

An Introduction to δ -Continuous Function in a Pentapartitioned Neutrosophic Topological Spaces

P. Revathi¹, S. Sudarmozhi²

¹Department of Mathematics, Government Polytechnic College, Kooduveli, Kattumannarkoil Taluk, Cuddalore, Tamil Nadu, India - 608305

²Department of Mathematics, Annamalai University, Annamalai Nagar, Chidambaram, Tamil Nadu, India - 608002

Received: 16th August, 2026; Revised: 8th September 2026; Accepted: 19th September, 2026; Available Online: 20th September, 2026

Abstract

In this paper, introduced and discussed about Pentapartitioned neutrosophic δ -continuous mappings and discussed some of their properties and its application.

Keywords:

How to cite this article: P. Revathi¹ S. Sudarmozhi². An Introduction to δ -Continuous Function in a Pentapartitioned Neutrosophic Topological Spaces. *Int J Drug Deliv Technol.* 2026;16(84s):1149-1153. DOI: 10.25258/ijddt.16.84s.123.

1. Introduction

Zadeh(1965)[1] was the first person in the field of logic and set theory to introduce the idea of a fuzzy set between real standard intervals in mathematics. Chang(1968)[4] took the general topology as a fuzzy topological space and framed it with a fuzzy set. Lewine.K (1970), define [7] Generalized closed sets in topology. Atanassov (1983) [3], introduced intuitionistic fuzzy sets, which combine membership and non-membership values. Abdel-Monsef, El-Deep and Mohmoud(1983) [1] introduced β -Open sets and β -Continuous mappings. Coker[6](1997), developed the intuitionistic fuzzy topological space. Chatterjee, Majumdar and Samanta[5](2016), contributed significantly to this development by introducing similarity measures and entropy for quadripartitioned single-valued neutrosophic sets. Arokianani, Dhavaseelan, Jafari and Parimala[2](2017), introduced a variety of new notions and functions in neutrosophic topological spaces. Smarandache[14](2012), introduced the idea of neutrosophic set at the start of the 20th century. Salama and Alblowi[13](2012), Neutrosophic set in a Neutrosophic Topological spaces. Vadivel, Seenivasan and John Sundar(2021), [16] introduce δ -open sets in a Neutrosophic topological spaces. Navuluri Mohanarao and Sathish kumar[8](2025), proposed a new operator based on β -Open sets within quadri partitioned neutrosophic topological spaces. Sai Anu and Arulselvam(2025), proposed on intuitionistic quadri-partitioned neutrosophic N-soft set model. In the context of Pentapartitioned neutrosophic topological spaces, we expand on the notion of δ -continuous mapping and its application.

2. Preliminaries

Definition 2.1. [15] Assume $\mathbb{Y}$ is a non-empty set. An object with the form $\mathcal{L} = \{y, \dot{T}_{\mathcal{L}}(y), \dot{C}_{\mathcal{L}}(y), \dot{I}_{\mathcal{L}}(y), \dot{U}_{\mathcal{L}}(y), \dot{F}_{\mathcal{L}}(y): y \in \mathbb{Y}\}$

where $\dot{T}_{\mathcal{L}} \rightarrow [0, 1]$ Truth, $\dot{C}_{\mathcal{L}} \rightarrow [0, 1]$ contradiction, $\dot{I}_{\mathcal{L}} \rightarrow [0, 1]$ ignorance, $\dot{U}_{\mathcal{L}} \rightarrow [0, 1]$ unknown, $\dot{F}_{\mathcal{L}} \rightarrow [0, 1]$ false and $0 \leq \dot{T}_{\mathcal{L}}(y) + \dot{C}_{\mathcal{L}}(y) + \dot{I}_{\mathcal{L}}(y) + \dot{U}_{\mathcal{L}}(y) + \dot{F}_{\mathcal{L}}(y) \leq 5$ for each $y \in \mathbb{Y}$.

Definition 2.2. [15], [12] Assume $\mathbb{Y}$ is a non-empty set and the pentapartitioned neutrosophic sets $\mathcal{L}$ and $\mathcal{M}$ in the form $\mathcal{L} = \{y, \dot{T}_{\mathcal{L}}(y), \dot{C}_{\mathcal{L}}(y), \dot{I}_{\mathcal{L}}(y), \dot{U}_{\mathcal{L}}(y), \dot{F}_{\mathcal{L}}(y): y \in \mathbb{Y}\}$, $\mathcal{M} = \{y, \dot{T}_{\mathcal{M}}(y), \dot{C}_{\mathcal{M}}(y), \dot{I}_{\mathcal{M}}(y), \dot{U}_{\mathcal{M}}(y), \dot{F}_{\mathcal{M}}(y): y \in \mathbb{Y}\}$, then

