Innovative Fuzzy Cube Difference Labeling Techniques for Enhancing Special Graphs

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Abstract
We are excited to present an innovative concept called FCDL (Fuzzy Cube Difference Labeling). Our findings confirm that several key graph structures specifically, path graph P_n, cycle graph C_n, star graph S_n, complete bipartite graph $K_{m,n}$ $(n \leq 3)$, complete graph k_n, wheel graph W_n, and Bistar graph $B_{n,n}$ successfully qualify as FCDG (fuzzy cube difference graphs). This discovery enhances our understanding of graph theory and opens up new avenues for research and application.

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1. Introduction

Fuzzy graph theory has many applications, including networking, data mining, communication, image capture, clustering, planning, and scheduling. Fuzzy labeling methods enhance flexibility, accuracy, and system compatibility compared to traditional fuzzy and classical models. Additionally, they have important uses in computer science, physics, chemistry, and other mathematical fields. Lotfi Zadeh [9] first used the phrase "fuzzy logic" in 1965, and Lukasiewicz and Tarski [4] have also analysed it. By examining fuzzy relations on fuzzy sets, Rosenfeld [5] created the notion of fuzzy graphs in 1975. He also explored fuzzy graphs' cut nodes, bridges, blocks, and fuzzy trees. A. Nagoor Gani [2, 3] proposed the idea of fuzzy labeling graphs and studied their unique characteristics. The fuzzy square difference labeling of some graphs was introduced by T. Bharathi, Jeya Rowena and P. Ashwini Sibiya Rani [6]. J. Shiama invented the idea of CDL (Cube difference labeling) [7].

2. Preliminaries

Definition 2.1 If V is a set and R is a relation on V, then the pair (V, E) is a graph. We can think of any fuzzy relation ρ on a fuzzy subset μ of a set V as a fuzzy graph with weight or strength $\rho(u,v) \in [0,1]$ on the edge $(u,v) \in V \times V$.

Definition 2.2 If an injection $f: V(G) \to \{0,1,\dots,p-1\}$ occurs such that the induced function $f^*: E(G) \to N$, which is provided by $f^*(uv) = |f(u)^3 - f(v)^3|$ for each $uv \in E(G)$, is distinct, then a graph G is said to have a **CDL**. A Cube difference graph is a graph that allows CDL.

Definition 2.3 A walk in a graph is a finite, non-null subspace with words that alternately represent vertices and edges, $W = v_0 e_1 v_1 e_2 \dots e_k v_k$. W is a (v_0, v_k) walk, as we say. When a walk W has distinct vertices, it is called a **path**. The term **"cycle"** refers to a closed trail with the same starting and ending points.

Definition 2.4 A tree with one internal node and n leaves is called a star S_n, which is the complete bipartite network $K_{1,n}$.

Definition 2.5 If v can be divided into two distinct subsets, X and Y so that each line of G connects a point of X to a point of Y, then the graph is referred to as a bigraph or bipartite graph. We refer to (X, Y) as a bipartition of G. G Is referred to as a complete **bigraph** or **complete bipartite graph** if it also includes every line connecting the points of X and Y.

Definition 2.6 If every pair of unique vertices in a graph G is adjacent to every other pair in G, then the graph is **complete**. K_p is the notation for a complete graph on p vertices. $\frac{p(p-1)}{2}$ is the size of $\boldsymbol{K_p}$.

Definition 2.7 A **wheel graph** with $n + 1$ vertices is the sum of C_n and K_1. It is abbreviated as W_n.

Definition 2.8 The graph known as a **Bistar** is created by connecting the apex vertices of two copies of the star $K_{1,n}$ by an edge. $B_{n,n}$ is its notation. It's also known as a double star.

Definition 2.9 A graph $G = (\sigma, \mu)$ is said to be a **fuzzy labeling graph**, if $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) < \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

3. Fuzzy cube difference labeling of graphs

Definition 3.1 Definition 1.1.35 A graph $G = (V, E)$ is considered to be an **FCDL** if it is a fuzzy labeling graph and satisfies the following conditions: $\mu(u,v) = |[\sigma(u)]^3 - [\sigma(v)]^3|$ for each $(u,v) \in E$ are distinct. An **FCDG** is a graph that allows for fuzzy cube difference labeling.

Example:

Figure 1

Theorem 3.2 Every path graph P_n admits FCDG.

Proof:

Assume that $\{v_j : 1 \le j \le n\}$ represents the set of vertices and $\{v_{j-1}v_j : 2 \le j \le n\}$ represents the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-1}$ for $n \le 9, \omega = 10^{-2}$ for $10 \le n \le 99, \omega = 10^{-3}$ for $100 \le n \le 999$ and so on.

$\sigma(v_i) = i * \omega, \quad i = 1,2,\dots,n$

$\mu(v_iv_{i+1}) = 7 * \omega^3, \quad i = 1$

$\mu(v_iv_{i+1}) = (6\omega^3 * i) + \mu(v_{i-1}v_i), \quad i = 2,3,\dots,n-1$

This label complies with FCDL's requirements. Therefore, all path graphs are FCDGs.

