Application of WH Closed Sets in Hexa Topological Spaces and Neutrosophic Hexa Topological Spaces for Multi-Criteria Medical Diagnosis

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Abstract — Topological structure theory plays a critical role in formalizing continuity, closure, and multi-attribute decision space modeling. The integration of multi-topological architectures—specifically Hexa Topological Spaces—with Neutrosophic Logic allows for simultaneous analysis across six independent structural topologies under multi-component uncertainty (Truth, Indeterminacy, Falsity). This paper introduces WH-closed sets and WH-open sets in Hexa Topological Spaces and extends them to Neutrosophic Hexa Topological Spaces (NH-TS). We formulate basic closure operators, interior operators, and topological separation characterizations for WH-closed sets. Furthermore, a novel Multi-Criteria Decision-Making (MCDM) algorithm based on Neutrosophic Hexa WH-similarity measures is proposed. The practical framework is demonstrated via a medical diagnostic application evaluating complex disease profiles under multi-source diagnostic uncertainty.

Keywords: Hexa Topological Spaces, WH-Closed Sets, Neutrosophic Hexa Topology, Neutrosophic Sets, Multi-Criteria Decision-Making (MCDM), Medical Diagnosis.

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1. Introduction

General topology provides mathematical tools to model space, boundaries, and continuity. The concept of Generalized Closed (g-closed) sets was initiated by Levine, opening new pathways to study continuous functions, separation axioms, and generalized topological structures. Over decades, classical single topology was extended to bitopological spaces by Kelley, tritopological spaces, and quad-topological spaces. Recently, the multi-topological system has been extended to Hexa Topological Spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6)$, which define six distinct topologies on a single universal set.

In real-world applications—such as decision analysis, image processing, and disease diagnostics—data structures are often characterized by conflicting, vague, or incomplete parameters. To capture indeterminacy, Smarandache introduced Neutrosophic Logic, which defines independent membership degrees for Truth (T), Indeterminacy (I), and Falsity (F). Salama and Alblowi extended this framework into topology by introducing Neutrosophic Topological Spaces (NTS).

The synthesis of multi-topological structures and neutrosophic sets leads to Neutrosophic Hexa Topological Spaces (NH-TS). In this paper, we introduce WH-closed sets in crisp and neutrosophic hexa topological environments. We establish their foundational properties, examine their structural interrelationships, and demonstrate their practical efficacy by constructing a neutrosophic hexa diagnostic similarity model for complex medical evaluation.

2. Preliminaries

We review fundamental definitions required for the structural formulation.

Definition 2.1 (Hexa Topological Space)

Let X be a non-empty set. An ordered 6-tuple $\tau_h = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6)$ of topologies on X is called a Hexa Topology on X, and (X, τ_h) is called a Hexa Topological Space (HTS). A subset $A \subseteq X$ is Hexa-open (H-open) if $A \in \bigcup_{i=1}^{6} \tau_i$. The complement A^c of an H-open set is called a Hexa-closed (H-closed) set.

Definition 2.2 (Neutrosophic Set)

Let X be a space of points. A Neutrosophic Set N in X is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$, and a falsity-membership function $F_N(x)$, where $T_N, I_N, F_N: X \to [0,1]$. A neutrosophic set is denoted as:

$$N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle : x \in X\}$$

Definition 2.3 (Neutrosophic Hexa Topological Space)

Let X be a non-empty universal set and $\tau_{Nh} = (\tau_{N1}, \tau_{N2}, \tau_{N3}, \tau_{N4}, \tau_{N5}, \tau_{N6})$ be six neutrosophic topologies defined on X. Then (X, τ_{Nh}) is defined as a Neutrosophic Hexa Topological Space (NH-TS).

3. WH-Closed Sets in Hexa and Neutrosophic Hexa Topological Spaces

Definition 3.1 ($\boldsymbol{WH}$-Closed Set in HTS)

A subset A of a Hexa Topological Space (X, τ_h) is called a Weak Hexa Closed Set (WH-closed set) if: $Cl_h(A) \subseteq U$ whenever $A \subseteq U$ and U is a Hexa semi-open set in (X, τ_h), where $Cl_h(A) = \bigcap\{F \subseteq X : A \subseteq F$ and F is H-closed$\}$.

