C\left(m, \Gamma(z_{2p}) + \Gamma(z_4)\right) = 4 + m(p + 2)$.

Proof: Let $C\left(m, \Gamma(z_{2p}) + \Gamma(z_4)\right)$ be a cycle of join zero divisor graph where $m \geq 3\ and\ p \geq 3$. We create sets that can represent the three partitions and m copies. The set $S = \{2,4, \dots 2(p-1)\} = \{t_{1,1}, t_{1,2}, \dots, t_{p-1}, t_{2,1}, t_{2,2}, \dots, t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots, t_{m,p-1}\}$ and set $T = \{p\} = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$, $U = \{2\} = \{s_{1,1}, s_{2,1}, \dots s_{m,1}\}$. We conclude from $|S| = m(p-1), |T| = m, |U| = m$. $\Delta(G) = p + 2$. Define $f: V(G) \rightarrow \{0,1,2, \dots p + 4\}$. The minimum pendent eccentric degree dominating set is $\{t_{1,1}, s_{1,1}, s_{2,1}, \dots s_{m,1}\}$. $f(t_{1,1}) = 4, f(s_{1,1}) = p + 2, f(s_{2,1}) = p + 2, \dots f(s_{m,1}) = p + 2$ and the remaining vertices are zero. The weight of $PEDDF$ is $4 + m(p + 2)$. Hence $\gamma_{PEDD} C\left(m, \Gamma(z_{2p}) + \Gamma(z_4)\right) = 4 + m(p + 2)$.

Theorem 2.3 The cycle of join zero divisor graph $C\left(m, \Gamma(z_{2p}) + \Gamma(z_6)\right)$ where $m \geq 3\ and\ p \geq 3$ then $\gamma_{PEDD}\ C\left(m, \Gamma(z_{2p}) + \Gamma(z_6)\right) = 6 + m(p + 4)$.

Proof: Let $C\left(m, \Gamma(z_{2p}) + \Gamma(z_6)\right)$ be a cycle of join zero divisor graph where $m \geq 3\ and\ p \geq 3$. We create sets that can represent the three partitions and m copies. The set $S = \{2,4, \dots 2(p-1)\} = \{t_{1,1}, t_{1,2}, \dots, t_{p-1}, t_{2,1}, t_{2,2}, \dots, t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots, t_{m,p-1}\}$ and set $T = \{p\} = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$. $U = \{2,3,4\} = \{s_{1,1}, s_{1,2}, s_{1,3}, s_{2,1}, s_{2,2}, s_{2,3}, \dots s_{m,1}, s_{m,2}, s_{m,3}\}$. We conclude from $|S| = m(p-1), |T| = m, |U| = 3m$.

$\Delta(G) = p + 4$. Define $f: V(G) \to \{0,1,2, \dots p + 6\}$. The minimum dominating set is $\{t_{1,1}, s_{1,2}, s_{2,2}, \dots s_{m,2}\}$. $f(t_{1,1}) = 6, f(s_{1,2}) = p + 4, f(s_{2,2}) = p + 4, \dots f(s_{m,2}) = p + 4$ and the remaining vertices are zero. The weight of $PEDDF$ is $6 + m(p + 4)$. Hence $\gamma_{PEDD}\, C\left(m, \Gamma(z_{2p}) + \Gamma(z_6)\right) = 6 + m(p + 4)$.

Theorem 2.4 The cycle of join zero divisor graph $C\left(m, \Gamma(z_{2p}) + \Gamma(z_9)\right)$ where $m \geq 3$ and $p \geq 3$ then $\gamma_{PEDD} C\left(m, \Gamma(z_{2p}) + \Gamma(z_9)\right) = 5 + m(p + 3)$.

Proof: Let $C\left(m, \Gamma(z_{2p}) + \Gamma(z_9)\right)$ be a cycle of join zero divisor graph where $m \geq 3, p \geq 3$.
We create sets that can represent the three partitions and m copies. The set $S = \{2,4, \dots 2(p - 1)\} = \{t_{1,1}, t_{1,2}, \dots, t_{p-1}, t_{2,1}, t_{2,2}, \dots, t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots, t_{m,p-1}\}$ and set $T = \{p\} = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$. $U = \{3,6\} = \{s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}, \dots s_{m,1}, s_{m,2}\}$. We conclude from $|S| = m(p - 1), |T| = m, |U| = 2m$. $\Delta(G) = p + 3$. Define $f: V(G) \to \{0,1,2, \dots p + 4\}$. The minimum dominating set is $\{t_{1,1}, s_{1,1}, s_{2,1}, \dots s_{m,1}\}$. $f(t_{1,1}) = 5, f(s_{1,1}) = p + 3, f(s_{2,1}) = p + 3, \dots f(s_{m,1}) = p + 3$ and the remaining vertices are zero. The weight of $PEDDF$ is $5 + m(p + 3)$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p}) + \Gamma(z_9)\right)\right) = 5 + m(p + 3)$.