Example:

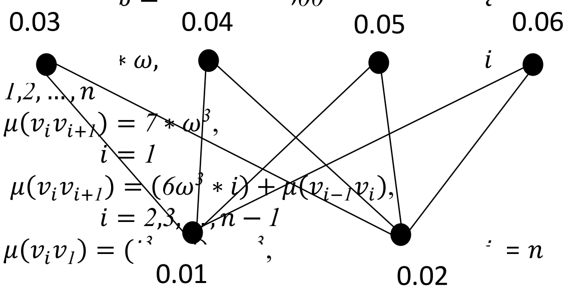
Figure 2: FCDG of P_9

Theorem 3.3 Every cycle graph C_n admits FCDG.

Proof:

Let $\{v_j : 1 \le j \le n\}$ be the set of vertices and $\{v_1v_2, v_2v_3, .., v_{n-1}v_n, v_nv_1\}$ be the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-1}$ for $n \le 9, \omega = 10^{-2}$ for $10 \le n \le 99, \omega = 10^{-3}$ for $100 \le$ ... $\omega = 10^{-4}$ for $1000 \le n \le 9999$ ...



... $* \omega, \quad i$ ... $1,2,\dots,n$

$\mu(v_iv_{i+1}) = 7 * \omega^3, \quad i = 1$

$\mu(v_iv_{i+1}) = (6\omega^3 * i) + \mu(v_{i-1}v_i), \quad i = 2,3,\dots,n-1$

$\mu(v_iv_1) = ($ ... $, \quad$ ... $= n$

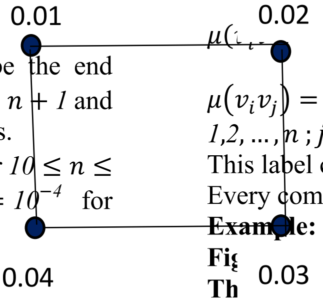
This label co... FCDL rec...

Every cycle graph is therefore an FCDG.

Example:

Figure 3: FCDG of C_4

Theorem 3.4 Every star graph S_n admits ...



Proof:

Let v_1 be the Central vertex and v_j be the end vertices, where $2 \le j \le n+1$ and $\{v_1v_2, v_1v_3, .., v_1v_{n+1}\}$ be the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-2}$ for $10 \le n \le 99, \omega = 10^{-3}$ for $100 \le n \le 999, \omega = 10^{-4}$ for $1000 \le n \le 9999$ and so on.

$\sigma(v_i) = i * \omega, \quad i =$

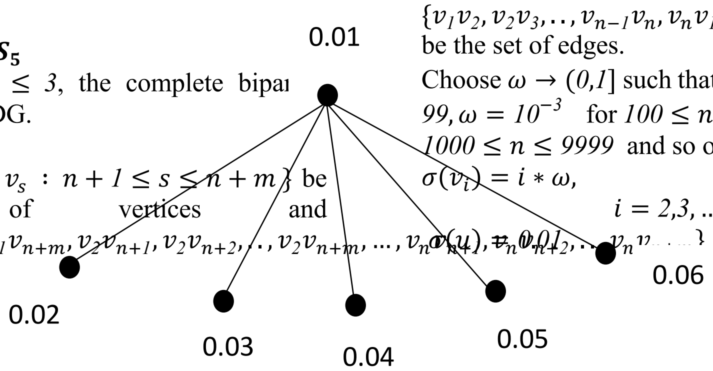
$\mu(v_1v_i) = (i^3 - 1) * \omega^3, \quad i =$

This label satisfies with FCDL requirements.

All star graphs are hence FCDGs.

Example:

Figure 4: FCDG of S_5



Theorem 3.5 For $n \le 3$, the complete bipa... graph $K_{n,m}$ is an FCDG.

Proof:

Let $\{v_r : 1 \le r \le n ; v_s : n+1 \le s \le n+m\}$ be the set of vertices and $\{v_1v_{n+1}, v_1v_{n+2}, .., v_1v_{n+m}, v_2v_{n+1}, v_2v_{n+2}, .., v_2v_{n+m}, \dots, v_nv_{n+1}, v_nv_{n+2}, .., v_nv_{\dots}\}$ be the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-2}$ for $10 \le n \le 99, \omega = 10^{-3}$ for $100 \le n \le 999, \omega = 10^{-4}$ for $1000 \le n \le 9999$ and so on.

