

DESIGN AND ANALYSIS OF TCSP - V CONTINUOUS SAMPLING PLANS FOR INDUSTRIAL QUALITY CONTROL

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Abstract:

MIL-STD-1235C establishes standard procedures for the selection and implementation of single and multi-level continuous sampling plans, such as CSP-1, CSP-2, CSP-T, and CSP-V. The TCSP-V plan represents a single-level continuous sampling approach in which inspection alternates between 100% inspection and sampling inspection. Whenever a defective unit is encountered during sampling, the procedure reverts to 100% inspection. During the sampling phase, once k inspected units are found that conform, stop sampling inspection and restart 100% inspection but with a lesser clearance number x , where each unit is inspected in the order of production. The performance measures such as the average outgoing quality, and the operating characteristic function are derived through a Markov chain modeling approach. Tables are given for the selection of TCSP - V plans for specified acceptable quality level or the limiting quality level and average outgoing quality limit.

Keywords: Acceptable quality level, Average outgoing limit, Continuous sampling plan, Limiting quality level

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1. Introduction

In modern industrial production, it is essential to ensure consistent product quality. Continuous sampling plans (CSPs) are helpful whenever we are in need of inspecting every single unit. These plans are designed for processes where products are manufactured in a continuous flow, such as in assembly lines or automated production systems. In 1943, Dodge introduced the first Continuous Sampling Plan, known as CSP-1, designed specifically for continuous production processes (like assembly lines). Dodge's contributions revolutionized industrial inspection methods and became the basis for many MIL-STD (Military

Standard) procedures, widely used in defense and manufacturing industries across the world.

CSPs are widely used due to their balance between inspection cost and quality assurance. They reduce the risk of full inspection while maintaining acceptable levels of outgoing quality. Among various CSP models, the CSP-V plan, introduced by Kandasamy, reduces the amount of inspection by lowering the clearance number. Stephens (1979) used a Markov chain model to find certain measures of performance for CSP - 1, CSP - 2 and CSP - 3 plans, assuming that the production process is in statistical quality control. In this paper, the formulation of Tightened CSP- V (TCSP-V) as a Markov chain is

given. A flow diagram showing the operation of the proposed sampling plan is given in fig 1.

Based on Stephen's approach. We derive the performance measures such as operating characteristics function (P_a), average outgoing quality (AOQ) and average fraction inspected (AFI). Tables are given for the selection TCSP - V plan. Table 1 provides TCSP-V plan with parameters with $=I = k = 2x$ and f , when the acceptable quality level (AQL) with producer's risk $\alpha = 0.05$ and average outgoing limit (AOQL) are specified. Table 2 can be used to obtain TCSP-V plan when the limiting quality level (LQL) with consumer's risk $\beta = 0.1$ and AOQL are specified.

The operating procedure of the proposed plan is given below:

(i) Inspect units one by one from the flow of production, when i successive units are found that conform, discontinue 100% inspection and start sampling inspection.

(ii) During sampling inspection, inspect only a fraction f of the units selected at random from the flow of production. Continue sampling inspection until a nonconforming unit is found. If a nonconforming item is found, then revert to 100% inspection.

(iii) During the sampling phase, once k inspected units are found that conform, stop sampling inspection and restart 100% inspection with a lesser clearance number x , where each unit is inspected in the order of production.

(iii) The inspection then shifts to 100% inspection with clearance number i when a nonconforming unit is found or to the sampling inspection when x consecutive units are found which conform.

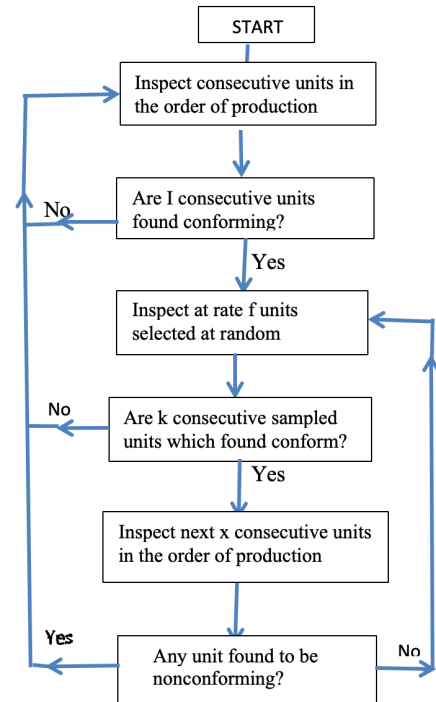


Fig 1: Flowchart for the operation of TCSP - V

2. The TCSP-V procedure as a Markov chain

Let $[X_m]$ ($m = 1, 2, \dots$) denote a discrete-parameter Markov chain with finite state space (S_j) ($j = 1, 2, \dots, i + 3k + x + 2$). The states of the process are defined, in a way similar to Roberts (1965) and Lasater (1970), as follows:

- $S_k = A_{k-1}$ ($k = 1, 2, \dots, i + 1$); 100% inspection with clearance number i is being conducted, and the last $(k-1)$ consecutive units inspected were found to conform;
- $S_{i+j+1} = Id_j$ ($j = 1, 2, \dots, k$); sampling inspection is in effect and, of the j units submitted for inspection, only the j th unit was found to be non-conforming;
- $S_{i+k+j+1} = In_j$ ($j = 1, 2, \dots, k$); sampling inspection is in effect and the j units submitted for inspection were all found to conform;
- $S_{i+2k+j+1} = N_j$ ($j = 1, 2, \dots, k$); sampling inspection is in effect and the $(j-1)$ units submitted for inspection were all found to conform, but the last unit was not selected for inspection;
- $S_{i+3k+j+2} = B_{j-1}$ ($j = 1, 2, \dots, x + 1$); 100% inspection with clearance number x is being conducted and the last $(j-1)$ consecutive units were found to conform.

The set of $(i + 3k + x + 2)$ states defined above completely describes the mutually exclusive phases of inspection for the TCSP-V plan. A flow chart describing the process by means of states and transitions is given in Fig. 1, and the one-step transition probability matrix for the process is given in Table 1. The transition probability matrix reveals that the process is a discrete-parameter, finite, recurrent, irreducible, aperiodic (DFRIA) Markov chain (Karlin, 1966; Lasater, 1970).

3. The performance measures of the proposed TCSP - V plan

1. The average number of units inspected under 100% inspection is

$$u = \frac{1-q^i+q^k(q^i-q^x)}{pq^i} \quad \text{-----}(1)$$

2. The average number of units passed under the sampling inspection is

$$v = \frac{f+q^{i-1}(1-q^k)+fp^2q^{i+k-1}}{fpq^{i-1}} \quad \text{-----}(2)$$

3. The average fraction of total production accepted on a sampling basis (the operating characteristic function) is

$$P_a(p) = \frac{fp(1-q^{x+k})+q^i(1-f)(1-q^k)-fq^{i+k}+fp^2q^{i+k}}{f(1-q^{x+k})+q^i(1-f)(1-q^k)+fp^2q^{i+k}} \quad \text{-----}(3)$$

4. The Average outgoing Quality (AOQ) is

$$AOQ(p) = \frac{(1-f)(1-q^k)pq^{i-1}}{f(1-q^{x+k})+q^i(1-f)(1-q^k)+fp^2q^{i+k}} \quad \text{-----}(4)$$

4. Selection of plans

Four tables are provided to enable the selection of TCSP-V ($k=i$) plans. Tables 2 and 4 can be used to select TCSP-V plans with parameters $i=2x$ and f . Tables 3 and 5 can be used to select TCSP-V plans with parameters $i=3x$ and f . The following examples illustrate the selection of the plans from these tables.

4.1 Example 1

Let us suppose that one requires a TCSP-V plan with parameters $i=2x$ and f satisfying the conditions $AQL=0.1\%$ with $a=0.05$ and $AOQL=0.13\%$. Table 2 is used for the selection of the plan. The ratio $AOQL/AQL$ is equal to 1.3. Corresponding to $AQL=0.1\%$ and $AOQL/AQL=1.3$, Table 2 gives the TCSP-V plan with $x=1301$ and $n=1/f=274$. The required CSP-V plan is $(x,n)=(1301, 274)$.

4.2 Example 2

Let us suppose that one requires a TCSP-V plan with parameters $i=2x$ and f satisfying the desired conditions $LQL=0.4\%$ with $B=0.10$ and $AOQL=0.16\%$. Table 2 is used for the selection of the plan. The ratio $LQL/AOQL$ is equal to 2.5. Corresponding to $LQL/AOQL=2.5$ and $LQL=0.4\%$, Table 3 gives the TCSP-V plan with $x=140$ and $n=1/f=894$. The required TCSP-V plan is $(x,n)=(140, 894)$.

4.3 Example 3

Let us suppose that one requires TCSP-V plan with parameters $i=3x$ and f satisfying the desired conditions $AQL=0.1\%$ with $a=5\%$ and $AOQL=0.13\%$. Table 3 is used for the selection of the plan. The ratio $AOQL/AQL$ is equal to 1.3. Corresponding to $AOQL/AQL=1.3$ and $AQL=0.1\%$, Table 3 gives the TCSP-V plan with $x=863$ and $n=268$. The desired TCSP-V plan is $(x, n) = (863, 268)$.

4.4 Example 4

Let us suppose that one requires a TCSP-V plan with parameters $i=3x$ and f satisfying the conditions $LQL=0.25\%$ with $B=0.10$ and $AOQL=0.1\%$. The ratio $LQL/AOQL$ is equal to 2.5. Corresponding to $LQL/AOQL=2.5$ and $LQL=0.25\%$, Table 4 gives the TCSP-V plan with $x=952$ and $n=1/f=139$. Thus, the desired CSP-V plan is $(x, n) = (952, 139)$.