- $0_{PN} = \langle y, 0, 0, 1, 1, 1 \rangle$ and $1_{PN} = \langle y, 1, 1, 0, 0, 0 \rangle$,
- $\mathcal{L} \subseteq \mathcal{M}$ iff $\dot{T}_{\mathcal{L}}(y) \leq \dot{T}_{\mathcal{M}}(y), \dot{C}_{\mathcal{L}}(y) \leq \dot{C}_{\mathcal{M}}(y), \dot{I}_{\mathcal{L}}(y) \leq \dot{I}_{\mathcal{M}}(y), \dot{U}_{\mathcal{L}}(y) \leq \dot{U}_{\mathcal{M}}(y), \dot{F}_{\mathcal{L}}(y) \leq \dot{F}_{\mathcal{M}}(y): y \in \mathbb{Y}$,
- $\mathcal{L} = \mathcal{M}$ iff $\mathcal{L} \subseteq \mathcal{M}$ and $\mathcal{M} \subseteq \mathcal{L}$,
- $\mathcal{L}^c = \{y, \dot{F}_{\mathcal{L}}(y), \dot{U}_{\mathcal{L}}(y), 1 - \dot{I}_{\mathcal{L}}(y), \dot{C}_{\mathcal{L}}(y), \dot{T}_{\mathcal{L}}(y): y \in \mathbb{Y}\}$,
- $\mathcal{L} \cup \mathcal{M} = \{y, \max(\dot{T}_{\mathcal{L}}(y), \dot{T}_{\mathcal{M}}(y)), \max(\dot{C}_{\mathcal{L}}(y), \dot{C}_{\mathcal{M}}(y)), \min(\dot{I}_{\mathcal{L}}(y), \dot{I}_{\mathcal{M}}(y)), \min(\dot{U}_{\mathcal{L}}(y), \dot{U}_{\mathcal{M}}(y)), \min(\dot{F}_{\mathcal{L}}(y), \dot{F}_{\mathcal{M}}(y)): y \in \mathbb{Y}\}$,
- $\mathcal{L} \cap \mathcal{M} = \{y, \min(\dot{T}_{\mathcal{L}}(y), \dot{T}_{\mathcal{M}}(y)), \min(\dot{C}_{\mathcal{L}}(y), \dot{C}_{\mathcal{M}}(y)), \max(\dot{I}_{\mathcal{L}}(y), \dot{I}_{\mathcal{M}}(y)), \max(\dot{U}_{\mathcal{L}}(y), \dot{U}_{\mathcal{M}}(y)), \max(\dot{F}_{\mathcal{L}}(y), \dot{F}_{\mathcal{M}}(y)): y \in \mathbb{Y}\}$.

Definition 2.3. [9, 10] A non-empty set possessing a pentapartitioned neutrosophic structure, there are pentapartitioned subsets of $\mathbb{Y}$ that satisfy the family $\mathcal{Q}_{PN}$.

- $0_{PN}, 1_{PN} \in \mathcal{Q}_{PN}$.
- $\mathcal{L}_1 \cap \mathcal{L}_2 \in \mathcal{Q}_{PN}$, for any $\mathcal{L}_1, \mathcal{L}_2 \in \mathcal{Q}_{PN}$
- $\cup \mathcal{L}_a \in \mathcal{Q}_{PN}, \forall \mathcal{L}_a: a \in A \subseteq \mathcal{Q}_{PN}$

Then $(\mathbb{Y}, \varrho_{PN})$ is called pentapartitioned neutrosophic topological space in $\mathbb{Y}$. In $\mathbb{Y}$, the φ elements are referred to as pentapartitioned neutrosophic open sets. If the complement $\mathcal{C}^c$ of a PN_{CS} is PN_{OS} , it is referred to as a pentapartitioned neutrosophic closed set.

Definition 2.4. [11] If $\mathcal{L}$ is a pentapartitioned neutrosophic set on and $(\mathbb{Y}, \varrho_{PN})$ is a pentapartitioned neutrosophic topological space on $\mathbb{Y}$, then the pentapartitioned neutrosophic closure of $\mathcal{L}$ and its pentapartitioned neutrosophic interior are defined as

$$PN_{int}(\mathcal{L}) = \bigcup \{T : T \subseteq \mathcal{L} \text{ and } T \text{ is a } PN_{OS} \text{ in } \mathbb{Y}\}$$

$$PN_{cl}(\mathcal{L}) = \bigcap \{T : \mathcal{L} \subseteq T \text{ and } T \text{ is a } PN_{CS} \text{ in } \mathbb{Y}\}.$$

Definition 2.5. [12] If $\mathcal{L}$ in a PN_S on $\mathbb{Y}$ and $(\mathbb{Y}, \varrho_{PN})$ is a PN_{ts} on $\mathbb{Y}$. Consequently, $\mathcal{L}$ is regarding as a pentapartition neutrosophic regular (resp. pre, semi, α and β) open set