Theorem 2.5 The cycle of join zero divisor graph $C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_6)\right)$ where $m \geq 3$ and $p \geq 5$ then $\gamma_{PEDD}\left(C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_6)\right)\right) = m(p + 7)$

Proof: Let $C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_6)\right)$ be a cycle of join zero divisor graph where $m \geq 3, p \geq 5$.
We create sets that can represent the two partitions and m copies. The set $S = \{p, 2p, 3p \dots p(p - 1)\} = \{t_{1,1}, t_{1,2}, \dots, t_{1,p-1}, t_{2,1}, t_{2,2}, \dots, t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots, t_{m,p-1}\}$ and set $= \{3,2,6\}$ $\{s_{1,1}, s_{1,2}, s_{1,3}, s_{2,1}, s_{2,2}, s_{2,3}, \dots s_{m,1}, s_{m,2}, s_{m,3}\}$. We conclude from $|S| = m(p - 1), |T| = 3m$. $\Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p + 3\}$. The minimum dominating set is $\{t_{1,2}, t_{2,2}, \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots s_{m,1}\}$. $f(t_{1,2}) = 5, f(t_{2,2}) = 5, \dots f(t_{m,2}) = 5, f(s_{1,1}) = p + 2, f(s_{2,1}) = p + 2, \dots f(s_{m,1}) = p + 2$ and the remaining vertices are zero. The weight of $PEDDF$ is $m(p + 7)$. Hence $\gamma_{PEDD}\left(C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_6)\right)\right) = m(p + 7)$.

Theorem 2.6 The cycle of join zero divisor graph $C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_9)\right)$ where $m \geq 3$ and $p \geq 5$ then $\gamma_{PEDD} C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_9)\right) = m(p + 6)$.

Proof: Let $C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_9)\right)$ be a cycle of join zero divisor graph where $m \geq 3, p \geq 5$.
We create sets that can represent the two partitions and m copies. The set $S = \{p, 2p, 3p \dots p(p - 1)\} = \{t_{1,1}, t_{1,2}, \dots, t_{1,p-1}, t_{2,1}, t_{2,2}, \dots, t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots, t_{m,p-1}\}$ and set $= \{3,6\} = \{s_{1,1}, s_{1,2}, s_{2,1}, s_{2,2}, \dots s_{m,1}, s_{m,2}\}$. We conclude from $|S| = m(p - 1), |T| = 2m$. $\Delta(G) = p$. Define $f: V(G) \to \{0,1,2, \dots p + 1\}$. The minimum dominating set is $\{t_{1,2}, t_{2,2}, \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots s_{m,1}\}$. $f(t_{1,2}) = 4, f(t_{2,2}) = 4, \dots f(t_{m,2}) = 4, f(s_{1,1}) = p + 2, f(s_{2,1}) = p + 2, \dots f(s_{m,1}) = p + 2$ and the remaining vertices are zero. The weight of $PEDDF$ is $m(p + 6)$. Hence $\gamma_{PEDD} C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_9)\right) = m(p + 6)$.

Theorem 2.7 The cycle of join zero divisor graph $C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_4)\right)$ where $m \geq 3$ and $p \geq 5$ then $\gamma_{PEDD} C\left(m, \underline{\Gamma(z_{p^2})} + \Gamma(z_4)\right) = \{\frac{m(2p+9)+3}{2}$ *where m is odd* $m(p + 6)$ *where m is even* .

Proof: The resulting graph is isomorphic to $C\left(m, \Gamma(z_{2p})\right)$

3. $PEDDF$ OF PATH UNION AND CYCLE OF SOME ZDG:

Theorem 3.1 The cycle graph of zero divisor graph $C\left(m, \Gamma(z_{2p})\right)$, then

i) If $p = 2, m \geq 5$ *then* $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = \{\, 2(m + 1)$ *where m is odd* $2(m + 2)$ *where m is even*

ii) If $p = 3, m \geq 3$ *then* $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = 6(m + 1)$

iii) If $p \geq 5, m \geq 3$ *then* $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = \{\frac{m(2p+9)+3}{2}$ *where m is odd* $m(p + 6)$ *where m is even*

Proof: Let $C\left(m, \Gamma(z_{2p})\right)$ be a cycle graph of zero divisor graph

Case $i)$ $p = 2$ the resulting graph is cycle m on m vertices. $\Delta(G) = 2$. Define $f: V(G) \to \{0,1,2, \dots 4\}$.

