

Investigating new generalized closed sets in ideal binary topological spaces.

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Abstract: In this work, we analyze the concepts of $\mathcal{K}^*$ - bl -sets, bl - $\mathcal{Z}$ -sets, $\mathcal{K}^{**}$ - bl -sets, I - $\mathcal{Z}_p^*$ -sets, $\mathcal{Z}^{**}$ - bl -sets and $\mathcal{Z}_p^*$ - bl -sets in an ideal binary topological spaces. Also the notions of pre * - bl -closed sets, α^* -closed sets, bl - t -sets, pre * - bl -regular sets, locally closed sets namely bl -LC sets, bl^* -LC sets, $\star$ - bl -LC sets, $\mathcal{L}$ - bl -LC sets are given their properties are studied. The Characterizations of a bl - t -set and a $\mathcal{K}^*$ - bl -set are established. Also new types of decompositions of $b \star$ -open sets (in five various forms) and α^* - bl -open sets are given.

Keywords: $\mathcal{K}^*$ - bl -sets, bl - $\mathcal{Z}$ -sets, $\mathcal{K}^{**}$ - bl -sets, I - $\mathcal{Z}_p^*$ -sets, $\mathcal{Z}^{**}$ - bl -sets and $\mathcal{Z}_p^*$ - bl -set

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1 Introduction and Preliminaries

The properties of α -open sets in classical topology were studied by Njastad [19], who showed that these sets form a topology. Semi-open sets were first described by Levine [14], who also talked about their associated topological characteristics. Furthermore, Mashhour et al. [16] discussed pre-open sets and examined pre-continuous function characteristics. α -continuous and α -open mappings were examined by Mashhour, et al.[17]. Abd Monsef, et al. presented and described β -open sets and β -continuous functions in [2]. The relationship between α -open sets, semi-open sets, and pre-open sets was discussed by Reilly et al.[21]. Andrijević [1] proposed and researched a novel generalized open set in a topological space called b -open sets. Kuratowski [12, 13] works with ideals in a topological space (X, τ) . An ideal space is a topological space denoted as $(X, \tau, \mathcal{I})$ that has an ideal $\mathcal{I}$. Furthermore, for each subset of X , he defined the local function with respect to an ideal $\mathcal{I}$ and τ . The local function of a subset A of X with respect to $\mathcal{I}$ and τ is represented by $A^*(\mathcal{I}, \tau)$, or simply A^* . In 1945, Vaidyanathaswamy [24, 25] extended the research on ideals and local functions. The features of the topology produced by the ideal $\mathcal{I}$ and τ , known as the $\star$ -topology, which is

finer than τ , denoted by τ^* , have been studied by Vaidyanathaswamy [25], Hashimoto [9, 10], Hayashi [8], and Samuels [22]. All of the results from the aforementioned research were gathered in 1990 by Jankovic and Hamlett [11]. Dontchev, Ganster, and Rose [7] introduced and studied $\mathcal{J}$ -dense sets and completely codense ideals in 1999. IAbd El-monsef, Lashien, and Nasef introduced α -open sets and I -continuous functions [3]. Modak Shyamapada et al. [23] Next, we explore the connections between binary topological spaces and provide some insights on ideal binary topological spaces. In this paper, we study the concepts of $\mathcal{K}^*$ - bl -sets, bl - $\mathcal{Z}$ -sets, $\mathcal{K}^{**}$ - bl -sets, I - $\mathcal{Z}_p^*$ -sets, $\mathcal{Z}^{**}$ - bl -sets, and $\mathcal{Z}_p^*$ - bl -sets in ideal binary topological spaces. Also the concepts of pre * - bl -closed sets, α^* -closed sets, bl - t -sets, pre * - bl -regular sets, and locally closed sets, namely bl -LC sets, bl^* -LC sets, $\star$ - bl -LC sets, $\mathcal{L}$ - bl -LC sets are given their properties are studied. A bl - t -set and a $\mathcal{K}^*$ - bl -set are characterized. Together with α^* - bl -open sets, five new forms of decompositions for $b \star$ -open sets are also introduced.