(briefly, PN_{ros} (resp. PN_{pos} , PN_{sos} , $PN_{\alpha os}$ and $PN_{\beta os}$))

if $\mathcal{L} = PN_{int}(PN_{cl}(\mathcal{L}))$ (resp. $\mathcal{L} \subseteq$

$$PN_{int}(PN_{cl}(\mathcal{L})), \mathcal{L} \subseteq PN_{cl}(PN_{int}(\mathcal{L})), \mathcal{L} \subseteq$$

$$PN_{int}(PN_{cl}(PN_{cl}(\mathcal{L}))) \text{ and } \mathcal{L} \subseteq$$

$$PN_{cl}(PN_{int}(PN_{cl}(\mathcal{L}))).$$

The complement of an PN_{pos} (resp. PN_{sos} , $PN_{\alpha os}$, PN_{ros}) and $PN_{\beta os}$ is called a pentapartitioned neutrosophic pre (resp. semi, α , regular and β) closed set (breif, PN_{cs} (resp. PN_{scs} , $PN_{\alpha cs}$, PN_{rcs} , $PN_{\beta cs}$)) in $\mathbb{Y}$.

The family of all PN_{os} (resp. PN_{pos} , PN_{sos} , $PN_{\alpha os}$, $PN_{\alpha cs}$, $PN_{\beta os}$ and $PN_{\beta cs}$) of $\mathbb{Y}$ is denoted by $PN_{os}(\mathbb{Y})$ (resp. $PN_{pos}(\mathbb{Y})$, $PN_{sos}(\mathbb{Y})$, $PN_{\alpha os}(\mathbb{Y})$, $PN_{\alpha cs}(\mathbb{Y})$, $PN_{\beta os}(\mathbb{Y})$ and $PN_{\beta cs}(\mathbb{Y})$).

Definition 2.6. [12] A set $\mathcal{L}$ is said to be a pentapartitioned neutrosophic

a) δ interior of $\mathcal{T}$ (short, $PN_{\delta int}(\mathcal{T})$) is defined by $PN_{\delta int}(\mathcal{T}) = \bigcup \{I : I \subseteq \mathcal{T} \text{ and } I \text{ is a } PN_{ros} \text{ in } \mathbb{Y}\}$

b) δ closure of $\mathcal{T}$ (short, $PN_{\delta cl}(\mathcal{T})$) is defined by $PN_{\delta cl}(\mathcal{T}) = \bigcap \{B : \mathcal{T} \subseteq B \text{ and } B \text{ is a } PN_{rcs} \text{ in } \mathbb{Y}\}$

Definition 2.7. [12] A set $\mathfrak{I}$ is said to be a pentapartitioned neutrosophic

a) δ - open set (breifly, $PN_{\delta os}$) if $\mathfrak{I} = PN_{\delta int}(\mathfrak{I})$.

b) δ - pre open set (breifly, $PN_{\delta pos}$) if $\mathfrak{I} \subseteq PN_{int}(PN_{\delta cl}(\mathfrak{I}))$.

c) δ - semi open set (breifly, $PN_{\delta sos}$) if $\mathfrak{I} \subseteq PN_{cl}(PN_{\delta int}(\mathfrak{I}))$.

d) $\delta\alpha$ - open set (breifly, $PN_{\delta \alpha os}$) if $\mathfrak{I} \subseteq PN_{int}(PN_{cl}(PN_{\delta int}(\mathfrak{I})))$.

e) $\delta\beta$ - open set (breifly, $PN_{\delta \beta os}$) if $\mathfrak{I} \subseteq PN_{cl}(PN_{int}(PN_{\delta cl}(\mathfrak{I})))$.

The complement of an $PN_{\delta os}$ – $PN_{\delta pos}$, $PN_{\delta sos}$, $PN_{\delta \alpha os}$ and $PN_{\delta \beta os}$ is called a

$PN_{\delta cs}$ – $PN_{\delta pcs}$, $PN_{\delta scs}$, $PN_{\delta \alpha cs}$ and $PN_{\delta \beta cs}$ in $\mathbb{Y}$.

$PN_{\delta pos}(\mathbb{Y})$, $PN_{\delta sos}(\mathbb{Y})$, $PN_{\delta \alpha cs}(\mathbb{Y})$ and $PN_{\delta \beta cs}(\mathbb{Y})$ are represents the family of all $PN_{\delta pos}$, $PN_{\delta pcs}$, $PN_{\delta sos}$, $PN_{\delta scs}$, $PN_{\delta \alpha os}$, $PN_{\delta \alpha cs}$, $PN_{\delta \beta os}$ and $PN_{\delta \beta cs}$ of $\mathbb{Y}$.