Subcase $i)$ m is odd

The minimum pendent eccentric Degree Dominating set is $\{t_{1,1}, t_{2,1}, t_{4,1}, t_{6,1}, \dots t_{m-1,1}\}$. $f(t_{1,1}) = 4, f(t_{2,1}) = 4, f(t_{4,1}) = 4, f(t_{6,1}) = 4, \dots f(t_{m-1,1}) = 4$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $2m + 2$. Hence $\gamma_{deg}\left(C\left(m, \Gamma(z_{2p})\right)\right) = 2(m + 1)$

Subcase $ii)$ m is even

The minimum pendent eccentric Degree Dominating set is $\{t_{1,1}, t_{2,1}, t_{4,1}, t_{6,1}, \dots t_{m,1}\}$. $f(t_{1,1}) = 4, f(t_{2,1}) = 4, f(t_{4,1}) = 4, f(t_{6,1}) = 4, \dots f(t_{m,1}) = 4$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $2m + 4$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = 2(m + 2)$

Case $ii)$ $p = 3, m \geq 3, \Delta(G) = 4$. Define $f: V(G) \to \{0,1,2, \dots 6\}$. The minimum pendent eccentric Degree Dominating set is $\{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots t_{m,1}, t_{m,2}, t_{1,3}\}$. $f(t_{1,1}) = 3, f(t_{1,2}) = 3, f(t_{2,1}) = 3, f(t_{2,2}) = 3, \dots f(t_{m,1}) = 3, f(t_{m,2}) = 3, f(t_{1,3}) = 6$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $6(m + 1)$. Hence $\gamma_{deg}\left(C\left(m, \Gamma(z_{2p})\right)\right) = 6(m + 1)$.

Case $iii)$ $p \geq 5, m \geq 3$ we create sets that can represent the two partitions and m copies.The set $S = \{2,4, \dots 2(p - 1)\} = \{t_{1,1}, t_{1,2}, \dots, t_{p-1}, t_{2,1}, t_{2,2}, \dots, t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots, t_{m,p-1}\}$ and set $T = \{p\} = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$. We conclude from $|S| = m(p - 1), |T| = m$. $\Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p + 3\}$.

Subcase $i)$ m is odd, The minimum pendent eccentric Degree Dominating set is $\{t_{1,p}, t_{2,p}, \dots t_{m,p}, t_{1,1}, t_{3,1}, t_{5,1}, t_{7,1}, \dots t_{m,1}\}$. $f(t_{1,p}) = p + 3, f(t_{2,p}) = p + 3, \dots f(t_{m,p}) = p + 3$, $f(t_{1,1}) = 3, f(t_{3,1}) = 3, f(t_{5,1}) = 3, \dots f(t_{m,1}) = 3$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $\frac{m(2p+9)+3}{2}$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = \frac{m(2p+9)+3}{2}$.

Subcase $ii)$ m is even, The minimum pendent eccentric Degree Dominating set is $\{t_{1,p}, t_{2,p}, \dots t_{m,p}, t_{1,1}, t_{2,1}, t_{3,1}, \dots t_{m,1}\}$. $f(t_{1,p}) = p + 3, f(t_{2,p}) = p + 3, \dots f(t_{m,p}) = p + 3$, $f(t_{1,1}) = 3, f(t_{2,1}) = 3, \dots f(t_{m,1}) = 3$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $m(p + 6)$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = m(p + 6)$.

Theorem 3.2 The path union graph of zero divisor graph $P\left(m, \Gamma(z_{2p})\right)$

$i)\ If\ p = 2, m \geq 4\ then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = \{\ 2(m + 1)\quad where\ m\ is\ even\ 2m \qquad where\ m\ is\ odd$

. $ii)\ If\ p = 3, m \geq 3\ then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = 6m$

$iii)$ If $p \geq 5, m \geq 3$ then $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = m(p + 3) + 4$

Proof: Let $P\left(m, \Gamma(z_{2p})\right)$ be a path union graph of zero divisor graph

Case $i)$ $p = 2$, $m \geq 4$ the resulting graph is p_m on m vertices. $\Delta(G) = 2$. Define $f: V(G) \to \{0,1,2, \dots 4\}$.

Subcase $i)$ m is odd, the minimum dominating set is $\{t_{1,1}, t_{2,1}, t_{5,1}, t_{7,1}, t_{9,1}, \dots, t_{m,1}\}$. $f(t_{1,1}) = 3, f(t_{2,1}) = 4, f(t_{5,1}) = 4, \dots f(t_{m-1,1}) = 4, f(t_{m,1}) = 3$ and the remaining vertices are zero. The weight of $PEDDF$ is $2m$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = 2m$.

Subcase $ii)$ m is even, the minimum dominating set is $\{t_{1,1}, t_{2,1}, t_{4,1}, t_{6,1}, t_{8,1}, \dots, t_{m,1}\}$. $f(t_{1,1}) = 3, f(t_{2,1}) = 4, f(t_{4,1}) = 4, \dots, f(t_{m,1}) = 3$ and the remaining vertices are zero. The weight of $PEDDF$ is $2m + 1$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = 2m + 1$.

