RESEARCH PAPER

BIG DATA ANALYSIS BY USING ODE ON SUSCEPTIBLE, INFECTED, RECOVER MATHEMATICAL MODEL

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Abstract: The well known Susceptible, Infected, Recover (SIR) models have been around for many years. Under some suitable assumptions, the models provide information about when does the epidemic occur and when it doesn't. The models can incorporate the birth, death, and immunization and analyze the outcome mathematically. In this paper we study of epidemic model using differential Equations and Mathematical Modelling. Epidemiology is a discipline, which deals with the study of infectious diseases in a population. It is concerned with all aspects of epidemic.

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Introduction: Based on some mathematical assumptions, it is known that epidemics[8,11,12,15] can be modeled mathematically in order to study the severity and prevention mechanism. This model (SIR) is used in epidemiology to compute the number of susceptible, infected, and recovered people in a population at any time. It can be used to explain the change in the number of people needing medical attention during an epidemic. The whole population is divided into three classes, *S;* the number of susceptible, *I;* the number of infected and *R;* the number of recovered during an epidemic. This model assumes that the total population remains the same with closed demography meaning that there is no birth and no natural death. Any disease related death, however, can be included in *R:*We study the basic SIR model with some reasonable assumptions.Then we include herd immunity, birth and death into the model. The constant vaccination at birth is also considered . The ultimate goal is to model the issue of saturated susceptible population, the time delay of infected to become infectious, the stability of equilibrium solutions and associated bifurcation.SIR models have been around for many years, for example [5, 9,10] and the references there in. The first one was introduced and published in 1927, in "Contribution to the Mathematical Theory of Epidemics", written by William Kermack and Anderson McKendrick. They introduced the important compartments, which make up the SIR model, S- susceptible, I - infected and R - recovered. They searched for a mathematical answer as to when the epidemic would terminate and observed that, in general whenever the population of susceptible individuals falls below a threshold value, which depends on several parameters, the epidemic terminates.

Basic Terminology

Population dynamics: The population dynamics is a description (and prediction) of the size and age composition of a group of individuals of one particular species, and how the number and age composition of individuals in a population change over time.

Variables: In mathematical modelling, variables are symbols that represent quantities that can change, such as time, distance, temperature, or population size.

Equations: The equations in mathematical model contain variables, which are values to input into the equation, and parameters, which are constants whose value depends on the particular model and situation.

Constraints: In mathematical modelling, constraints are the conditions that a solution to an optimization problem must satisfy

Susceptible – Infective (SI) Model: The SI model is a simple mathematical model that describes the spread of an infectious disease through a population that is divided into two groups: susceptible and infected.

Susceptible - Infected-Recovered (SIR) Model: In a standard SIR model, the host population is divided into susceptible, infected and recovered individuals, denoted by S(t), I(t) and R(t), respectively.

The Differential Equation Model

As the first step in the modeling process, we identify the independent and dependent variables. The independent variable is time t, measured in days. We consider two related sets of dependent variables. The first set of dependent variables counts people in each

of the groups, each as a function of time: S = S(t) is the number of susceptible individuals, I = I(t) is the number of infected individuals, and R = R(t) is the number of recovered individuals. The second set of dependent variables represents the fraction of the total population in each of the three categories. So, if N is the total population (7,900,000 in our example),

we have $s(t) = \frac{S(t)}{N}$ the susceptible fraction of the let there be n susceptible and one infected person in the system, so that,

$$S(t) + I(t) = n + 1, S(0) = n, I(0) = 1 \quad (1)$$

A susceptible person gets infected when he comes in contact with an infected one. Mathematically, we can say that the rate of increase of the infected class is proportional to the product of the susceptible and infected persons. Hence, the susceptible class also decreases at the same rate.

So that we get the system of differential equations:

$$\frac{ds}{dt} = \beta SI, \quad \frac{dI}{dt} = \beta SI \quad (2)$$

By method of separable of variables and by using partial fraction, integration method we get

$$-\ln(n + 1 - S) + \ln(S) = -(n + 1)\beta t + A(\text{constant}).$$

$$at\ t = 0, S(0) = n\ which\ gives\ A = \ln(n)$$

$$\ln\left[\frac{S(n)}{n + 1 - s}\right] = -(n + 1)\beta t$$

$$\beta t\ gives \quad \frac{S(n)}{n + 1 - s} = e^{-(n+1)\beta t}$$

$$S(t) = \frac{n(n + 1)}{n + e^{(n+1)\beta t}}$$

And $$I(t) = \frac{(n+1)}{1+ne^{-(n+1)\beta t}}$$

$$\therefore \lim_{n\to\infty} S(t) = 0, \lim_{n\to\infty} I(t) = n + 1,$$

This shows that as time increases, all the susceptible persons will become infected.

Example1 . If the contact rate (β) be 0.001 and the number of susceptible (n) be 2000 initially, we determine:

a. The number of susceptible left after 3 weeks;

b. The density of susceptible when the rate of appearance of new cases is maximum;

c. The time (in weeks) at which the rate of appearance of new cases is a maximum.

d. The maximum rate of appearance of new cases and

e. The epidemic curve.

a.The number of susceptible left after 3 weeks is:

$$S(t) = \frac{(n + 1)}{1 + ne^{-(n+1)\beta t}} = \frac{2000 + 1}{1 + 2000e^{-(2000+1)0.001*3}} = 1664$$

population, $i(t) = \frac{I(t)}{N}$ the infected fraction of the population, and $r(t) = \frac{R(t)}{N}$ the recovered fraction of the population.