3. Pentapartitioned neutrosophic δ - continuous mapping

Definition. 3.1.[15] A g is said to be a pentapartitioned neutrosophic δ (resp. $\delta\alpha$, $\delta\mathbb{S}$, $\delta\mathbb{P}$ and $\delta\beta$, e^*)- continuous (briefly, $PN_{\delta \alpha Cts}$ ($PN_{\delta \mathbb{S} Cts}$, $PN_{\delta \mathbb{P} Cts}$ and $PN_{\delta \beta Cts}$ or $PN_{\delta e^* Cts}$)) if the inverse image of every PN_{OS} in $(\mathbb{Z}, \varrho_{PN})$ is a $PN_{\delta OS}$ (resp. $PN_{\delta \alpha OS}$, $PN_{\delta \mathbb{S} OS}$, $PN_{\delta \mathbb{P} OS}$ and $PN_{\delta \beta OS}$ or $PN_{\delta \alpha OS}$) in $(\mathbb{Y}, \varrho_{PN})$.

Theorem 3.2. The claims are hold for PN_{ts} .

- (i) All $PN_{\delta Cts}$ are PN_{Cts} ,
- (ii) All PN_{Cts} are $PN_{\delta \mathbb{S} Cts}$,
- (iii) All PN_{Cts} are $PN_{\delta \mathbb{P} Cts}$,
- (iv) All $PN_{\delta \mathbb{S} Cts}$ are $PN_{\delta \beta Cts}$,
- (v) All $PN_{\delta \mathbb{P} Cts}$ are $PN_{\delta \beta Cts}$,
- (vi) All $PN_{\alpha Cts}$ are $PN_{\delta \mathbb{S} Cts}$,
- (vii) All $PN_{\alpha Cts}$ are $PN_{\delta \mathbb{P} Cts}$.

But not converse.

Proof. Let g be a $PN_{\delta Cts}$ and $\mathcal{K}$ is a $PN_{\delta OS}$ in $\mathbb{Z}$. Then $g^{-1}(\mathcal{K})$ is PN_{OS} in $\mathbb{Z}$ Since $\forall PN_{\delta OS}$ is PN_{OS} , $g^{-1}(\mathcal{K})$ is PN_{OS} in $\mathbb{Z}$.

$\therefore g$ is PN_{Cts} .

The proofs of other cases are similar.

Example 3.4. Let $\mathbb{Y} = \{u\}$ and defined PN 's $\mathbb{Y}_1$ and $\mathbb{Y}_2$ in $\mathbb{Y}$ are

$$\mathbb{Y}_1 = \langle \mathbb{Y}, (0.2, 0.4, 0.5, 0.6, 0.8) \rangle, \mathbb{Y}_2 = \langle \mathbb{Y}, (0.5, 0.5, 0.5, 0.5, 0.5) \rangle.$$

Then we have g be an identity mapping, then g is PN_{Cts} but not $PN_{\delta Cts}$, the set $g^{-1}(\mathbb{Y}_1) = \mathbb{Y}_1$ is a PN_{OS} but not a $PN_{\delta OS}$.

Example 3.5. Let $\mathbb{Y} = \{l, m, n\}$ are defined PN 's $\mathbb{Y}_1$, $\mathbb{Y}_2$ and $\mathbb{Y}_3$ in $\mathbb{Y}$ are,

$$\mathbb{Y}_1 = \langle \mathbb{Y}, (0.8, 0.7, 0.5, 0.5, 0.2) \rangle,$$

$$(0.9, 0.7, 0.5, 0.6, 0.4), (0.7, 0.6, 0.5, 0.2, 0.1) \rangle,$$

$$\mathbb{Y}_2 = \langle \mathbb{Y}, (0.8, 0.7, 0.5, 0.5, 0.2), (0.8, 0.6, 0.5, 0.6, 0.4), (0.7, 0.6, 0.5, 0.3, 0.2) \rangle,$$

$$\mathbb{Y}_3 = \langle \mathbb{Y}, (0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1) \rangle.$$

Then we have $\varsigma = \{0_{PN}, \mathbb{Y}_1, \mathbb{Y}_2, \mathbb{Y}_3, I_{PN}\}$ then $\mathbb{Z} = \{l, m, n\}$ and define $PN_{\mathbb{Z}}$ in $\mathbb{Z}$ is

$$\mathbb{Z}_1 = \langle \mathbb{Z}, (0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1) \rangle.$$

Here $\varrho_{PN} = \{0_{PN}, \mathbb{Z}_1, I_{PN}\}$. Let $h : (\mathbb{Y}, \varrho_{PN}) \rightarrow (\mathbb{Z}, \varrho_{PN})$ be an identity mapping. Then h is,

(a) $PN_{\delta \mathbb{S} Cts}$ but not PN_{Cts} (resp. $PN_{\delta \alpha Cts}$), the set $h^{-1}(\mathbb{Z}_1) = \mathbb{Y}_3$ is a $PN_{\delta \mathbb{S} OS}$ but not a PN_{OS} (resp. $PN_{\delta \alpha OS}$).

(b) $PN_{\delta \beta Cts}$ but not $PN_{\delta \mathbb{P} Cts}$, the set $h^{-1}(\mathbb{Z}_1) = \mathbb{Y}_3$ is a $PN_{\delta \beta OS}$ but not a PN_{OS} (resp. $PN_{\delta \mathbb{P} OS}$).

