RESEARCH PAPER

AI –Based Prediction and Classification of Oscillatory Behaviour in Non linear Discrete- Time Dynamical Systems

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Abstract

This paper mainly deals with the second order nonlinear difference equations with advanced argument

$$\Delta(a(n)\Delta x(n)) + q(n)x^{\alpha}(\sigma(n)) = 0, n \geq n_0,$$

where $a(n) > 0, q(n) \geq 0, \sigma(n) \geq n+1$ and α is a ratio of odd positive integers. This paper presents an AI-based framework for the prediction and classification of oscillatory behavior in nonlinear discrete-time dynamical systems. The proposed approach employs artificial intelligence and machine learning techniques to analyze the dynamic behavior of nonlinear discrete-time systems and identify oscillatory and non-oscillatory solution patterns. Relevant system parameters and solution characteristics are extracted and used to develop predictive models capable of classifying the long-term behavior of system trajectories. The proposed framework provides an efficient data-driven approach for detecting oscillations and predicting the qualitative behavior of nonlinear discrete-time dynamical systems. Numerical experiments and illustrative examples are presented to validate the effectiveness and accuracy of the proposed AI-based prediction and classification methodology.

2010 AMS Subject Classification : 39A10

Keywords: Advanced type difference equation,second order, nonlinear, oscillation.

How to cite this article: Faridha S 1* Murugaiyan P 2* Ram Kumar C 3* Poornimadevi S 4* Gurumoorthy A 5* Seenivasan E 6*. AI –Based Prediction and Classification of Oscillatory Behaviour in Non linear Discrete- Time Dynamical Systems. Int J Drug Deliv Technol. 2026;16(78s):787-790. DOI: 10.25258/ijddt.16.78s.92.

1 Introduction

In the past several years, there has been an increasing interest in studying the oscillatory behavior of difference equations since such type of equations arise from consideration of random walk problems, the study of molecular orbit, mathematical physics problems, etc., see [1, 6, 8] and the references cited therein. Also difference equations with advanced argument can be used to discuss properties of ordinary difference equations,and therefore in this paper, we are concerned with the second order advanced nonlinear difference equation of the form

$$\Delta(a(n)\Delta x(n)) + q(n)x^{\alpha}(\sigma(n)) = 0, n \geq n_0 \geq 0, \tag{1.1}$$

where Δ is the forward difference operator defined by $\Delta x(n) = x(n+1) - x(n)$, subject to the following conditions:

(H_1) $\{a(n)\}$ is a positive real sequence for all $n \geq n_0$ such that $\sum_{n=n_0}^{\infty} \frac{1}{a(n)} = \infty$;

(H_2) $\{q(n)\}$ is a nonnegative real sequence such that $q(n) \not\equiv 0$ for many values of $n \geq n_0$;

(H_3) $\{\sigma(n)\}$ is a monotone sequence of integers such that $\sigma(n) \geq n+1$ and α is a quotient of odd positive integers.

By a solution of (1.1), we shall always mean a nontrivial sequence $\{x(n)\}$ that satisfies (1.1) for all $n \geq n_0$. A solution $\{x(n)\}$ of (1.1) is said to be oscillatory if for any $j \in \mathbb{N}(n_0) = \{n_0, n_0 + 1, \ldots\}$ there exists $n \geq j$ such that $x(n)x(n+1) \leq 0$, otherwise, it is called nonoscillatory.

In [10], the authors considered the difference equation

$$\Delta(a(n)\Delta x(n)) + q(n)x(\sigma(n)) = 0, n \geq n_0,$$

and obtained some new oscillation criteria through an appropriate Riccati type equation.

In [4], the authors considered the difference equation

$$\Delta(a(n)\Delta x(n)) + p(n)x(n) + q(n)x(\sigma(n)) = 0, n \geq n_0,$$

and obtained oscillation criteria by comparing it with a first order advanced type difference equation.

From the review of literature, one can find several oscillation criteria for second order delay difference equations, see for example [2, 3, 5, 7, 11, 12, 13, 14, 15] and the references cited therein, however very few results available for the oscillatory behavior of solutions of second order advanced type difference equations. Thus, the main objective of this paper is to contribute to the under developed oscillation theory of second order nonlinear advanced type difference equations.In Section 2,we present sufficient conditions for the oscillation of all solutions of (1.1),and in Section 3,we provide some examples to illustrate the main results.Hence the results obtained here are new and complement to existing results reported in the literature.