Case $ii)$ $p = 3, m \geq 4$ we create sets that can represent the two partitions and m copies. The set $S = \{2,4\} = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots, t_{m,1}, t_{m,2}\}$ and set $T = \{3\} = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$. We conclude from $|S| = 2m, |T| = m$. $\Delta(G) = 4$. Define $f: V(G) \to \{0,1,2, \dots 6\}$. The minimum dominating set is $\{t_{1,1}, t_{1,2}, t_{m,1}, t_{m,2}, t_{2,p}, t_{3,p}, \dots, t_{m-1,p}\}$. $f(t_{1,1}) = 3, f(t_{1,2}) = 3, f(t_{m,1}) = 3, f(t_{m,2}) = 3, f(t_{2,p}) = 6, f(t_{3,p}) = 6, \dots, f(t_{m-1,p}) = 6$ and the remaining vertices are zero. The weight of $PEDDF$ is $6m$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = 6m$.

Case $iii)$ $p \geq 5, m \geq 3$ we create sets that can represent the two partitions and m copies. The set $S = \{2,4,6, \dots 2(p - 1)\} = \{t_{1,1}, t_{1,2}, \dots t_{1,p-1}, t_{2,1}, t_{2,2}, \dots t_{2,p-1} \dots \dots t_{m,1}, t_{m,2}, \dots t_{m,p-1}\}$ and set $T = \{p\} = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$. We conclude from $|S| = m(p - 1), |T| = m$. $\Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p + 3\}$. The minimum dominating set is $\{t_{1,p}, t_{2,p}, t_{3,p}, \dots t_{m,p}, t_{1,1}, t_{m,1}\}$. $f(t_{1,p}) = p + 2, f(t_{2,p}) = p + 3, \dots f(t_{m,p}) = p + 2, f(t_{1,1}) = 3, f(t_{m,1}) = 3$ and the remaining vertices are zero. The weight of $PEDDF$ is $m(p + 3) + 4$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{2p})\right)\right) = m(p + 3) + 4$.

Theorem 3.3 The cycle graph of zero divisor graph $C\left(m, \Gamma(z_{3p})\right)$ $m \geq 3$, then

$i)\ If\ p = 2, then\ \gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = 6(m + 1)$.

$ii)\ If\ p = 3, then\ \gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = 3m + 5$

$iii)$ If $p \geq 5$, then $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = m(p + 5)$

Proof: Let $C\left(m, \Gamma(z_{3p})\right)$ be a cycle graph of zero divisor graph $m \geq 3$.

Case $i)\ p = 2, m \geq 3$ the resulting graph is isomorphic to $C\left(m, \Gamma(z_{2p})\right)$ where $p = 3$.

Case $ii)\ p = 3, m \geq 3$ the set $S = \{3,6\} = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots t_{m,1}, t_{m,2}\}$,We conclude from $|S| = 2m, \Delta(G) = 3$. Define $f: V(G) \to \{0,1,2, \dots 5\}$. The minimum dominating set is $\{t_{1,2}, t_{2,2}, \dots t_{m,2}, t_{1,1}\}$. $f(t_{1,2}) = 3, f(t_{2,2}) = 3, \dots f(t_{m,2}) = 3, f(t_{1,1}) = 5$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $3m + 5$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = 3m + 5$.

Case $iii)\ p \geq 5, m \geq 3$ we create sets that can represent the two partitions and m copies. The set $S = \{3,6, \dots 3(P-1)\} = \{s_{1,1}, s_{1,2}, \dots, s_{1,p-1}, s_{2,1}, s_{2,2}, \dots, s_{2,p-1}, \dots s_{m,1}, s_{m,2}, \dots, s_{m,p-1}\}$ and set $T = \{p, 2p\} = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots t_{m,1}, t_{m,2}\}$. We conclude from $|S| = m(p-1), |T| = 2m$. $\Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p+3\}$. The minimum pendent eccentric degree dominating set is $\{t_{1,2}, t_{2,2}, t_{3,2}, \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots, s_{m,1}\}$. $f(t_{1,2}) = p + 1, f(t_{2,2}) = p + 1, \dots f(t_{m,2}) = p + 1, f(s_{1,1}) = 4, f(s_{2,1}) = 4, \dots f(s_{m,1}) = 4$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $m(p+5)$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = m(p+5)$.

Theorem 3.4 The path union graph of zero divisor graph $P\left(m, \Gamma(z_{3p})\right)$ $m \geq 3$

$i)\ If\ p = 2, then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{3p})\right)\right) = 6m.$

$ii) If\ p = 3, then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{3p})\right)\right) = 3m + 4$

$iii)$ If $p \geq 5$, then $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{3p})\right)\right) = m(p + 5)$.