It may seem more natural to work with population counts, but some of our culations will be simpler if we use the fractions instead.

The two sets of dependent variables are proportional to each other, so either set will give us the same information about the progress of the epidemic.

For $\beta > 0, \frac{ds}{dt} + \frac{dI}{dt} = 0$

Integrating, we get,

$$S(t) + I(t) = \text{constant}$$

When t = 0

$$S(0) + I(0) = n + 1 \quad (3)$$

And

$$\frac{ds}{dt} = -\beta SI(n + 1 - S), \quad (4)$$

$$\frac{dI}{dt} = \beta SI(n + 1 - I) \quad (5)$$

b.When the rate of appearance of new cases is maximum, the

density of susceptible is $x = \frac{n+12}{2} = \frac{2012}{2} \approx 1001$

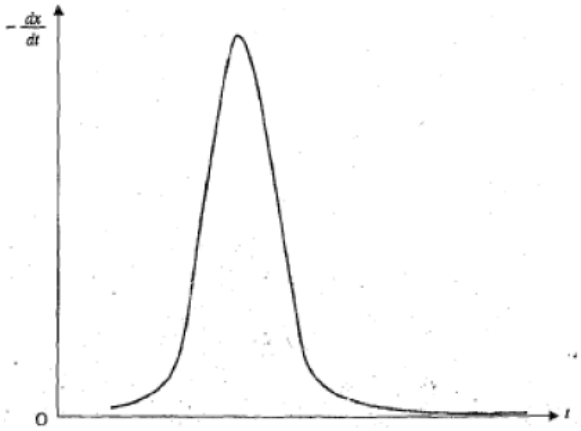
c. The time at which the rate of appearance of new cases is a maximum is

$$t = \frac{\ln(n)}{\beta(n + 1)} = \frac{\ln(2000)}{2.001} = 1.6\ weeks$$

d.The maximum rate of appearance of new cases is

$$-\frac{dS}{dt} = \beta(n + 12)2 = (0.001)(1000.5) \approx 1001$$

We draw the curve by using the values β=0.001,n=2000 and t=1,2,3,4,5,6,7,8,9,10.



Example2 . If the contact rate (β) be 0.001 and the number of susceptible (n) be 3000for 5 weeks initially

a.The number of susceptible left after 3 weeks is:

$$S(t) = \frac{(n+1)}{1+ne^{-(n+1)\beta t}} = \frac{3000+1}{1+3000e^{-(3000+1)0.001*5}} \approx 2999$$

b.When the rate of appearance of new cases is maximum, the

density of susceptible is $x = \frac{n+12}{2} = \frac{3012}{2} \approx 1506$

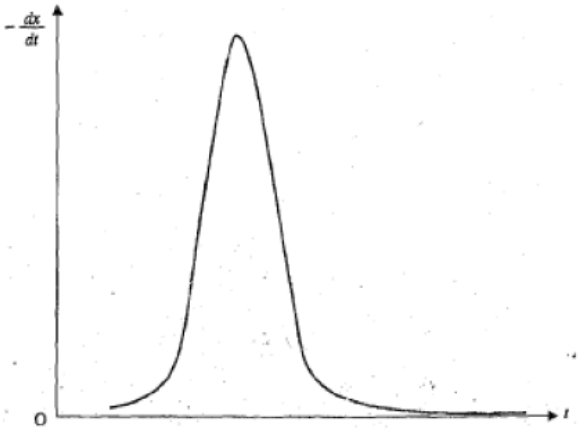
c. The time at which the rate of appearance of new cases is a maximum is

$$t = \frac{\ln(n)}{\beta(n+1)} = \frac{\ln(3000)}{0.001*5*3001} = 0.5\ weeks$$

d.The maximum rate of appearance of new cases is

$$-\frac{dS}{dt} = \beta(n+12)2 = (0.001)(1000.5) \approx 9072.144 \approx 9073$$

We draw the curve by using the values β=0.001,n=3000 and t=1,2,3,4,5,6,7,8,9,10.



Conclusion

The present research work demonstrates the usefulness of mathematical modelling and differential equations in studying the spread of infectious diseases, From the mathematical analysis, it is observed that the contact rate (β) and the initial susceptible population have a significant influence on the progression of an epidemic. As the contact rate increases, susceptible individuals become infected more rapidly, resulting in a faster rise in the number of infected cases. Our study confirms that differential-equation-based epidemic models are valuable tools for understanding epidemic dynamics and supporting decisions related to prevention, vaccination, healthcare-resource planning, and epidemic control. The basic SIR model uses simplifying assumptions, it provides an important foundation for more realistic models incorporating birth and death rates, vaccination, immunity, population movement, and time delays. Thus, the SIR model can be used to estimate the peak of an epidemic, the timing of maximum infection, and the decline of disease transmission. In conclusion, the SIR model provides a simple yet powerful mathematical approach for understanding, analyzing, and controlling the spread of infectious diseases in a population.

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