Definition 1.1 [23] Let X and Y be any two non empty sets. A binary ideal from X to Y is a binary structure $\mathcal{I} \subseteq \wp(X) \times \wp(Y)$ that satisfies the

following axioms:

1. $(A, B) \in \mathcal{J}$ and $(C, D) \subseteq (A, B)$ implies $(CD) \in \mathcal{J}$ (hereditary)
2. $(A_1, B_1) \in \mathcal{J}$ and $(A_2, B_2) \in \mathcal{J}$ implies $(A_1 \cup A_2, B_1 \cup B_2) \in \mathcal{J}$ finite additivity).

Definition 1.2 [23] An binary ideal I on $\wp(X) \times \wp(Y)$ is called ideal binary topological space and it is denoted as $(X, Y, \mathcal{M}, \mathcal{J})$. For a binary subset (A, B) of $X \times Y$, we define the following set operator: $(\cdot)^*: \wp(X) \times \wp(Y) \rightarrow \wp(X) \times \wp(Y)$, is called a binary local function with respect to $\mathcal{M}$ and $\mathcal{J}$ is defined as follows: $(A, B)^*(\mathcal{J}, \mathcal{M}) = \{(x, y) \in (X, Y) : (U \cap A, V \cap B) \notin \mathcal{J} \text{ for every } (U, V) \in \mathcal{M}(x, y) \text{ where } \mathcal{M}(x, y) = \{(U, V) \in \mathcal{M} : (x, y) \in (U, V)\}\}$. In case there is no confusion $(A, B)^*(\mathcal{J}, \mathcal{M})$ is briefly denoted by $(A, B)^*$ and is called Binary local function of (A, B) with respect to $\mathcal{J}$ and $\mathcal{M}$. From [23], we have $C^*: \wp(X) \times \wp(Y) \rightarrow \wp(X) \times \wp(Y)$ is a Kuratowski closure operator. Therefore $\{(U, V) \subseteq (X, Y) : C^*[(X, Y) \setminus (U, V)] = (X, Y) \setminus (U, V)\}$ forms a binary topology on $X \times Y$, and it is denoted as $\mathcal{M}^*$.

Theorem 1.3 [23] Let $(X, Y, \mathcal{M}, \mathcal{J})$ be an ideal binary topological space. Then $\beta(\mathcal{M}, \mathcal{J}) = (V1, V2) \setminus I : (V1, V2)$ is a binary open set of $(X, Y, \mathcal{M})$, $I \in \mathcal{J}$ is a basis for $\mathcal{M}^*$.

Theorem 1.4 [4] Let $(\mathcal{B}^{\mathcal{M}}, I)$ be a space and (A, B) and (C, D) subsets of $\mathcal{M}$. Then

1. $(A, B) \subseteq (C, D) \Rightarrow (A, B)_b^* \subseteq (C, D)_b^*$,
2. $I \subseteq I' \Rightarrow (A, B)_b^*(I') \subseteq (A, B)_b^*(I)$,
3. $(A, B)_b^* = b-cl(A, B)_b^* \subseteq b-cl(A, B)[(A, B)_b^*]$ is b -closed of $b-cl(A, B)$,
4. $((A, B)_b^*)_b^* \subseteq (A, B)_b^*$,
5. $(A, B)_b^* \cup (C, D)_b^* = ((A, B) \cup (C, D))_b^*$,
6. $(U, V) \in \mathcal{M} \Rightarrow (U, V) \cap (A, B)_b^* = (U, V) \cap ((U, V) \cap (A, B))_b^* \subseteq ((U, V) \cap (A, B))_b^*$.

Definition 1.5 [4] Let $\mathcal{B}^{\mathcal{M}}$ be a binary topological space with an ideal I on (X, Y) . The operator $b-cl^*$ is called a binary $\star$ -closure and is defined as $b-cl^*(A, B) = (A, B) \cup (A, B)_b^*$ for $A \subseteq (X, Y)$.

Theorem 1.6 [4] The set operator $b-cl^*$ satisfies the following Properties:

1. $(A, B) \subseteq b-cl^*(A, B)$,
2. $b-cl^*(\phi, \phi) = \phi$ and $b-cl^*(X, Y) = (X, Y)$,
3. If $(A, B) \subset (C, D)$ then $b-cl^*(A, B) \subseteq b-cl^*(C, D)$,
4. $b-cl^*(A, B) \cup b-cl^*(C, D) = b-cl^*((A, B) \cup (C, D))$,
5. $b-cl^*(b-cl^*(A, B)) = b-cl^*(A, B)$.