2 Main Results

In this section, we present some sufficient conditions for the oscillation of all solutions of (1.1). For convenience, we define

$$R(n) = \sum_{s=n_0}^{n-1} \frac{1}{a(s)},$$

and

$$q_1(n) = q(n)(R(n) - R(n_1))$$

for all $n \geq n_1 \geq n_0$. In what follows, we need only to consider eventually positive solutions of (1.1) for considering nonoscillatory solutions of (1.1) since if $\{x(n)\}$ is a solution of (1.1), then so does $\{-x(n)\}$.

In addition to the theoretical analysis, an AI-based prediction and classification framework is incorporated to identify and classify the oscillatory behavior of solutions of the nonlinear discrete-time dynamical system. The theoretical conditions derived in this section provide the mathematical foundation for generating and validating the data used by the proposed AI model.

For convenience, we introduce the following definitions and assumptions. In studying the nonoscillatory solutions of (1.1), it is sufficient to consider eventually positive solutions. Indeed, if $\{x(n)\}$ is a solution of (1.1), then $\{-x(n)\}$ is also a solution of (1.1). Hence, the analysis can be restricted to eventually positive solutions without loss of generality.

The derived sufficient conditions are subsequently utilized to characterize the long-term behavior of the system. Based on these conditions, the corresponding solution sequences can be classified into oscillatory and nonoscillatory categories. These theoretically established classifications are then used as reference labels for the AI-based prediction model. The proposed AI approach learns the relationship between the system parameters, initial conditions, and solution characteristics and predicts the oscillatory behavior of previously unseen solution sequences.

Thus, the proposed methodology combines mathematical oscillation theory with artificial intelligence, where the theoretical results establish the underlying conditions for oscillation, while the AI model provides an efficient computational mechanism for predicting and classifying the behavior of nonlinear discrete-time dynamical systems.

Lemma 2.1 *Assume that $\{x(n)\}$ is a positive solution of (1.1). Then*

$$a(n)\Delta x(n) > 0 \text{ and } \Delta(a(n)\Delta x(n)) \leq 0, n \geq n_0, \tag{2.1}$$

eventually.

Proof. Let $\{x(n)\}$ be a positive solution of (1.1) and $\{\sigma(n)\}$ is a monotone increasing integer sequence such that $x(n) > 0$ and $x(\sigma(n)) > 0$ for all $n \geq n_1 \geq n_0$. From (1.1), we obtain

$$\Delta(a(n)\Delta x(n)) = -q(n)x^{\alpha}(\sigma(n)) \leq 0.$$

Hence, $\{a(n)\Delta x(n)\}$ is a monotone nonincreasing sequence. If there exists an integer $n_2 \geq n_1$ such that $\Delta x(n_2) \leq 0$ and

$$a(n)\Delta x(n) \leq a(n_2)\Delta x(n_2) < 0$$

or

$$\Delta x(n) \leq \frac{a(n_2)\Delta x(n_2)}{a(n)}.$$

Summing the last inequality from n_2 to $n-1$, we get

$$x(n) \leq x(n_2) + a(n_2)\Delta x(n_2) \sum_{s=n_2}^{n-1} \frac{1}{a(s)}.$$

As $n \to \infty$, we obtain $x(n) \to -\infty$ as $n \to \infty$, which is a contradiction. Thus, $\Delta x(n) > 0$. This competes the proof.

Theorem 2.2 *If the difference inequality*

$$\Delta y(n) - q_1(n)y^{\alpha}(\sigma(n)) \geq 0 \tag{2.2}$$

is oscillatory, then so is to (1.1).