Example 3.6. Let $\mathbb{Y} = \{l, m, n\}$ are defined $PN'S \mathbb{Y}_1, \mathbb{Y}_2, \mathbb{Y}_3$ and $\mathbb{Y}_4$ in $\mathbb{Y}$ are,

$$\begin{aligned}\mathbb{Y}_1 &= \langle y, (0.8, 0.7, 0.5, 0.3, 0.2), \\ &\quad (0.9, 0.7, 0.5, 0.4, 0.3), (0.8, 0.6, 0.5, 0.3, 0.2) \rangle, \\ \mathbb{Y}_2 &= \langle y, (0.8, 0.7, 0.5, 0.2, 0.1), (0.8, 0.7, 0.5, 0.4, 0.3), \\ &\quad (0.7, 0.5, 0.5, 0.2, 0.1) \rangle, \\ \mathbb{Y}_3 &= \langle y, (0.8, 0.7, 0.5, 0.2, 0.1), (0.9, 0.7, 0.5, 0.4, 0.3), \\ &\quad (0.8, 0.6, 0.5, 0.2, 0.1) \rangle, \\ \mathbb{Y}_4 &= \langle y, (0.8, 0.6, 0.5, 0.3, 0.2), (0.7, 0.7, 0.5, 0.5, 0.4), \\ &\quad (0.7, 0.5, 0.5, 0.3, 0.2) \rangle.\end{aligned}$$

Then we have $\varsigma = \{0_{PN}, \mathbb{Y}_1, \mathbb{Y}_2, 1_{PN}\}$ then $\mathbb{Z} = \{l, m, n\}$ and define $PN \mathbb{Z}_l$ in $\mathbb{Z}$ is

$$\mathbb{Z}_l = \langle z, (0.8, 0.6, 0.5, 0.3, 0.2), (0.7, 0.7, 0.5, 0.5, 0.4), (0.7, 0.5, 0.5, 0.3, 0.2) \rangle.$$

Then we have $\varrho = \{0_{PN}, \mathbb{Z}_l, 1_{PN}\}$. Let $\mathfrak{h} : (\mathbb{Y}, \varsigma_{PN}) \rightarrow (\mathbb{Z}, \varrho_{PN})$ be an identity mapping. Then $\mathfrak{h}$ is,

- $PN\delta\mathbb{P}\mathbb{C}ts$ but not $PN\mathbb{C}ts$ (resp. $PN\delta\alpha\mathbb{C}ts$), the set $\mathfrak{h}^{-1}(\mathbb{Z}_l) = \mathbb{Y}_4$ is a $PN\delta\mathbb{P}_{OS}$ but not a PN_{OS} (resp. $PN\delta\alpha_{OS}$).
- $PN\delta\beta\mathbb{C}ts$ but not $PN\delta\mathbb{S}\mathbb{C}ts$, the set $\mathfrak{h}^{-1}(\mathbb{Z}_l) = \mathbb{Y}_4$ is a $PN\delta\beta_{OS}$ but not $PN\delta\mathbb{S}_{OS}$.

Theorem 3. 7. A map $\mathfrak{h}$ satisfies the conditions,

- $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$.
- The inverse $\mathfrak{h}^{-1}(\mathcal{K})$ of all $PN\delta_{OS} \mathcal{K}$ in $\mathbb{Z}$ is $PN\delta\beta_{OS}$ in $\mathbb{Y}$ are equivalent.
The proof is directly, since $\mathfrak{h}^{-1}(\overline{\mathcal{K}}) = \overline{\mathfrak{h}^{-1}(\mathcal{K})}$ for all $PN\delta_{OS} \mathcal{K}$ of $\mathbb{Z}$.

Theorem 3. 8. A map $\mathfrak{h}$ satisfies the conditions,

- $\mathfrak{h}(PN\delta\beta cl(\mathcal{L})) \subseteq PN\delta cl(\mathfrak{h}(\mathcal{L}))$, for all $PN_{CS} \mathcal{L}$ in $\mathbb{Y}$.
- $PN\delta\beta cl(\mathfrak{h}^{-1}(\mathcal{K})) \subseteq \mathfrak{h}^{-1}(PN\delta cl(\mathcal{K}))$, for all $PN_{CS} \mathcal{K}$ in $\mathbb{Z}$ are equivalent.

Proof.