Now, $\mathcal{M}^*(I, \mathcal{M}) = \{(U, V) \subseteq (X, Y) : b-cl^*((X, Y) - (U, V)) = (X, Y) - (U, V)\}$. $\mathcal{M}^*(I, \mathcal{M})$ is called binary $\star$ -topology which is finer than $\mathcal{M}$ (we can write $\mathcal{M}^*$ for $\mathcal{M}^*(I, \mathcal{M})$). The elements of $\mathcal{M}^*(I, \mathcal{M})$ are called binary $\star$ -open (simply, $b\star$ -open) and the complement of an $b\star$ -open is called

binary $b\star$ -closed (simply $b\star$ -closed). Here $b-cl^*(A, B)$ and $b-int^*(A, B)$ will denote the closure and interior of (A, B) in $((X, Y), \mathcal{M}^*)$.

Theorem 1.7 [4] Let $(\mathcal{B}^{\mathcal{M}}, I)$ be a space and (A, B) and (C, D) subsets of $\mathcal{M}$. Then

1. $(A, B) \subseteq (C, D) \Rightarrow (A, B)_b^* \subseteq (C, D)_b^*$,
2. $I \subseteq I' \Rightarrow (A, B)_b^*(I') \subseteq (A, B)_b^*(I)$,
3. $(A, B)_b^* = b-cl(A, B)_b^* \subseteq b-cl(A, B)[(A, B)_b^*]$ is b -closed of $b-cl(A, B)$,
4. $((A, B)_b^*)_b^* \subseteq (A, B)_b^*$,
5. $(A, B)_b^* \cup (C, D)_b^* = ((A, B) \cup (C, D))_b^*$,
6. $(U, V) \in \mathcal{M} \Rightarrow (U, V) \cap (A, B)_b^* = (U, V) \cap ((U, V) \cap (A, B))_b^* \subseteq ((U, V) \cap (A, B))_b^*$.

Definition 1.8 [23] Let $(X, Y, \mathcal{M}, I)$ be an ideal binary topological space. Then the subset (A, B) of $X \times Y$ is said to be $\star$ -dense in itself if $(A, B) \subseteq (A, B)^*$.

Lemma 1.9 [23] Let $(X, Y, \mathcal{M}, I)$ be an ideal binary topological space and $(A, B) \subseteq X \times Y$. If (A, B) is $b\star$ -dense in itself, then $(A, B)^* = b-cl(A, B)^* = b-cl(A, B) = cl^*(A, B)$.

Definition 1.10 [4] A subset (A, B) of an ideal binary topological space $(\mathcal{B}^{\mathcal{M}}, I)$ is $b\star$ -dense [23] int itself (resp. $b\star$ -perfect) if $(A, B) \subseteq (A, B)_b^*$ (resp. $(A, B) = (A, B)_b^*$).

Remark 1.11 [4] A subset (A, B) of an ideal binary topological space $(\mathcal{B}^{\mathcal{M}}, I)$ is $b\star$ -closed if and only if $(A, B)_b^* \subseteq (A, B)$.

Definition 1.12 [5] A subset (A, B) of space $(\mathcal{B}^{\mathcal{M}}, I)$ is said to be

1. binary α - I -open (briefly α - bI -open) if $(A, B) \subseteq b-int(b-cl^*(b-int(A, B)))$,
2. binary semi- I -open (briefly semi- bI -open) if $(A, B) \subseteq b-cl^*(b-int(A, B))$,
3. binary pre- I -open (briefly pre- bI -open) if $(A, B) \subseteq b-int(b-cl^*(A, B))$.

The complements of the above mentioned sets are called their respective closed sets.

Theorem 1.13 [5] A subset (V, W) of a space $(\mathcal{B}^{\mathcal{M}}, I)$ is semi- bI -open if and only if $b-cl^*(V, W) = b-cl^*(n-int(V, W))$.

Lemma 1.14 [5] Let $(\mathcal{B}^{\mathcal{M}}, I)$ be a space and (V, W) a subset of (X, Y) . If (G, H) is b -open in $(\mathcal{B}^{\mathcal{M}}, I)$, then $(G, H) \cap b-cl^*(V, W) \subseteq b-cl^*((G, H) \cap (V, W))$.