5. Construction of tables:

$$f = \frac{q^i(1-q^k)(1-P_a(p))}{P_a(p)(1-q^{x+k}-q^i(1-q^k)+p^2q^{i+k})-p(1-q^{x+k})+q^i(1-p^2q^k)} \quad \text{-----}(5)$$

When $k=i=2x$, then the above equation becomes

$$f = \frac{q^{2x}(1-q^{2x})(1-P_a(p))}{P_a(p)(1-q^{3x}-q^{2x}(1-q^{2x})+p^2q^{4x})-p(1-q^{3x})+q^i(1-p^2q^{2x})} \quad \text{-----}(6)$$

When $i=k=3x$, then the equation becomes

$$f = \frac{q^{3x}(1-q^{3x})(1-P_a(p))}{P_a(p)(1-q^{4x}-q^{3x}(1-q^{3x})+p^2q^{6x})-p(1-q^{4x})+q^i(1-p^2q^{3x})} \quad \text{-----}(7)$$

The $AOQL(p)$ is given by the maximum of $AOQ(p)$ overall the possible values of p .

(1) For the integer x in the interval $(2,2500)$, find f and the $AOQL p_L$ as follows:

(a) If the AQL condition is specified, take $p = AQL$ and $P_a(p) = 0.95$; if the LQL condition is specified, take $p = LQL$ and $P_a(p) = 0.10$;

(b) Find f using equation (6) if $i = 2x$ is considered, find f using (7) if $i = 3x$ is considered.

(c) Use any search procedure and find p_L , which is the maximum of $AOQ(p)$ given in equation (4), when p takes values from 0 to 1 in steps of 0.0001;

(2) For the desired AOQL, find x and $n = 1/f$ for which $D(x, n) = (AOQL - p_L)$ is non-negative and minimum.

(3) Take $n = \text{int}(n+0.5)$ and revised $f = 1/n$. Increase or decrease x such that the pair (x, n) satisfies the following conditions:

(A) $P_a(AQL)$ is at least 0.95 if AQL condition is given or $P_a(LQL)$ is less than or equal to 0.10;

(B) $D(x, n)$ is non-negative.

These x and n values are given in the tables. Tables 2 and 4 give TCSP-V plans for preferred AQL values with $\alpha = 5\%$ and fixed values of AOQL/AQL. Tables 3 and 5 give TCSP-V plans for preferred LQL values with $\beta = 10\%$ and fixed values of LQL/AOQL.

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Table1: TCSP-V Transition Probability Matrix

	A ₀	A ₁	A ₂	...	A _i	Id ₁	In ₁	N ₁	Id ₂	In ₂	N ₂	..	Id _k	In _k	N _k	B ₀	B ₁	B ₂	...	B _x
A ₀	p	q	0	...	0	0	0	0	0	0	0		0	0	0	0	0	0	...	0
A ₁	p	0	q	...	0	0	0	0	0	0	0		0	0	0	0	0	0	...	0
A ₂	p	0	0	...	0	0	0	0	0	0	0		0	0	0	0	0	0	...	0
.	...	...	...	...	...	...	...	...	...	...	...	.	...	...	...	...	...	...	...	..
A _{i-1}	p	0	0	...	q	0	0	0	0	0	0	.	0	0	0	0	0	0	...	0
A _i	0	0	0	...	0	fp	fq	1-f	0	0	0	.	0	0	0	0	0	0	...	0
Id ₁	p	q	0	...	0	0	0	0	0	0	0	.	0	0	0	0	0	0	...	0
In ₁	0	0	0	...	0	0	0	0	fp	fq	1-f	.	0	0	0	0	0	0	...	0

N ₁	0	0	0	...	0	fp	fq	1-f	0	0	0	.	0	0	0	0	0	...	0
Id ₂	p	q	0	...	0	0	0	0	0	0	0	.	0	0	0	0	0	...	0
In ₂	0	0	0	0	0	0	0	0	0	0	0	.	0	0	0	0	0	...	0
N ₂	0	0	0	...	0	0	0	0	fp	fq	1-f	.	0	0	0	0	0	...	0
...	...	...	...	...	...	...	...	...	...	...	...	..	0	...	...	...	...	...	...
Id _k	p	q	0	...	0	0	0	0	0	0	0	..	0	0	0	0	0	...	0
In _k	0	0	0	0	0	0	0	0	0	0	0	.	0	0	0	p	q	0	...
N _k	0	0	0	0	0	0	0	0	0	0	0	.	fp	fq	1-f	0	0	0	...
B ₀	p	q	0	0	0	0	0	0	0	0	0	.	0	0	0	0	0	...	0
B ₁	0	0	0	0	0	0	0	0	0	0	0	,	0	0	0	p	0	q	...
...	...	...	...	...	...	...	...	...	...	...	...	.	...	...	...	...	...	...	...
B _{x-1}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	p	0	0	...
B _x	0	0	0	0	0	fp	fq	1-f	0	0	0	0	0	0	0	0	0	0	...