$\sigma(v_i) = i * \omega, \quad i = 1,2,\dots,n,n+1,\dots,n+m$

Case (i): When $n = 1$

$\mu(v_1v_i) = (i^3 - 1) * \omega^3, \quad i = 2,3,\dots,n+m$

Case (ii): When $n = 2$

$\mu(v_1v_i) = (i^3 - 1) * \omega^3, \quad i = 3,4,\dots,n+m$

$\mu(v_2v_i) = (i^3 - 8) * \omega^3, \quad i = 3,4,\dots,n+m$

Case (iii): When $n = 3$

$\mu(v_1v_i) = (i^3 - 1) * \omega^3, \quad i = 4,5,\dots,n+m$

$\mu(v_2v_i) = (i^3 - 8) * \omega^3, \quad i = 4,5,\dots,n+m$

$\mu(v_3v_i) = (i^3 - 27) * \omega^3, \quad i = 4,5,\dots,n+m$

This label complies with FCDL requirements. Therefore, for $n \le 3$, the complete bipartite graph $K_{n,m}$ is an FCDG.

Example:

Figure 5: FCDG of $K_{2,4}$

Theorem 3.6 Every complete graph K_n is an FCDG.

Proof:

Let $\{v_1, v_2, \dots, v_n\}$ be the set of vertices and $\{v_rv_s / 1 \le r, s \le n\ (r \ne s)\}$ be the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-2}$ for $10 \le n \le 99, \omega = 10^{-3}$ for $100 \le n \le 999, \omega = 10^{-4}$ for $1000 \le n \le 9999$ and so on.

$\sigma(v_i) = i * \omega, \quad i = 1,2,\dots,n$

$\mu(v_iv_{i+1}) = 7 * \omega^3, \quad i = 1$

$\mu(v_iv_{i+1}) = (6\omega^3 * i) + \mu(v_{i-1}v_i), \quad i = 2,3,\dots,n-1$

$\mu($ ... $i^3 - 1) * \omega^3, \quad i = n$

$\mu(v_iv_j) = |i^3 - j^3| * \omega^3, \quad i = 1,2,\dots,n ; j = 3,4,\dots,n-1$ and $i \ne j$

This label complies with FCDL requirements.

Every complete graph is therefore an FCDG.

Example:

Fig... CDG of K_5

Th... 7 Every wheel graph W_n is an FCDG.

Proof:

Let $\{v_r : 1 \le r \le n\}$ be the set of vertices and $\{v_1v_2, v_2v_3, .., v_{n-1}v_n, v_nv_1\} \cup \{uv_1, uv_2, \dots, uv_n\}$ be the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-2}$ for $10 \le n \le 99, \omega = 10^{-3}$ for $100 \le n \le 999, \omega = 10^{-4}$ for $1000 \le n \le 9999$ and so on.

$\sigma(v_i) = i * \omega, \quad i = 2,3,\dots,n$

$\sigma(u) = 0.01,$

$\mu(v_i v_{i+1}) = 19 * \omega^3,$
$i = 1$
$\mu(v_i v_{i+1}) = (6\omega^3 * (i+1)) + \mu(v_{i-1} v_i),$
$i = 2,3,\dots,n-1$
$\mu(v_1 v_i) = 56 * \omega^3,$
$i = n$ if $n = 3$
$\mu(v_1 v_i) = (3(i-1)^2 + 9(i-1) + 7) * \omega^3 + \mu(v_1 v_{i-1}),$ $i = n$ if $n > 3$
$\mu(u v_{i-1}) = (i^3 - 1) * \omega^3,$
$i = 2,3,\dots,n+1$

This label complies with the FCDL requirements. Hence the wheel graph is an FCDG.

Example:

Figure 7: FCDG of W_5

Theorem 3.8 Every Bistar graph $B_{n,n}$ admits FCDG.

Proof:

Let $\{v_r, v, u, u_s : 1 \le r \le n, 1 \le s \le n\}$ be the set of vertices, where v, u are apex vertices and $\{vv_1, vv_2, \dots, vv_n, uu_1, uu_2, \dots, uu_n\}$ be the set of edges.

Choose $\omega \to (0,1]$ such that $\omega = 10^{-2}$ for $10 \le n \le 99$, $\omega = 10^{-3}$ for $100 \le n \le 999$, $\omega = 10^{-4}$ for $1000 \le n \le 9999$ and so on.

The vertex labels are,
$\sigma(u) = 1 * \omega$
$\sigma(v) = 2 * \omega$
$\sigma(u_i) = (2i+1) * \omega,$ $i = 1,2,\dots,n$
$\sigma(v_i) = (2i+2) * \omega,$ $i = 1,2,\dots,n$

The induced edge labels are,
$\mu(uu_i) = (8i^3 + 12i^2 + 6i) * \omega^3,$ $i = 1,2,\dots,n$
$\mu(vv_i) = (8i^3 + 24i^2 + 24i) * \omega^3,$ $i = 1,2,\dots,n$
$\mu(uv) = 7 * \omega^3$

This labeling obey the conditions of FCDL.
Thus the Bistar graph $B_{n,n}$ is an FCDG.

Example:

Figure 8: FCDG of $B_{5,5}$

4. Conclusion

In this research, we looked at seven new fuzzy cube difference graphs. The findings presented in this research are new. For a better understanding of the labeling pattern defined in each theorem, examples are provided at the end of each theorem.

Conflict of interests

The authors proudly affirm that they have no conflicts of interest to declare concerning the publication of this paper.

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