- Since $PN\delta cl(\mathfrak{h}(\mathcal{L}))$ is a $PN\delta_{CS}$ in $\mathbb{Z}$ and $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$, then $\mathfrak{h}^{-1}PN\delta cl(\mathfrak{h}(\mathcal{L}))$ is $PN\delta\beta$ in $\mathbb{Y}$. Now, $\mathcal{L} \subseteq \mathfrak{h}^{-1}(\mathcal{K})$, $PN\delta\beta cl(\mathcal{L}) \subseteq \mathfrak{h}^{-1}(PN\delta cl(\mathfrak{h}(\mathcal{L})))$. Therefore, $\mathfrak{h}(PN\delta\beta cl(\mathcal{L})) \subseteq (PN\delta cl(\mathfrak{h}(\mathcal{L})))$.
- By replacing $\mathcal{L}$ with $\mathcal{K}$ in (a), we obtain $\mathfrak{h}(PN\delta\beta cl(\mathfrak{h}^{-1}(\mathcal{K}))) \subseteq PN\delta cl(\mathfrak{h}(\mathfrak{h}^{-1}(\mathcal{K}))) \subseteq PN\delta cl(\mathcal{K})$. Hence, $PN\delta\beta cl(\mathfrak{h}^{-1}(\mathcal{K})) \subseteq \mathfrak{h}^{-1}(PN\delta cl(\mathcal{K}))$.

Remark 3.9. If $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}$ mapping. Then

- $\mathfrak{h}(PN\delta\beta cl(\mathcal{L}))$ is not necessarily equal to $PN\delta cl(\mathfrak{h}(\mathcal{L}))$ where $\mathcal{L} \subseteq \mathbb{Y}$.
- $PN\delta\beta cl(\mathfrak{h}^{-1}(\mathcal{K}))$ is not necessarily equal to $\mathfrak{h}^{-1}(PN\delta cl(\mathcal{K})) \in \mathbb{Z}$.

Example 3. 10.. In example 3.4, $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$.

- Let $\mathcal{L} = \{(0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1)\}$. Then, $PN\delta\beta cl(\mathbb{Z}) = \{(0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1)\}$. But, $PN\delta cl(\mathcal{L}) = \{(0.1, 0.2, 0.5, 0.6, 0.8), (0.1, 0.3, 0.5, 0.5, 0.7), (0.1, 0.2, 0.5, 0.8, 0.9)\}$. Thus $\mathfrak{h}(PN\delta\beta cl(\mathcal{L})) \neq PN\delta cl(\mathfrak{h}(\mathcal{L}))$.
- Let $\mathcal{K} = \{(0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1)\}$. Then $\mathfrak{h}(PN\delta\beta cl(\mathfrak{h}^{-1}(\mathcal{K}))) = \{(0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1)\}$. But, $\mathfrak{h}^{-1}(PN\delta cl(\mathcal{K})) = \{(0.1, 0.2, 0.5, 0.6, 0.8), (0.1, 0.3, 0.5, 0.5, 0.7), (0.1, 0.2, 0.5, 0.8, 0.9)\}$. Thus $\mathfrak{h}(PN\delta\beta cl(\mathfrak{h}^{-1}(\mathcal{K}))) \neq \mathfrak{h}^{-1}(PN\delta cl(\mathcal{K}))$.

Theorem 3.11. If $PN\delta\beta\mathbb{C}ts$, then $\mathfrak{h}^{-1}(PN\delta int(\mathcal{L})) \subseteq PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{L}))$, for all $PN_{CS} \mathcal{L}$ in $\mathbb{Z}$.

Proof.

If $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$ and $L \subseteq \mathbb{Z}$. $PN\delta int(L)$ is $PN\delta O$ in $\mathbb{Z}$.

Hence, $\mathfrak{h}^{-1}(PN\delta\beta int(L))$ is $PN\delta\beta O$ in $\mathbb{Y}$.

Therefore $PN\delta\beta int(\mathfrak{h}^{-1}(PN\delta int(\mathcal{L}))) = \mathfrak{h}^{-1}(PN\delta int(\mathcal{L}))$.

Also, $PN\delta int(\mathcal{L}) \subseteq \mathcal{L} \Rightarrow \mathfrak{h}^{-1}(PN\delta int(\mathcal{L})) \subseteq \mathfrak{h}^{-1}(\mathcal{L})$.

Therefore $PN\delta\beta int(\mathfrak{h}^{-1}(PN\delta int(\mathcal{L}))) \subseteq PN\delta\beta(\mathfrak{h}^{-1}(\mathcal{L}))$.

That is, $\mathfrak{h}^{-1}(PN\delta int(L)) \subseteq PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{L}))$.

Conversely, let $\mathfrak{h}^{-1}(PN\delta int(L)) \subseteq PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{L}))$ for all subset $\mathcal{L}$ of $\mathbb{Z}$. If $\mathcal{L}$ is $PN\delta O$ in $\mathbb{Z}$, then $PN\delta int(\mathcal{L}) = \mathcal{L}$. By assumption, $\mathfrak{h}^{-1}(PN\delta int(\mathcal{L})) \subseteq PN\delta\beta(\mathfrak{h}^{-1}(\mathcal{L}))$. Thus $\mathfrak{h}^{-1}(\mathcal{L}) \subseteq PN\delta\beta(\mathfrak{h}^{-1}(\mathcal{L}))$. But $PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{L})) \subseteq \mathfrak{h}^{-1}(\mathcal{L})$. Therefore, $PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{L})) = \mathfrak{h}^{-1}(\mathcal{L})$. That is, $\mathfrak{h}^{-1}(\mathcal{L})$ is $PN\delta\beta O$ in $\mathbb{Y}$, for all $PN\delta_{OS} \mathcal{L}$ in $\mathbb{Z}$. Therefore, $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$ on $\mathbb{Y}$.