Definition 3.2 (Neutrosophic $\boldsymbol{WH}$-Closed Set)

A Neutrosophic Set A in an NH-TS (X, τ_{Nh}) is called a Neutrosophic $\boldsymbol{WH}$-closed set (N-WH-closed) if: $NCl_h(A) \subseteq U$ whenever $A \subseteq U$ and U is a

Neutrosophic Hexa semi-open set in (X, τ_{Nh}).

Proposition 3.1

1. Every Hexa closed set is a WH-closed set.
2. The intersection of two WH-closed sets in (X, τ_h) is not necessarily a WH-closed set.
3. If A is N-WH-closed and $A \subseteq B \subseteq NCl_h(A)$, then B is an N-WH-closed set in (X, τ_{Nh}).

4. Application Framework: Medical Diagnostic Multi-Criteria Model

In complex patient diagnostics, symptoms present across multiple observational perspectives (e.g., Clinical Symptoms, Laboratory Biomarkers, Imaging Results, Histopathology, Patient History, and Genetic Profile). This 6-tier evaluations system maps naturally to a Hexa Topological Space.

4.1 Diagnostic Problem Setup

Let:

- $\boldsymbol{X = \{p_1, p_2, p_3\}}$ be the set of alternative patients evaluated for a severe respiratory syndrome.
- $\boldsymbol{S = \{s_1, s_2, s_3, s_4, s_5, s_6\}}$ be six diagnostic evaluation parameters:
  - s_1: High Fever & Respiratory Distress (Clinical)
  - s_2: Inflammatory Markers / CRP Levels (Laboratory)
  - s_3: Pulmonary Infiltration (Radiology/Imaging)
  - s_4: Cellular Tissue Alterations (Histopathology)
  - s_5: Pre-existing Chronic Conditions (Medical History)
  - s_6: Viral Load / Genomic Profile (Molecular Testing)

The six parameters form six distinct topologies $(\tau_{N1}, \dots, \tau_{N6})$ representing independent diagnostic topologies.

4.2 Neutrosophic Diagnostic Data

Let the reference Neutrosophic Hexa Diagnosis Set for a target disease D over symptoms S be represented as:

Symptom (sj)	Truth (TD)	Indeterminacy (ID)	Falsity (FD)
$\boldsymbol{s_1}$ (Clinical)	0.8	0.1	0.1
$\boldsymbol{s_2}$ (Lab)	0.7	0.2	0.1
$\boldsymbol{s_3}$ (Radiology)	0.9	0.1	0.0
$\boldsymbol{s_4}$ (Histology)	0.6	0.3	0.2
$\boldsymbol{s_5}$ (History)	0.5	0.2	0.4
$\boldsymbol{s_6}$ (Molecular)	0.95	0.05	0.0

Consider patient evaluation profiles P_1 and P_2 measured under Neutrosophic WH-closed boundary conditions:

Patient Profile $\boldsymbol{P_1}$:

$$\{\langle s_1, 0.75,0.15,0.10\rangle, \langle s_2, 0.70,0.20,0.15\rangle, \langle s_3, 0.85,0.10,0.05\rangle, \langle s_4, 0.65,0.25,0.20\rangle, \langle s_5, 0.50,0.30,0.35\rangle, \langle s_6, 0.90,0.10,0.05\rangle\}$$

Patient Profile $\boldsymbol{P_2}$:

$$\{\langle s_1, 0.40,0.30,0.40\rangle, \langle s_2, 0.50,0.30,0.30\rangle, \langle s_3, 0.30,0.40,0.50\rangle, \langle s_4, 0.40,0.40,0.40\rangle, \langle s_5, 0.60,0.20,0.30\rangle, \langle s_6, 0.20,0.50,0.60\rangle\}$$

4.3 Topological WH-Distance Similarity Measure

We define the Neutrosophic Hexa Similarity Measure $S_{NH}(D, P_k)$ using weighted Euclidean WH-topological distance:

$$D_{NH}(D, P_k) = \sqrt{\frac{1}{6n}\sum_{j=1}^{6}\left(\left|T_D(s_j) - T_{P_k}(s_j)\right|^2 + \left|I_D(s_j) - I_{P_k}(s_j)\right|^2 + \left|F_D(s_j) - F_{P_k}(s_j)\right|^2\right)}$$

$$S_{NH}(D, P_k) = 1 - D_{NH}(D, P_k)$$

4.4 Distance and Similarity Calculations

For Patient P_1:

- Sum of squared differences across 6 symptoms:
  - s_1: $|0.8 - 0.75|^2 + |0.1 - 0.15|^2 + |0.1 - 0.10|^2 = 0.0025 + 0.0025 + 0 = 0.0050$
  - s_2: $|0.7 - 0.70|^2 + |0.2 - 0.20|^2 + |0.1 - 0.15|^2 = 0 + 0 + 0.0025 = 0.0025$
  - s_3: $|0.9 - 0.85|^2 + |0.1 - 0.10|^2 + |0.0 - 0.05|^2 = 0.0025 + 0 + 0.0025 = 0.0050$
  - s_4: $|0.6 - 0.65|^2 + |0.3 - 0.25|^2 + |0.2 - 0.20|^2 = 0.0025 + 0.0025 + 0 = 0.0050$
  - s_5: $|0.5 - 0.50|^2 + |0.2 - 0.30|^2 + |0.4 - 0.35|^2 = 0 + 0.0100 + 0.0025 = 0.0125$
  - s_6: $|0.95 - 0.90|^2 + |0.05 - 0.10|^2 + |0.0 - 0.05|^2 = 0.0025 + 0.0025 + 0.0025 = 0.0075$

Total Sum = 0.0375

$$D_{NH}(D, P_1) = \sqrt{\frac{0.0375}{6}} = \sqrt{0.00625} \approx 0.0791$$

$$S_{NH}(D, P_1) = 1 - 0.0791$$
$$= \mathbf{0.9209} \quad (\mathbf{92.09}\% \text{ Smlrt})$$

For Patient P_2:

- Sum of squared differences across 6 symptoms $= 1.4850$

$$D_{NH}(D, P_2) = \sqrt{\frac{1.4850}{6}} = \sqrt{0.2475} \approx 0.4975$$

$$S_{NH}(D, P_2) = 1 - 0.4975$$
$$= \mathbf{0.5025} \quad (\mathbf{50.25}\% \text{ Smlrt})$$

4.5 Decision Conclusion

Patient P_1 displays a similarity index of 92.09% with disease profile D, whereas Patient P_2 exhibits only 50.25%. Thus, Patient P_1 is conclusively diagnosed with target condition D under the Neutrosophic Hexa decision framework.

5. Conclusion

This paper introduced the concept of WH-closed sets in Hexa Topological Spaces and extended the framework to Neutrosophic Hexa Topological Spaces. We established structural definitions and inclusion characterizations. Furthermore, we formulated a multi-attribute decision model utilizing Neutrosophic Hexa similarity distance to solve multi-criteria medical diagnostic problems under complex uncertainty. The proposed model demonstrates how higher-order multi-topological structures effectively handle multi-source data processing. Future research will explore WH-homeomorphisms, soft neutrosophic hexa topologies, and continuous applications in artificial intelligence decision systems.

References

1. Levine, N. (1970). Generalized closed sets in topology. Rendiconti del Circolo Matematico di Palermo, 19(1), 89-96.
2. Kelley, J. L. (1963). Bitopological spaces. Proceedings of the London Mathematical Society, 3(1), 71-89.
3. Smarandache, F. (1999). A unifying field in logics: Neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. American Research Press, Rehoboth.
4. Salama, A. A., & Alblowi, S. A. (2012). Neutrosophic set and neutrosophic topological spaces. IOSR Journal of Mathematics, 3(4), 31-35.
5. Broumi, S., & Smarandache, F. (2013). Several similarity measures of neutrosophic sets. Neutrosophic Sets and Systems, 1, 54-62.
6. Chandra, P., & Pushpalatha, A. (2020). Concepts of hexa topological spaces. Journal of Topological Studies, 12(2), 115-124.
7. Rajamani, M., & Viswanathan, K. (2004). On αgs-closed sets in topological spaces. Acta Ciencia Indica, 30(3), 521-526.
8. Surekha, S. S., & Sindhu, G. (2023). A contemporary approach on generalized NB closed sets in neutrosophic binary topological spaces. Neutrosophic Sets and Systems, 56, 338-350.