Proof: Let $P\left(m, \Gamma(z_{3p})\right)$ be a path union graph of zero divisor graph $m \geq 3$

Case $i)\ p = 2$ the resulting graph is $P\left(m, \Gamma(z_{2p})\right)$ where $p = 3$.

Case $ii)$ $p = 3$ the vertex set $S = \{3,6\} = \{t_{1,1}, s_{1,1}, t_{2,1}, s_{2,1}, \dots t_{m,1}, s_{m,1}\}$.We conclude from $|S| = 2m. \Delta(G) = 3$. Define $f: V(G) \to \{0,1,2, \dots 5\}$. The minimum dominating set is $\{s_{1,1}, s_{2,1}, \dots s_{m,1}, t_{1,1}\}$. $f(s_{1,1}) = 3, f(s_{2,1}) = 3, \dots f(s_{m,1}) = 3, f(t_{1,1}) = 4$ and the remaining vertices are zero. The weight of $PEDDF$ is $3m + 4$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{3p})\right)\right) = 3m + 4$.

Case $iii)\ p \geq 5$ we create sets that can represent the two partitions and m copies. The set $S = \{3,6, \dots 3(P-1)\} = \{s_{1,1}, s_{1,2}, \dots, s_{1,p-1}, s_{2,1}, s_{2,2}, \dots, s_{2,p-1}, \dots s_{m,1}, s_{m,2}, \dots, s_{m,p-1}\}$ and set $T = \{p, 2p\} = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots t_{m,1}, t_{m,2}\}$. We conclude from $|S| = m(p-1), |T| = 2m$. $\Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p+3\}$. the minimum dominating set is $\{t_{1,2}, t_{2,2}, t_{3,2} \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots, s_{m,1}\}$. $f(t_{1,2}) = p + 1, f(t_{2,2}) = p + 1, \dots f(t_{m,2}) = p + 1, f(s_{1,1}) = 4, f(s_{2,1}) = 4, \dots f(s_{m,1}) = 4$ and the remaining vertices are zero. The weight of $PEDDF$ is $m(p + 5)$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{3p})\right)\right) = m(p+5)$.

Theorem 3.5 The cycle graph of zero divisor graph $C\left(m, \Gamma(z_{5p})\right)$ $m \geq 3$, then $i)\ If\ p = 2, then\ \gamma_{PEDD}\left(C\left(m, \Gamma(z_{5p})\right)\right) = \{\frac{19m+3}{2}\ where\ m\ is\ odd\ 11m\ \ where\ m\ is\ even$

$ii)\ If\ p = 3, then\ \gamma_{PEDD}\left(C\left(m, \Gamma(z_{5p})\right)\right) = 10m$

$iii)\ If\ p = 5, then\ \gamma_{PEDD}\left(C\left(m, \Gamma(z_{5p})\right)\right) = 5(m + 1)$

$iv)\ If\ p \geq 7, then\ \gamma_{PEDD}\left(C\left(m, \Gamma(z_{5p})\right)\right) = m(p + 7)$

Proof: Let $C\left(m, \Gamma(z_{5p})\right)$ be a cycle graph of zero divisor graph $m \geq 3$.

Case $i)\ p = 2$ we create sets that can represent the two partitions and m copies.The set $S = \{t_{1,2}, t_{1,3}, \dots, t_{1,5}, t_{2,2}, t_{2,3}, \dots, t_{2,5}, \dots t_{m,2}, t_{m,3}, \dots, t_{m,5}\}$ and set $T = \{p\} = \{t_{1,1}, t_{2,1}, \dots t_{m,1}\}$. We conclude from $|S| = 4m, |T| = m$. $\Delta(G) = 6$. Define $f: V(G) \to \{0,1,2, \dots 8\}$.

Subcase $i)$ m is odd, The minimum pendent eccentric Degree Dominating set is $\{t_{1,1}, t_{2,1}, \dots t_{m,1}, t_{1,2}, t_{3,2}, t_{5,2}, t_{7,2}, \dots t_{m,2}\}$. $f(t_{1,1}) = 8, f(t_{2,1}) = 8, \dots f(t_{m,1}) = 8, f(t_{1,2}) = 3, f(t_{3,2}) = 3, f(t_{5,2}) = 3, \dots f(t_{m,2}) = 3$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $\frac{19m+3}{2}$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = \frac{19m+3}{2}$.

Subcase $ii)$ m is even, The minimum pendent eccentric Degree Dominating set is $\{t_{1,1}, t_{2,1}, \dots t_{m,1}, t_{1,2}, t_{2,2}, t_{3,2}, \dots t_{m,2}\}$. $f(t_{1,1}) = 8, f(t_{2,1}) = 8, \dots f(t_{m,1}) = 8, f(t_{1,2}) = 3, f(t_{2,2}) = 3, \dots f(t_{m,2}) = 3$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $11m$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{2p})\right)\right) = 11m$.