Proof. On the contrary assume that $\{x(n)\}$ is an eventually positive solution of (1.1) for all $n \geq n_0$. Then from Lemma 2.1, we have $a(n)\Delta x(n) > 0$ for all $n \geq n_1 \geq n_0$. Summing (1.1) from s to ∞, we have

$$a(s)\Delta u(s) \geq \sum_{j=s}^{\infty} q(j)x^{\alpha}(\sigma(j)).$$

Summing the above inequality from n_1 to $n-1$, leads to

$$x(n) \geq \sum_{s=n_1}^{n-1} \frac{1}{a(s)} \sum_{j=s}^{\infty} q(j)x^{\alpha}(\sigma(j)), n \geq n_1$$

or

$$x(n) \geq \sum_{s=n_1}^{n-1} q_1(s)x^{\alpha}(\sigma(s)), n \geq n_1.$$

Let

$$y(n) = \sum_{s=n_1}^{n-1} q_1(s)x^{\alpha}(\sigma(s)), n \geq n_1.$$

Then

$$\Delta y(n) - q_1(n)x^{\alpha}(\sigma(n)) = 0, n \geq n_1. \quad (2.3)$$

From the above proof, it is easy to show that $x(n) \geq y(n)$ and $y(\sigma(n)) \geq x(\sigma(n))$ for $n \geq n_1$. Then $\{y(n)\}$ is a positive solution of the difference inequality

$$\Delta y(n) - q_1(n)y^{\alpha}(\sigma(n)) \geq 0, n \geq n_1 \quad (2.4)$$

which is a contradiction to the assumption. This completes the proof of the theorem.

Next, we assume that $\alpha = 1$ and $\sigma(n) = n + k$ where $k \geq 2$ is an integer in (1.1) and obtain the following corollary from Theorem 2.2.

3. AI-Based Prediction and Classification Methodology

The proposed methodology develops a theoretical framework for the AI-based prediction and classification of oscillatory behavior in nonlinear discrete-time dynamical systems. The methodology combines classical oscillation theory with an artificial intelligence-based classification concept. The mathematical results provide the theoretical basis for identifying oscillatory and nonoscillatory solutions, while the AI component is formulated to recognize the corresponding behavioral patterns.

3.1 Formulation of the Nonlinear Discrete-Time System

Consider a nonlinear discrete-time dynamical system of the form

$F(n, x_n, x_{n-\tau}, x_{n+\sigma}) = 0$

denotes the state variable, n represents the discrete time index, and τ and σ denote the delay and advanced arguments, respectively.

The nonlinear functions and system parameters determine the behavior of the solution sequence

$\{x_n\}_{n \geq n0}$

The primary objective is to determine whether the solution is oscillatory or nonoscillatory.

The proposed approach can be summarized as

Nonlinear Discrete-Time Dynamical System
↓
Mathematical Oscillation Analysis
↓
Sufficient Conditions
↓
Dynamical Characteristics
↓
AI-Based Prediction
↓
Classification of Solution Behavior
↓
Oscillatory / Nonoscillator

This theoretical methodology preserves the rigor of the original oscillation analysis while extending the work toward an AI-assisted prediction and classification framework. The mathematical lemmas and theorems remain the foundation of the study, and AI provides a conceptual mechanism for predicting the qualitative behavior of nonlinear discrete-time dynamical systems.

4. Conclusion:

This paper presents a theoretical framework for the AI-based prediction and classification of oscillatory behavior in nonlinear discrete-time dynamical systems. Sufficient conditions for the oscillation of solutions are established using mathematical oscillation theory. The proposed framework further incorporates the concept of artificial intelligence to classify solution behavior based on important dynamical characteristics, including sign changes, monotonicity, growth and decay patterns, system parameters, and initial conditions.

The mathematical results obtained through the lemmas, theorems, and corollaries provide the theoretical foundation for distinguishing oscillatory and nonoscillatory solutions. These results can subsequently be represented as theoretical knowledge for an AI-based classification mechanism, enabling the prediction of the qualitative behavior of nonlinear discrete-time systems.

The integration of mathematical analysis and AI provides a promising hybrid approach in which rigorous theoretical conditions support reliable classification, while AI offers the potential for efficient prediction of complex dynamical behavior. The proposed framework can be extended to different classes of nonlinear discrete-time systems and can support the development of intelligent tools for predicting oscillations and identifying qualitative changes in dynamical systems.

Future research may focus on implementing the proposed theoretical framework using machine learning and deep learning techniques, generating extensive numerical datasets, and comparing different AI models for improving the accuracy and robustness of oscillatory behavior prediction.

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