TABLE 2 : TCSP-V(i=2x)plansindexed by AQLand AOQL/AQL($\alpha=0.05$)

AQL(%)
0.065 0.1 0.15 0.25 1.0 1.5 2.5 4.0

$\frac{AOQL}{AQL}$	x	n	x	n	x	n	x	n	x	n	x	n	x	n	x	n
1.2	203	28	132	28	88	28	53	28	13	30	9	310	56	332	36	361
9	5	5	3	5	4	7	1	8	5	1	1					
1.3	200	27	130	27	86	27	52	27	13	29	8	293	55	317	35	334
	0	3	1	4	8	4	2	6	3	0	9					
1.3	196	26	127	26	85	26	51	26	13	27	8	285	54	302	35	334
1	6	2	9	3	2	2	3	5	0	4	8					
1.3	193	25	125	25	83	25	50	25	12	26	8	269	53	288	34	309
2	0	1	7	2	9	3	4	4	8	4	6					
1.3	189	24	123	24	82	24	49	24	12	25	8	261	52	275	33	286
3	4	0	4	2	5	3	6	5	6	4	5					
1.3	185	23	121	23	81	23	48	23	12	24	8	247	51	262	33	286
4	8	0	3	3	2	5	8	6	4	5	3					
1.3	182	22	119	22	79	22	48	22	12	23	8	240	50	250	32	265
5	9	2	6	5	8	6	0	7	2	6	2					
1.3	180	21	117	21	78	21	47	22	12	22	8	234	49	238	32	265
6	6	6	9	8	7	9	3	0	0	7	1					
1.3	178	21	116	21	77	21	46	21	11	21	8	227	49	238	31	246
7	2	0	1	1	5	2	6	3	8	9	0					
1.3	175	20	114	20	76	20	45	20	11	21	7	215	48	227	31	246
8	8	4	4	5	3	5	9	6	6	1	8					
1.3	173	19	112	19	75	19	45	20	11	20	7	209	47	217	30	228
9	4	9	6	8	2	9	2	0	5	7	7					
1.4	170	19	110	19	74	19	44	19	11	20	7	203	46	207	30	228

	9	3	9	2	2	4	6	4	3	0	6					
1.4	168	18	109	18	73	18	44	18	11	19	7	198	46	207	29	211
1	4	7	5	8	1	8	0	9	1	3	5					
1.4	165	18	108	18	72	18	43	18	11	18	7	193	45	198	29	211
2	9	2	1	3	1	3	4	4	0	9	4					
1.4	163	17	106	17	71	17	42	17	10	18	7	187	45	198	29	211
3	3	7	7	8	1	8	8	9	8	2	3					
1.4	161	17	105	17	70	17	42	17	10	17	7	182	44	189	28	196
4	0	2	2	4	2	4	2	5	7	9	2					
1.4	159	16	103	16	69	17	41	17	10	17	7	178	43	181	28	196
5	2	8	8	9	3	0	7	1	6	6	1					
1.4	157	16	102	16	68	16	41	16	10	17	7	173	43	181	27	182
6	4	5	4	5	4	6	1	6	4	0	0					
1.4	155	16	101	16	67	16	40	16	10	16	6	168	42	173	27	182
7	6	2	2	2	5	2	6	3	3	7	9					
1.4	153	15	100	15	66	15	40	15	10	16	6	164	42	173	26	169
8	8	8	0	8	7	9	1	9	2	4	8					
1.4	152	15	988	15	65	15	39	15	10	15	6	164	41	165	26	169
9	0	5		5	9	5	6	6	0	8	8					
1.5	150	15	976	15	65	15	39	15	99	15	6	159	41	165	26	169
	1	2		2	1	2	2	3		5	7					
1.5	148	14	964	14	64	14	38	15	98	15	6	155	40	158	26	169
1	2	9		9	3	9	7	0		3	6					
1.5	146	14	952	14	63	14	38	14	97	15	6	151	40	158	26	169
2	3	5		6	6	6	3	7		0	5					
1.5	144	14	942	14	62	14	37	14	96	14	6	147	39	151	25	157
3	4	2		3	9	4	8	4		7	4					
1.5	142	14	932	14	62	14	37	14	95	14	6	147	39	151	25	157
4	9	0		1	2	1	4	1		5	4					
1.5	141	13	922	13	61	13	37	13	94	14	6	144	38	145	25	157
5	5	8		8	5	9	0	9		2	3					
1.5	140	13	912	13	60	13	36	13	93	14	6	140	38	145	24	147
6	1	6		6	8	6	6	7		0	2					
1.5	138	13	902	13	60	13	36	13	92	13	6	140	38	145	24	147
7	7	4		4	2	4	2	4		8	2					
1.5	137	13	891	13	59	13	35	13	91	13	6	136	37	138	24	147
8	3	2		1	6	2	8	2		5	1					
1.5	135	13	883	13	58	13	35	13	90	13	6	133	37	138	24	147
9	9	0		0	9	0	4	0		3	0					
1.6	134	12	874	12	58	12	35	12	89	13	6	133	37	138	23	137
	4	8		8	3	8	1	8		1	0					
1.6	132	12	866	12	57	12	34	12	88	12	5	130	36	133	23	137
1	9	6		6	8	6	7	6		9	9					
1.6	131	12	857	12	57	12	34	12	87	12	5	126	36	133	23	137
2	5	4		4	2	4	4	5		7	8					
1.6	130	12	848	12	56	12	34	12	86	12	5	126	35	127	23	137
3	0	2		2	6	3	0	3		4	8					
1.6	128	12	839	12	56	12	33	12	85	12	5	123	35	127	22	128
4	9	0		1	0	1	7	1		2	7					
1.6	127	11	832	11	55	11	33	12	84	12	5	123	35	127	22	128
5	8	9		9	5	9	4	0		0	7					