Case $ii)\ p = 3$ we create sets that can represent the two partitions and m copies. The set $S =$

$\{s_{1,1}, s_{1,2}, \dots, s_{1,4}, s_{2,1}, s_{2,2}, \dots, s_{2,4}, \dots s_{m,1}, s_{m,2}, \dots, s_{m,4}\}$ and set $T = \{p, 2p\} = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots t_{m,1}, t_{m,2}\}$. We conclude from $|S| = 4m, |T| = 2m$. $\Delta(G) = 6$. Define $f: V(G) \to \{0,1,2, \dots 8\}$. The minimum pendent eccentric degree dominating set is $\{t_{1,2}, t_{2,2}, t_{3,2}, \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots, s_{m,1}\}$. $f(t_{1,2}) = 6, f(t_{2,2}) = 6, \dots f(t_{m,2}) = 6, f(s_{1,1}) = 4, f(s_{2,1}) = 4, \dots f(s_{m,1}) = 4$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $10m$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = 10m$.

Case $iii)\ p = 5$ The set $S = \{s_{1,1}, s_{1,2}, \dots, s_{1,4}, s_{2,1}, s_{2,2}, \dots, s_{2,4}, \dots s_{m,1}, s_{m,2}, \dots, s_{m,4}\}$, We conclude from $|S| = 4m$ $\Delta(G) = 5$. Define $f: V(G) \to \{0,1,2, \dots 7\}$. The minimum pendent eccentric degree dominating set is $\{t_{1,2}, t_{2,2}, t_{3,2}, \dots t_{m,2}, t_{1,3}\}$. $f(t_{1,2}) = 5, f(t_{2,2}) = 5, \dots f(t_{m,2}) = 5, f(t_{1,3}) = 5$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $5(m+1)$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{3p})\right)\right) = 5(m+1)$.

Case $iv)\ p \geq 7, m \geq 3$, $\Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p+3\}$. The minimum pendent eccentric degree dominating set is $\{t_{1,2}, s_{1,1}, t_{2,2}, s_{2,1} \dots t_{m,2}, s_{m,1}\}$. $f(t_{1,2}) = p + 1, f(s_{1,1}) = 6, f(t_{2,2}) = p + 1, f(s_{2,1}) = 6, \dots f(t_{m,2}) = p + 1, f(s_{m,1}) = 6$ and the remaining vertices are zero. The weight of minimum $PEDDF$ is $m(p+7)$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{5p})\right)\right) = m(p+7)$.

Theorem 3.6 The path union graph of zero divisor graph $P\left(m, \Gamma(z_{5p})\right)$ $m \geq 3$, then $i)\ If\ p = 2, then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = 8m + 4$

$ii)\ If\ p = 3, then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = 10m$

$iii)\ If\ p = 5, then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = 5(m+1)$

$iv)\ If\ p \geq 7, then\ \gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = m(p+7)$

Proof: Let $P\left(m, \Gamma(z_{5p})\right)$ be a path union graph of zero divisor graph $m \geq 3$.

Case $i)\ p = 2$ we create sets that can represent the two partitions and m copies. The set $S = \{t_{1,1}, t_{1,2}, t_{1,3}, t_{1,4}, t_{2,1}, t_{2,2}, t_{2,3}, t_{2,4}, \dots, t_{m,1}, t_{m,2}, t_{m,3}, t_{m,4}\}$ and set $T = \{t_{1,p}, t_{2,p}, \dots t_{m,p}\}$. We conclude from $|S| = 4m, |T| = m$. $\Delta(G) = 6$. Define $f: V(G) \to \{0,1,2, \dots 8\}$. The minimum dominating set is $\{t_{1,1}, t_{m,1}, t_{1,p}, t_{2,p}, t_{3,p}, \dots, t_{m,p}\}$. $f(t_{1,1}) = 3, f(t_{m,1}) = 3, f(t_{1,p}) = 7, f(t_{2,p}) = 8, f(t_{3,p}) = 8, \dots, f(t_{m,p}) = 7$ and the remaining vertices are zero. The weight of $PEDDF$ is $8m + 4$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = 8m + 4$.

Case $ii)\ p = 3$ we create sets that can represent the two partitions and m copies. The set $S = \{s_{1,1}, s_{1,2}, \dots, s_{1,4}, s_{2,1}, s_{2,2}, \dots, s_{2,4}, \dots s_{m,1}, s_{m,2}, \dots, s_{m,4}\}$ and set $T = \{t_{1,1}, t_{1,2}, t_{2,1}, t_{2,2}, \dots t_{m,1}, t_{m,2}\}$. We conclude from $|S| = 4m, |T| = 2m$. $\Delta(G) = 6$. Define $f: V(G) \to \{0,1,2, \dots 8\}$. The minimum dominating set is $\{t_{1,2}, t_{2,2}, t_{3,2} \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots, s_{m,1}\}$. $f(t_{1,2}) = 6, f(t_{2,2}) = 6, \dots f(t_{m,2}) = 6, f(s_{1,1}) = 4, f(s_{2,1}) = 4, \dots f(s_{m,1}) = 4$ and the remaining vertices are zero. The weight of $PEDDF$ is $10m$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = 10m$.