Proposition 1.15 [5] Intersection of a pre- bI -open sets and b -open sets is pre- bI -open.

Definition 1.16 [6] A subset (A, B) of an ideal binary space $(\mathcal{B}^{\mathcal{M}}, I)$ is called binary semi- $\star$ - I -open (briefly semi- $\star$ - bI -open) if $(A, B) \subseteq b-cl(b-int^*(A, B))$. The complement of a semi- $\star$ - bI -open set is said to be semi- $\star$ - bI -closed.

Definition 1.17 [18] A subset (A, B) of an ideal binary topological space $(\mathcal{B}^{\mathcal{M}}, I)$ is called a binary t - I -set (briefly t - bI -set) if $b-int(A, B) = b-int(b-$

$cl^*(A, B)$.

Nithyanantha Jothi, et al.[20] introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. Let X and Y be any two nonempty sets. A binary topology [20] from X to Y is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1. (ϕ, ϕ) and $(X, Y) \in \mathcal{M}$,
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and
3. If $\{(A_\alpha, B_\alpha) : \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha \in \delta} A_\alpha, \bigcup_{\alpha \in \delta} B_\alpha) \in \mathcal{M}$.

If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

Definition 1.18 [20] Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in \mathbb{P}(X) \times \mathbb{P}(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

Definition 1.19 [20] Let $(X, Y, \mathcal{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary closed in $(X, Y, \mathcal{M})$ if $(X \setminus A, Y \setminus B) \in \mathcal{M}$.

Proposition 1.20 [20] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$.

Proof:

Let $(A, B)^{1*} = \bigcap \{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A, B)^{2*} = \bigcap \{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A, B)^{1*}, (A, B)^{2*})$ is binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$.

Proposition 1.21 [20] Let $(X, Y, \mathcal{M})$ be a binary topological space and $(A, B) \subseteq (X, Y)$.

Proof:

Let $(A, B)^{1*} = \bigcup \{A_\alpha : (A_\alpha, B_\alpha) \text{ is binary open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$ and $(A, B)^{2*} = \bigcup \{B_\alpha : (A_\alpha, B_\alpha) \text{ is binary open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}$.

Definition 1.22 [20] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called the binary closure of (A, B) , denoted by $b-cl(A, B)$ in the binary space $(X, Y, \mathcal{M})$ where $(A, B) \subseteq (X, Y)$.

Definition 1.23 [20] The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ defined in proposition 1.21 is called the binary interior of (A, B) , denoted by $b-int(A, B)$. Here $((A, B)^{1*}, (A, B)^{2*})$ is binary open and $((A, B)^{1*}, (A, B)^{2*}) \subseteq (A, B)$.

Definition 1.24 [20] Let $(X, Y, \mathcal{M})$ be a binary

topological space and let $(x, y) \subseteq (X, Y)$. The binary open set (A, B) is said to be a binary neighbourhood of (x, y) if $x \in A$ and $y \in B$.

Proposition 1.25 [20] Let $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and $(X, Y, \mathcal{M})$ be a binary topological space. Then, the following statements hold:

1. $b-int(A, B) \subseteq (A, B)$.
2. If (A, B) is binary open, then $b-int(A, B) = (A, B)$.
3. $b-int(A, B) \subseteq b-int(C, D)$.
4. $b-int(b-int(A, B)) = b-int(A, B)$.
5. $(A, B) \subseteq b-cl(A, B)$.
6. If (A, B) is binary closed, then $b-cl(A, B) = (A, B)$.
7. $b-cl(A, B) \subseteq b-cl(C, D)$.
8. $b-cl(b-cl(A, B)) = b-cl(A, B)$.

Definition 1.26 [26] Let (A, B) be a subset of a binary topological space $(X, Y, \mathcal{M})$ is called a binary dense set, if $b-cl((A, B)) = (X, Y)$.

Definition 1.27 [26] A binary topological space $(X, Y, \mathcal{M})$ is called a binary submaximal space if every binary dense set is a binary open set.