1.6 6	126 7	11 8	824	11 8	55 0	11 8	33 1	11 8	84	12 0	5 6	120	34	122	22	128
1.6 7	125 6	11 6	817	11 6	54 5	11 6	32 7	11 6	83	11 8	5 6	120	34	122	22	128
1.6 8	124 4	11 5	809	11 5	54 0	11 5	32 4	11 5	82	11 7	5 5	117	34	122	22	128
1.6 9	123 3	11 4	802	11 4	53 5	11 4	32 1	11 4	81	11 5	5 5	117	33	117	21	119
1.7 1	122 1	11 2	794	11 2	53 0	11 2	31 9	11 3	81	11 5	5 4	114	33	117	21	119
1.7 1	120 9	11 1	787	11 1	52 5	11 1	31 6	11 1	80	11 3	5 4	114	33	117	21	119
1.7 2	119 7	10 9	780	11 0	52 1	11 0	31 3	11 0	79	11 1	5 3	112	32	112	21	119
1.7 3	118 7	10 8	774	10 9	51 6	10 9	31 0	10 9	79	11 1	5 3	112	32	112	21	119

TABLE 3 : TCSP-V($i=3x$)plans indexed by AQL and AOQL/AQL($\alpha=0.05$)

AQL(%)

0.065 0.1 0.15 0.25 1.0 1.5 2.5 4.0

$\frac{AOQ}{AQL}$	x	n	x	n	x	n	x	n	x	n	x	n	x	n	x	n
1.2 9	135 0	27 9	877	27 8	58 6	28 0	35 2	28 1	90	29 8	6 0	298	37	322	24	358
1.3	132 7	26 7	863	26 8	57 6	26 8	34 6	26 9	88	28 1	5 9	286	36	299	23	318
1.3 1	130 3	25 6	848	25 6	56 5	25 6	34 0	25 8	86	26 6	5 8	274	36	299	23	318
1.3 2	127 9	24 5	833	24 6	55 6	24 6	33 4	24 7	85	25 8	5 7	262	35	278	23	318
1.3 3	125 5	23 4	818	23 6	54 7	23 7	32 9	23 9	83	24 4	5 6	251	34	259	22	283
1.3 4	123 1	22 4	804	22 6	53 8	22 8	32 3	22 9	82	23 7	5 5	241	34	259	22	283
1.3 5	121 2	21 6	793	22 0	52 9	22 0	31 8	22 1	81	23 1	5 4	231	33	241	21	252
1.3 6	119 6	21 0	781	21 2	52 1	21 3	31 3	21 3	79	21 8	5 4	231	33	241	21	252
1.3 7	110 0	20 4	769	20 5	51 3	20 6	30 9	20 7	78	21 2	5 3	221	32	224	21	252
1.3 8	116 4	19 8	757	19 9	50 5	19 9	30 4	20 0	77	20 6	5 2	212	32	224	20	225
1.3 9	114 8	19 3	745	19 2	49 8	19 3	29 9	19 3	76	20 0	5 1	203	31	209	20	225
1.4	113 1	18 7	734	18 6	49 1	18 8	29 5	18 8	75	19 5	5 0	195	31	209	20	225
1.4 1	111 4	18 1	725	18 2	48 4	18 2	29 1	18 3	74	18 9	5 0	195	30	195	20	225
1.4 2	109 7	17 6	715	17 7	47 7	17 7	28 7	17 8	73	18 4	4 9	187	30	195	19	201
1.4 3	108 0	17 1	706	17 3	47 0	17 2	28 3	17 3	72	17 9	4 8	179	30	195	19	201
1.4 4	106 5	16 6	696	16 8	46 4	16 8	27 9	16 9	71	17 4	4 8	179	29	182	19	201
1.4 5	105 3	16 3	686	16 4	45 8	16 4	27 6	16 5	70	17 0	4 7	172	29	182	18	179
1.4 6	104 1	15 9	677	16 0	45 2	16 0	27 2	16 1	69	16 5	4 6	165	28	170	18	179
1.4 7	102 9	15 6	669	15 6	44 6	15 6	26 9	15 8	68	16 1	4 6	165	28	170	18	179
1.4 8	101 7	15 3	661	15 3	44 1	15 3	26 5	15 3	67	15 6	4 5	158	28	170	18	179
1.4 9	100 4	14 9	653	15 0	43 6	15 0	26 2	15 0	66	15 2	4 5	158	27	158	18	179
1.5	992	14 6	645	14 6	43 0	14 6	25 9	14 7	66	15 2	4 4	152	27	158	17	160
1.5 1	979	14 3	637	14 3	42 5	14 4	25 6	14 4	65	14 8	4 4	152	27	158	17	160
1.5 2	966	14 0	629	14 0	42 0	14 1	25 3	14 2	64	14 4	4 3	146	26	148	17	160
1.5	953	13	622	13	41	13	25	13	63	14	4	146	26	148	17	160