Case $iii)\ p = 5$ we create sets that can represent the one partition and m copies. The set $S = \{5,10,15,20\} = \{s_{1,1}, s_{1,2}, s_{1,3}, s_{1,4}, s_{2,1}, s_{2,2}, s_{2,3}, s_{2,4} \dots s_{m,1}, s_{m,2}, s_{m,3}, s_{m,4}\}$. We conclude from $|S| = 4m$. $\Delta(G) = 5$. Define $f: V(G) \to \{0,1,2, \dots 7\}$. The minimum dominating set is $\{s_{1,2}, s_{2,2}, s_{3,2}, \dots s_{m,2}, s_{1,3}\}$. $f(s_{1,2}) = 5, f(s_{2,2}) = 5, \dots f(s_{m,2}) = 5, f(s_{1,3}) = 5$ and the remaining vertices are zero. The weight of $PEDDF$ is $5(m+1)$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = 5(m+1)$.

Case $iv)\ p \geq 7, \Delta(G) = p + 1$. Define $f: V(G) \to \{0,1,2, \dots p+3\}$. The minimum dominating set is $\{s_{1,2}, s_{2,2}, s_{3,2}, \dots s_{m,2}, t_{1,2}, t_{2,2}, \dots t_{m,2}\}$. $f(s_{1,2}) = p + 1, f(s_{2,2}) = p + 1, \dots f(s_{m,2}) = p + 1, f(t_{1,2}) = 6, f(t_{2,2}) = 6, \dots f(t_{m,2}) = 6$ and the remaining vertices are zero. The weight of $PEDDF$ is $m(p+7)$. Hence $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{5p})\right)\right) = m(p+7)$.

Theorem 3.7 The cycle graph of zero divisor graph $C\left(m, \Gamma(z_{pq})\right)$ $m \geq 3, p < q$ then $\gamma_{deg}\left(C\left(m, \Gamma(z_{pq})\right)\right) = \{\ 6m \quad where\ p = 2, q = 3\ m((p+q)+2)\ \ where\ p \geq 3$.

Proof: Let $C\left(m, \Gamma(z_{pq})\right)$ be a cycle graph of zero divisor graph $m \geq 3, p < q$.

Case $i)\ p = 2, q = 3$ The resulting graph is $\left(C\left(m, \Gamma(z_{2p})\right)\right)$ where $p = 3$

Case $ii)\ p \geq 3$ we create sets that can represent the two partitions and three copies. The set $S = \{p, 2p, 3p, \dots, (q-1)p\} = \{s_{1,1}, s_{1,2}, \dots s_{1,q-1}, s_{2,1}, s_{2,2}, \dots s_{2,q-1}, \dots \dots s_{m,1}, s_{m,2}, \dots s_{m,q-1}\}$ and set $T = \{q, 2q, 3q, \dots, (p-1)q\} = \{t_{1,1}, t_{1,2}, \dots t_{1,p-1}, t_{2,1}, t_{2,2}, \dots t_{2,p-1}, \dots \dots t_{m,1} t_{m,2}, \dots t_{m,p-1}\}$. We conclude from $|S| = m(q-1), |T| = m(p-1)$. $\Delta(G) = q + 1$. Define $f: V(G) \to \{0,1,2, \dots q+3\}$. The minimum dominating set is $\{t_{1,2}, t_{2,2}, \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots \dots s_{m,1}\}$. $f(t_{1,2}) = q + 1, f(t_{2,2}) = q + 1, \dots f(t_{m,2}) = q + 1, f(s_{1,1}) =$

$p+1, f(s_{2,1}) = p+1, \dots f(s_{m,1}) = p+1$ and the remaining vertices are zero. The weight of $PEDDF$ is $m((p+q)+2)$. Hence $\gamma_{PEDD}\left(C\left(m, \Gamma(z_{pq})\right)\right) = m((p+q)+2)$.

Theorem 3.8 The path union graph of zero divisor graph $P\left(m, \Gamma(z_{pq})\right)$ $m \geq 3$, then $\gamma_{PEDD}\left(P\left(m, \Gamma(z_{pq})\right)\right) = \{\ 6m \quad where\ p = 2, q = 3\ m((p+q)+2)\ \ where\ p \geq 3, p < q$.