Remark 3.12. If $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$, then $PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{K}))$ is not necessarily equal to $\mathfrak{h}^{-1}(PN\delta int(\mathcal{K}))$ where $\mathcal{K} \in \mathbb{Z}$.

Example 3.13. In example 3.4, $\mathfrak{h}$ is $PN\delta\beta\mathbb{C}ts$.

Let $\mathcal{K} = \{(0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1)\}$. Then $PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{K})) = \{(0.9, 0.8, 0.5, 0.4, 0.2), (0.9, 0.7, 0.5, 0.5, 0.3), (0.9, 0.8, 0.5, 0.2, 0.1)\}$. But $\mathfrak{h}^{-1}(PN\delta int(\mathcal{K})) = \{(0.1, 0.2, 0.5, 0.6, 0.8), (0.1, 0.3, 0.5, 0.5, 0.7), (0.1, 0.2, 0.5, 0.8, 0.9)\}$. Thus $PN\delta\beta int(\mathfrak{h}^{-1}(\mathcal{K})) \neq \mathfrak{h}^{-1}(PN\delta int(\mathcal{K}))$.

4. Application

Disease diagnosis often involves uncertain, incomplete and contradictory medical information. Pentapartitioned neutrosophic sets provide an effective framework to handle such complexity by considering truth, indeterminacy, falsity, contradiction and unknown information. In this application, patients are evaluated based on different symptoms and their medical information is represented using pentapartitioned neutrosophic values. The results are used to determine and rank the possibility of a particular disease among the patients

4.1 Problem Description

Consider three patients $\mathbb{P}_1, \mathbb{P}_2$ and $\mathbb{P}_3$, who are examined for a particular disease $\mathbb{D}$. The diagnosis is based on three symptoms: fever, cough and breathing difficulty. Each medical

observation is represented by five components:

- * Truth degree
- * Indeterminacy degree
- * Falsity degree
- * Contradiction degree
- * Unknown degree

Thus, the medical condition of each patient is analysed by considering all these five types of information.

4.2 Medical Data

The medical information of the three patients is given below.

Patient	Fever	Cough	Breathing difficulty
$\mathbb{P}_1$	(0.8, 0.7, 0.5, 0.3, 0.1)	(0.5, 0.6, 0.5, 0.2, 0.2)	(0.6, 0.3, 0.5, 0.2, 0.1)
$\mathbb{P}_2$	(0.5, 0.3, 0.2, 0.3, 0.1)	(0.6, 0.4, 0.2, 0.1, 0.3)	(0.2, 0.3, 0.5, 0.2, 0.1)
$\mathbb{P}_3$	(0.6, 0.1, 0.2, 0.5, 0.3)	(0.7, 0.2, 0.5, 0.5, 0.1)	(0.3, 0.5, 0.3, 0.2, 0.1)

4.3 Methodology

The average medical value of each patient is calculated by taking the average of the corresponding five components of the three symptoms.

Step 1: Calculating the average pentapartitioned neutrosophic value

For each patient,

$$A(\mathbb{P}_i) = \frac{1}{3} \left[\sum_{j=1}^3 S_j(\mathbb{P}_i) \right]$$

For $\mathbb{P}_1$,
 $A(\mathbb{P}_1) = \left(\frac{0.8 + 0.5 + 0.6}{3}, \frac{0.7 + 0.6 + 0.3}{3}, \frac{0.5 + 0.5 + 0.5}{3}, \frac{0.3 + 0.2 + 0.2}{3}, \frac{0.1 + 0.2 + 0.1}{3} \right)$
 $A(\mathbb{P}_1) = (0.6, 0.5, 0.5, 0.2, 0.1)$

For $\mathbb{P}_2$,
 $A(\mathbb{P}_2) = \left(\frac{0.5 + 0.6 + 0.2}{3}, \frac{0.3 + 0.4 + 0.3}{3}, \frac{0.2 + 0.2 + 0.5}{3}, \frac{0.3 + 0.1 + 0.2}{3}, \frac{0.1 + 0.3 + 0.1}{3} \right)$
 $A(\mathbb{P}_2) = (0.5, 0.3, 0.3, 0.4, 0.2)$

For $\mathbb{P}_3$,
 $A(\mathbb{P}_3) = \left(\frac{0.6 + 0.7 + 0.3}{3}, \frac{0.1 + 0.2 + 0.5}{3}, \frac{0.2 + 0.5 + 0.3}{3}, \frac{0.5 + 0.5 + 0.2}{3}, \frac{0.3 + 0.1 + 0.1}{3} \right)$
 $A(\mathbb{P}_3) = (0.5, 0.3, 0.3, 0.4, 0.2)$

Step 2: Compare the truth degree

The average truth degrees are

$$\tilde{T}(\mathbb{P}_1) = 0.6, \tilde{T}(\mathbb{P}_2) = 0.4 \text{ and } \tilde{T}(\mathbb{P}_3) = 0.5.$$

Step 3: Results

The final average values of the three patients are shown below.