3		7		8	6	9	0	9		1	3					
1.5	944	13	616	13	41	13	24	13	63	14	4	140	25	138	16	144
4		5		6	1	6	7	6		1	2					
1.5	935	13	609	13	40	13	24	13	62	13	4	140	25	138	16	144
5		3		3	6	3	4	4		7	2					
1.5	925	13	602	13	40	13	24	13	61	13	4	135	25	138	16	144
6		1		1	2	1	2	2		3	1					
1.5	916	12	595	12	39	12	23	12	61	13	4	135	25	138	16	144
7		9		8	8	9	9	9		3	1					
1.5	906	12	588	12	39	12	23	12	60	13	4	130	25	138	16	144
8		6		6	3	7	6	7		0	0					
1.5	897	12	583	12	38	12	23	12	59	12	4	130	24	129	16	144
9		5		5	9	5	4	5		7	0					
1.6	887	12	577	12	38	12	23	12	59	12	3	125	24	129	15	129
		3		3	5	3	1	3		7	9					
1.6	877	12	571	12	38	12	22	12	58	12	3	125	24	129	15	129
1		1		1	1	1	9	1		3	9					
1.6	867	11	565	11	37	11	22	12	57	12	3	125	24	129	15	129
2		9		9	7	9	7	0		0	9					
1.6	858	11	559	11	37	11	22	11	57	12	3	120	23	121	15	129
3		7		7	3	7	4	7		0	8					
1.6	851	11	554	11	37	11	22	11	56	11	3	120	23	121	15	129
4		6		6	0	6	2	6		7	8					
1.6	843	11	549	11	36	11	22	11	56	11	3	116	23	121	15	129
5		4		4	6	4	0	5		7	7					
1.6	836	11	544	11	36	11	21	11	55	11	3	116	23	121	15	129
6		3		3	3	3	8	3		4	7					
1.6	828	11	538	11	35	11	21	11	55	11	3	116	22	114	14	116
7		1		1	9	1	6	2		4	7					
1.6	820	11	533	11	35	11	21	11	54	11	3	111	22	114	14	116
8		0		0	6	0	4	0		1	6					
1.6	812	10	528	10	35	10	21	10	54	11	3	111	22	114	14	116
9		9		9	3	9	2	9		1	6					
1.7	805	10	523	10	34	10	21	10	53	10	3	111	22	114	14	116
		7		7	9	7	0	8		9	6					
1.7	797	10	519	10	34	10	20	10	53	10	3	107	22	114	14	116
1		6		6	6	6	8	6		9	5					
1.7	789	10	514	10	34	10	20	10	52	10	3	107	21	107	14	116
2		5		5	3	5	6	5		6	5					
1.7	782	10	510	10	34	10	20	10	52	10	3	107	21	107	14	116
3		4		4	0	4	4	4		6	5					

TABLE 4: TCSP-V($i=2x$)plans indexed by LQL and LQL/AOQL($\beta=0.1$)

LQL (%)