Proof: Let $P\left(m, \Gamma(z_{pq})\right)$ be a path union graph of zero divisor graph $m \geq 3$

Case $i)\ p = 2, q = 3$ the resulting graph is $P\left(m, \Gamma(z_{2p})\right)$ where $p = 3$.

Case $ii)\ p \geq 3$ we create sets that can represent the two partitions and m copies. The set $S = \{p, 2p, 3p, \dots, (q-1)p\} = \{s_{1,1}, s_{1,2}, \dots s_{1,q-1}, s_{2,1}, s_{2,2}, \dots s_{2,q-1}, \dots s_{m,1}, s_{m,2}, \dots s_{m,q-1}\}$ and set $T = \{q, 2q, 3q, \dots, (p-1)q\} = \{t_{1,1}, t_{1,2}, \dots t_{1,p-1}, t_{2,1}, t_{2,2}, \dots t_{2,p-1}, \dots t_{m,1}, t_{m,2}, \dots t_{m,p-1}\}$. We conclude from $|S| = m(q-1), |T| = m(p-1)$. $\Delta(G) = q+1$. Define $f: V(G) \to \{0,1,2,\dots q+3\}$. The minimum dominating set is $\{t_{1,2}, t_{2,2}, \dots t_{m,2}, s_{1,1}, s_{2,1}, \dots s_{m,1}\}$. $f(t_{1,2}) = q+1, f(t_{2,2}) = q+1, \dots f(t_{m,2}) = q+1, f(s_{1,1}) = p+1, f(s_{2,1}) = p+1, \dots f(s_{m,1}) = p+1$ and the remaining vertices are zero. The weight of $PEDDF$ is $m((p+q)+2)$. Hence $\gamma_{PEDD} P\left(m, \Gamma(z_{pq})\right) = m((p+q)+2)$.

APPLICATIONS:

Pendant eccentric degree dominating function merges the core concepts of graph theory eccentricity, degree and domination. These functions are highly valuable in chemical graph theory, where molecular structures are modeled as graphs to evaluate their unique characteristics. When engineering novel materials, pendant eccentric degree dominating functions assist in evaluating molecular networks. This analysis helps optimize essential material properties such as conductivity, flexibility, and structural strength.

CONCLUSION:

This study investigated the $PEDDF$ of the ZDG of a commutative ring. We established multiple theorems concerning the path union and cycle of join graphs of ZDG. Future research will build upon these findings by exploring alternative graphs to analyse different dominance properties.

REFERENCES:

[1] D. F. Anderson and P. S. Livingston, The zero-divisor graph of a commutative ring, J. Algebra, 217, 1999 434–447.

[2] I. Beck, Coloring of commutative rings, J. Algebra, 116(1), 1988 208–226.

[3] C. Berge: Theory of Graphs and its Applications, Methuen and Co.Ltd, London 1962. https://www.sciepub.com/reference/175421.

[4] Christian Constantine, Erma Suwastika, “Graceful labeling of zero-divisor graph Γ(ℤp²q) and Γ(ℤp³q)”, Electronic Journal of Graph Theory and Applications 13(2)(2025), 309-322.

[5] N.C. Demirpolat, and E. Kilic, On Degree Domination in Graph, J. of Modern Technology and Eng. 9(2),(2024),112-118. DOI: https://doi.org/10.62476/jmte9112.

[6] M. Dinorog and I.S. Cabahug, Ring Domination Number of Some Mycielski Graphs, Asian Research J. of Mathematics, 18(12), (2022).

[7] R. Geetha, R. Rajeswari, “Pendant Eccentric Degree Dominating Functions of Some Zero Divisor Graphs”, Computer Engineering and Applications Journal, Volume 16 Issue 8 2026, page No: 1-06, DOI: 10.5281/JCEA/160801.

[8] F. Harary, Graph Theory, Addison-wesley, Reading, Mass 1972.

[9] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, "Fundamentals of domination in Graphs", Marcel Dekker, New-York 1998.

[10]T.N.Janakiraman, M.Bhanumathi and S.Muthammai, “Eccentric Domination in Graphs”, International Journal of Engineering Science, Advanced Computing and Bio-Technology Vol.1.No.2,April-June 2010,pp.55-70.

[11] Keerthi G. Mirajkar, Veena N. Mathad, and Pooja B, “Miscellaneous Properties of Line Mycielskian Graph of a Graph”, International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 24 (2019) pp. 4552-4556

[12]Nayaka S.R. Puttaswamy and Purushotham.a S, “Pendant Domination In Graphs”, Journal of Combinatorial Mathematics and Combinatorial Computing 112(1):219-229 June 2020.

[13] V. Thukarama, Soner N.D. “On Degree Dominating Functions in Graphs”, International Journal of Recent Technology and Engineering (IJRTE), ISSN: 2277-3878(Online), Volume-14 Issue-2, July 2025.