Patient	Average Pentapartitioned neutrosophic value	Truth degree	Rank
$\mathbb{P}_1$	(0.6, 0.5, 0.5, 0.2, 0.1)	(0.6)	(1)
$\mathbb{P}_2$	(0.4, 0.3, 0.3, 0.2, 0.2)	(0.4)	(3)
$\mathbb{P}_3$	(0.5, 0.3, 0.3, 0.4, 0.2)	(0.5)	(2)

The truth degree of patient $\mathbb{P}_1$ is 0.6, which is the highest among the three patients. Patient $\mathbb{P}_3$ has a truth degree of 0.5, while patient $\mathbb{P}_2$ has the lowest truth degree of 0.4. Therefore, the patients are ranked as follows, $\mathbb{P}_1$ is ranked first, $\mathbb{P}_3$ is ranked second and $\mathbb{P}_2$ is ranked third.

5. Conclusion

In this paper, the concept of pentapartitioned neutrosophic δ -continuous mappings has been introduced and studied in detail. Various fundamental properties, characterizations and relationships with other forms of continuity have been established. Several theorems and propositions

have been proved to describe the behavior of these mappings under different topological conditions. Suitable examples have also been presented to illustrate the proposed concepts and results. Thus, this chapter provides a foundation for further investigations of continuity and related mappings in pentapartitioned neutrosophic topological spaces. And also the application provide the analysis, patient $\mathbb{P}_1$ has the highest possibility of having the considered disease, followed by patient $\mathbb{P}_2$ and $\mathbb{P}_3$.

References

- [1] Abdel-Monsef.M.E, El-Deep.S.N and Mohmound.R.A(1983), β -open sets and β -Continuous mappings, Bull.Fac.Sci.,Assint Univ., 12, 77-90.
- [2] Arokiarani.I, Dhavaseelan.R, Jafari S and Parimala M 2017. On some new notions and functions in neutrosophic topological spaces neutrosophic sets and systems 16 pp 16-9.
- [3] Atanosssov k. 1986. Intuitionistic fuzzy sets Fuzzy sets and systems 20 pp 87-96.
- [4] Chang C.L. 1968 Fuzzy topological spaces J.Math.Anal.Appl. 24 pp 182-90.
- [5] Chatterjee, R., Majumdar,P., Samanta,S.K. (2016). On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets. Journal of Intelligent and Fuzzy systems, 30(4), 2475-2485.
- [6] Coker.D 1997 An introduction to intuitionistic fuzzy topological spaces Fuzzy sets and systems 88 p 81-9.
- [7] Lewine.K, (1970), Generalized closed sets in topology, Rend.circ.mat.palermo, 19(2), 89-96.
- [8] Mohanarao Navuluri and Sathish kumar.V, New operators using β -open sets in a quadripartitioned neutrosophic topological spaces, Communications on Applied Non linear Analysis, 32(45)(2025), 421-431(SCOPUS) .
- [9] Pramanik. S, (2024), Recent Trends in Pentapartitioned Neutrosophic Topology and Decision Analysis, Neutrosophic Sets and Systems, 58, 45-63.
- [10] Pramanik. S and Mallick. R, (2022), Advances in Pentapartitioned Neutrosophic Sets and Their Applications Neutrosophic Sets and Systems, 50, 1-18.
- [11] Radha. R and Stanis Arul Marry. A, (2021), Pentapartitioned Neutrosophic Generalized Semi-closed sets, Journal of Computational Mathematics, 5(1), 123-131.
- [12] Rama Mallick and Surapati Pramanik, (2020), pentapartitioned neutrosophic set and its properties, Neutrosophic Sets and Systems, 36, 184-192.
- [13] Salama and Alblowi in 2012, neutrosophic set in a neutrosophic topological spaces.

- [14] Smarandache introduction the idea of neutrosophic set at the start of the 20th Century.
- [15] Suman Das and Binod Chandra Tripathy. Pentapartitioned NTS, Neutrosophic sets and system, **45**.
- [16] Vadivel. A, Seenivasan. M and John sundar. C, (2021), An Introduction to δ -Open sets in a Neutrosophic Topological spaces, Journal of physics: conference series, 1724, 012007.
- [17] Zadeh. L. A, 1965 Fuzzy sets Information and control. 8 pp 338-53.