0.15 0.25 0.4 0.65 1.0 1.5 2.5 4.0

$\frac{LQL}{AOQL}$	X	n	x	n	x	n	x	n	x	n	x	n	x	n	x	n
2.4	2495	197	1496	195	937	196	579	199	378	201	255	211	156	226	101	255
2.45	2431	163	1461	164	914	163	565	166	370	171	249	176	152	184	99	217
2.50	2373	137	1429	140	894	139	552	140	361	143	243	147	149	159	97	184
2.55	2319	117	1397	119	874	119	540	120	353	122	238	127	146	136	95	157
2.60	2270	101	1366	102	855	102	529	104	346	106	233	109	143	117	93	133
2.65	2223	88	1337	89	838	89	518	90	339	92	228	94	140	101	91	113
2.70	2180	77	1310	77	821	78	508	79	332	80	224	83	138	91	90	105
2.75	2139	68	1285	68	804	68	498	70	326	71	220	74	135	79	88	89
2.80	2100	61	1261	61	709	61	489	62	320	63	216	66	133	71	87	82
2.85	2060	54	1239	55	777	55	480	55	315	57	212	58	131	64	85	70
2.90	2029	49	1218	49	764	50	472	50	309	51	209	53	128	55	84	65
2.95	1996	45	1198	45	751	45	465	46	304	46	205	47	127	53	83	60
3.01	1964	41	1178	41	739	41	457	41	300	43	202	43	125	48	81	51
3.05	1934	37	1160	37	727	37	450	38	295	39	199	40	123	43	80	47
3.10	1906	35	1143	34	716	34	443	35	290	35	196	36	121	39	79	43
3.15	1878	32	1126	32	705	31	437	32	286	32	193	33	119	36	78	40
3.20	1851	29	1110	29	695	29	430	29	282	30	191	31	118	34	77	37
3.25	1826	27	1094	27	685	27	424	27	278	28	188	29	116	31	76	34
3.30	1800	25	1079	25	676	25	418	25	274	26	185	26	115	29	75	31
3.35	1774	24	1064	23	668	24	413	24	271	24	183	25	113	27	75	31
3.40	1750	22	1050	22	658	22	407	22	267	23	181	24	112	25	74	29
3.45	1727	21	1036	20	649	20	402	21	264	21	179	22	110	23	73	27
3.50	1704	19	1023	19	641	19	397	20	261	20	176	20	109	22	72	25
3.55	1682	18	1010	18	634	18	392	18	258	19	174	19	108	21	71	23
3.60	1661	17	997	17	626	17	388	17	254	18	172	18	107	20	70	21
3.65	1641	16	985	16	619	16	383	16	252	17	170	17	106	19	70	21
3.70	1622	15	974	15	611	15	379	16	249	16	168	16	104	17	69	20
3.75	1603	15	963	15	604	15	375	15	246	15	167	16	103	16	68	18
3.80	1584	14	952	14	598	14	370	14	243	14	165	15	102	16	68	18
3.85	1567	13	941	13	591	13	366	13	241	14	163	14	101	15	67	17
3.90	1549	13	931	13	585	13	362	13	238	13	161	13	100	14	66	16
3.95	1532	12	921	12	578	12	359	12	236	13	160	13	99	14	66	16
4.00	1514	11	911	11	572	12	355	12	233	12	158	12	98	13	65	15

TABLE 5 : TCSP-V($i=3x$)plans indexed by LQL and LQL/AOQL($\beta=0.1$)

	0.15		0.25		0.4		0.65		1.0		1.5		2.5		4.0	
	LQL(%)															
$\frac{LQL}{AOQL}$	x	n	x	n	x	n	x	n	x	n	x	n	x	n	x	n
2.4	1662	196	997	194	624	195	386	199	252	201	170	211	104	226	67	245
2.45	1620	162	973	163	609	163	376	164	246	168	166	176	102	194	66	217
2.50	1581	136	952	139	595	138	368	140	241	144	162	147	99	155	65	192
2.55	1545	116	931	119	582	118	360	120	235	121	158	123	97	133	63	151
2.60	1511	100	910	102	570	102	352	103	230	104	155	100	95	114	62	133
2.65	1480	87	890	88	558	89	345	90	226	92	152	94	93	98	61	118
2.70	1451	76	872	77	547	78	338	78	221	79	149	82	92	91	60	105
2.75	1424	68	855	68	536	68	332	70	217	71	146	72	90	79	59	93
2.80	1398	60	839	60	526	61	326	62	213	63	144	66	88	68	58	82
2.85	1373	54	822	53	517	55	320	55	209	56	141	58	87	63	57	73
2.90	1350	49	810	49	508	49	314	49	206	51	139	53	86	58	56	64
2.95	1328	44	787	44	500	45	309	45	202	45	137	48	84	50	55	57
3.01	1307	40	784	40	491	40	304	41	199	41	134	42	83	47	54	51
3.05	1287	37	772	37	483	37	299	37	196	38	132	39	82	43	54	51
3.10	1267	34	760	34	476	34	295	34	193	35	130	35	80	37	53	45
3.15	1249	31	749	31	469	31	290	31	190	32	128	32	79	35	52	40
3.20	1231	29	738	29	462	29	286	29	188	30	127	31	78	32	52	40
3.25	1213	27	727	27	455	26	282	27	185	28	125	28	77	30	51	35
3.30	1195	25	716	25	449	25	278	25	182	25	123	26	76	28	50	31
3.35	1178	23	706	23	443	23	274	23	180	24	122	25	75	26	50	31
3.40	1162	22	697	21	437	22	271	22	178	23	120	23	74	24	49	28
3.45	1146	20	688	20	431	20	267	20	175	21	118	21	73	22	48	25
3.50	1131	19	677	19	426	19	264	19	173	20	117	20	72	21	48	25
3.55	1116	18	670	18	421	18	260	18	171	18	116	19	72	21	47	22
3.60	1102	17	662	17	414	17	257	17	169	17	114	18	71	19	47	22
3.65	1088	16	654	16	410	16	254	16	167	16	113	17	70	18	46	20
3.70	1075	15	646	15	405	15	251	15	165	16	112	16	69	17	46	20
3.75	1062	14	638	14	400	14	248	14	163	15	110	15	69	17	45	18
3.80	1050	13	631	13	396	14	246	14	161	14	109	14	68	16	45	18
3.85	1038	13	624	13	392	13	243	13	160	14	108	14	67	15	45	18
3.90	1026	12	617	12	386	12	240	12	158	13	107	13	66	14	44	16
3.95	1015	12	610	12	382	12	238	12	156	12	106	13	66	14	44	16
4.00	1004	11	603	11	379	11	235	11	154	11	105	12	65	13	43	14