2 On new subsets of $(\mathcal{B}^{\mathcal{M}}, I)$

Definition 2.1 A subset (V, W) of a space $(\mathcal{B}^{\mathcal{M}}, I)$ is called a

1. binary pre $^*-I$ -closed set (briefly pre $^*-bI$ -closed) if $b-cl^*(b-int(V, W)) \subseteq (V, W)$.
2. binary α^*-I -closed set (briefly α^*-bI -closed) if $b-cl^*(b-int(b-cl^*(V, W))) \subseteq (V, W)$.
3. binary $I-t$ -set (briefly $bI-t$ -set) if $b-int^*(V, W) = b-int^*(b-cl(V, W))$.
4. binary pre $^*-I$ -regular set (briefly pre $^*-bI$ -regular) if (V, W) is pre- bI -open and pre $^*-bI$ -closed.
5. binary $\mathcal{X}^*-I$ -set (briefly, $\mathcal{X}^*-bI$ -set) if $(V, W) = (L, M) \cap (N, O)$, where (L, M) is b^* -open set and (N, O) is $bI-t$ -set.

The complements of the above mentioned sets (1) and (2) are called their respective open sets.

In future an ideal binary topological spaces $(X, Y, \mathcal{M}, I)$ is referred as an ideal binary topological space $(\mathcal{B}^{\mathcal{M}}, I)$.

Example 2.2 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{2\}), (\{a\}, \{1\}), (\{a\}, Y), (X, \{1\}), (X, Y)\}$. Let the ideal be $I = \{(\phi, \phi), (\{a\}, \phi), (\{b\}, \phi), (X, \phi)\}$. In the space $(\mathcal{B}^{\mathcal{M}}, I)$, then

- pre $^*-bI$ -closed sets = $\{(\phi, \phi), (\phi, \{1\}), (\{a\}, \phi), (\{b\}, \phi), (\{b\}, \{1\}), (X, \phi), (X, \{2\}), (X, Y)\}$.
- α^*-bI -closed sets = $\{(\phi, \phi), (\{a\}, \phi), (\{b\}, \phi), (X, \phi), (X, \{2\}), (X, Y)\}$.
- $bI-t$ -sets = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\{a\}, \{1\}), (\{b\}, \phi), (\{b\}, \{1\}), (\{b\}, \{2\}), (X, \{1\}), (X, Y)\}$.
- $\mathcal{X}^*-bI$ -sets = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\phi, Y), (\{a\}, \{1\}), (\{a\}, Y), (\{b\}, \phi)\}$.

$(\{b\}, \{1\}), (\{b\}, \{2\}), (\{b\}, Y), (X, \{1\}), (X, Y)\}$.
 • pre $\star$ -bl-regular sets = $\{(\phi, \phi), (\phi, \{1\}), (X, \{2\}), (X, Y)\}$.

Proposition 2.3 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is $b \star$ -open $\Rightarrow (V, W)$ is pre $\star$ -bl-open,
2. (V, W) is b -open $\Rightarrow (V, W)$ is pre $\star$ -bl-open.

Proof:

1. Let (V, W) be a $b \star$ -open subset in (X, Y) . We have $(V, W) = b\text{-int}^*(V, W) \subseteq b\text{-int}^*(b\text{-cl}(V, W))$. Thus (V, W) is pre $\star$ -bl-open in (X, Y) .

2. Let (V, W) be a b -open subset. Then (V, W) is $b \star$ -open. By (1), (V, W) is pre $\star$ -bl-open.

Remark 2.4 The converse of (1) and (2) in Proposition 2.3 is need not to be true as shown in the following Example.

Example 2.5 In Example 2.2,

1. $(\{a\}, \{2\})$ is pre $\star$ -bl-open but not $b \star$ -open.
2. $(\phi, \{1\})$ is pre $\star$ -bl-open but not b -open.

Proposition 2.6 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, every bp-open set is pre $\star$ -bl-open. Let (V, W) be a bp-open subset in (X, Y) . We have $(V, W) \subseteq b\text{-int}(b\text{-cl}(V, W)) \subseteq b\text{-int}^*(b\text{-cl}(V, W))$. Thus (V, W) is pre $\star$ -bl-open in (X, Y) .

Remark 2.7 The converse of Proposition 2.6 is need not to be true as shown in the following Example.