Derivation of Performance measures

The formulation of the TCSP-V procedure as a Markov chain with states S_t ($t=1,2,\dots,i+3k+x+3$) implies that it is a DFRIA Markov chain. For an infinite number of items submitted for inspection under this plan, a unique vector of steady state probabilities exists, all elements of which are positive, and the probability of being in any state after a very large number of transitions is independent of the initial state. These steady state probabilities $P(S_t)$ satisfy the following conditions:

$$\begin{aligned} P(S_t \geq 0), & \quad t = 1, 2, 3, \dots, i + 3k + x + 3 \\ P(S_t) = \sum_{k=1}^{i+3k+x+3} P(S_k) p_{kt} & \quad t = 1, 2, 3, \dots, i + 3k + x + 3 \\ \sum_{t=1}^{i+3k+x+3} P(S_t) = 1 \end{aligned}$$

These three conditions give the following equations:

$$\begin{aligned} P(A_0) &= p \left[\sum_{t=0}^{i-1} P(A_t) + \sum_{t=1}^k P(Id_t) \right] + P(B_0) \\ P(A_1) &= q \left[P(A_0) + \sum_{t=1}^k P(Id_t) + P(B_0) \right] \\ P(A_t) &= q^{t-1} P(A_1), \quad t = 2, 3, \dots, i \\ P(Id_1) &= fp (P(A_i) + P(N_1) + P(B_x)) \\ P(In_1) &= fq (P(A_i) + P(N_1) + P(B_x)) \\ P(N_1) &= \frac{1-f}{f} (P(A_i) + P(B_x)) \\ P(Id_t) &= pq^{t-1} P(In_1), \quad t = 2, 3, \dots, k \\ P(In_t) &= q^{t-1} P(In_1), \quad t = 2, 3, \dots, k \\ P(N_t) &= \frac{1-f}{f} q^{t-2} (P(In_1)), \quad t = 2, 3, \dots, k \\ P(B_0) &= p \left[P(In_k) + \sum_{t=1}^{x-1} P(B_t) \right] \\ P(B_t) &= q^t P(In_k) \end{aligned}$$

$$\sum_{t=0}^i P(A_t) + \sum_{t=1}^k [P(Id_t) + P(In_t) + P(N_t)] + \sum_{t=0}^x P(B_t) = 1$$

The above equations can be written as follows:

$$\begin{aligned} P(Id_1) &= \frac{pq^{i-1}}{1-q^{x+k}} P(A_1) \\ P(In_1) &= \frac{q^i}{1-q^{x+k}} P(A_1) \\ P(N_1) &= \frac{(1-f)q^{i-1}}{f(1-q^{x+k})} P(A_1) \\ P(A_1) &= \frac{qP(A_0)}{1-q^i} \end{aligned}$$

Solving

$$\begin{aligned} P(A_0) &= \frac{1-q^i}{q} P(A_1) \\ P(A_0) &= \frac{(1-q^i)}{D} fp(1-q^{x+k}) \end{aligned}$$

Where $D = f(1-q^{x+k}) + q^i(1-f)(1-q^k) + fp^2q^{i+k}$

$$\begin{aligned} P(A_1) &= \frac{fpq(1-q^{x+k})}{D} \\ P(Id_t) &= \frac{fp^2q^{i+t}}{D}, \quad t = 1, 2, \dots, k \\ P(In_t) &= \frac{fpq^{i+t}}{D}, \quad t = 1, 2, \dots, k \\ P(N_t) &= \frac{(1-f)pq^{i+t-1}}{D}, \quad t = 1, 2, \dots, k \\ P(B_0) &= \frac{fp(1-q^x)q^{i+k}}{D} \\ P(B_t) &= \frac{fpq^{x+i+t}}{D}, \quad t = 1, 2, \dots, x \end{aligned}$$

Let $P(100) = \sum_{t=0}^i P(A_t) + \sum_{t=0}^x P(B_t)$

Then we have

$$u = \frac{P(100)}{P(A_i) + P(B_x)}$$

$$u = \frac{1-q^i + q^k(q^i - q^x)}{pq^i}$$

$$P_a(p) = 1 - P(100)$$

$$P_a(p) = \frac{fp(1-q^{x+k}) + q^i(1-f)(1-q^k) - fq^{i+k} + fp^2q^{i+k}}{D}$$

$$v = \frac{1 - P(100)}{P(A_i) + P(B_x)}$$

$$v = \frac{f + q^{i-1}(1-q^k) + fp^2q^{i+k-1}}{fpq^{i-1}}$$

Disclosure statement

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