Example 2.8 Let $X = \{a, b, c\}$, $Y = \{1, 2\}$ and $\mathcal{A} = \{(\phi, \phi), (\{a\}, \{1\}), (\{b\}, \phi), (\{b\}, \{2\}), (\{a, b\}, \{1\}), (\{a, b\}, Y), (X, Y)\}$. Let the ideal be $I = \{(\phi, \phi), (\{a\}, \{1\}), (\{c\}, \phi), (\{a, c\}, \{1\})\}$. In the space $(\mathcal{B}^{\mathcal{A}}, I)$, then the subset $(\{b, c\}, \{2\})$ is pre $\star$ -bl-open but not bp-open.

Theorem 2.9 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, a subset (V, W) of (X, Y) is semi-bl-closed in $(X, Y) \Leftrightarrow (V, W)$ is bl-t-set in (X, Y) .

Proof:

(V, W) is semi-bl-closed in $(X, Y) \Leftrightarrow (X, Y) - (V, W)$ is semi-bl-open in $(X, Y) \Leftrightarrow b\text{-cl}^*((X, Y) - (V, W)) = b\text{-cl}^*(b\text{-int}((X, Y) - (V, W)))$ by Theorem 1.13 $\Leftrightarrow (X, Y) - b\text{-int}^*(V, W) = b\text{-cl}^*((X, Y) - b\text{-cl}(V, W)) = (X, Y) - b\text{-int}^*(b\text{-cl}(V, W)) \Leftrightarrow b\text{-int}^*(V, W) = b\text{-int}^*(b\text{-cl}(V, W)) \Leftrightarrow (V, W)$ is bl-t-set in (X, Y) .

Remark 2.10 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results are hold for a (V, W) of (X, Y) .

1. (V, W) is $b \star$ -open $\Rightarrow$ a $\mathcal{X}^*$ -bl-set.
2. (V, W) is a bl-t-set $\Rightarrow$ a $\mathcal{X}^*$ -bl-set.

Remark 2.11 The converses of Remark 2.10 is need not to be true as shown in the following Example.

Example 2.12 In Example 2.2,

1. $(\{b\}, \{1\})$ is a $\mathcal{X}^*$ -bl-set but not $b \star$ -open.

2. $(\{a\}, Y)$ is a $\mathcal{X}^*$ -bl-set but not a bl-t-set.

Theorem 2.13 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results are equivalent.

1. (V, W) is $b \star$ -open.
2. (V, W) is pre $\star$ -bl-open and a $\mathcal{X}^*$ -bl-set.

Proof:

(1) $\Rightarrow$ (2): (2) follows by Proposition 2.3(1) and Remark 2.10(1).

(2) $\Rightarrow$ (1): Given (V, W) is $\mathcal{X}^*$ -bl-set. So $(V, W) = (L, M) \cap (N, O)$ where (L, M) is $b \star$ -open and $b\text{-int}^*(N, O) = b\text{-int}^*(b\text{-cl}(N, O))$. We have $(V, W) \subseteq (L, M) = b\text{-int}^*(L, M)$. Furthermore (V, W) is pre $\star$ -bl-open implies $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(V, W)) \subseteq b\text{-int}^*(b\text{-cl}(N, O)) = b\text{-int}^*(N, O)$ by assumption. Therefore $(V, W) \subseteq b\text{-int}^*(L, M) \cap b\text{-int}^*(N, O) = b\text{-int}^*((L, M) \cap (N, O)) = b\text{-int}^*(V, W)$ and hence (V, W) is $b \star$ -open.

Remark 2.14 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, the family of pre $\star$ -bl-open sets and the family of $\mathcal{X}^*$ -bl-sets are independent as shown in the following Example.

Example 2.15 In Example 2.2,

1. $(X, \{2\})$ is pre $\star$ -bl-open but not a $\mathcal{X}^*$ -bl-set.
2. $(\{b\}, \{1\})$ is a $\mathcal{X}^*$ -bl-set but not pre $\star$ -bl-open.

Proposition 2.16 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is $b \star$ -open $\Rightarrow (V, W)$ is α^* -bl-open.
2. (V, W) is b -open $\Rightarrow (V, W)$ is α^* -bl-open.

Proof:

1. Let (V, W) be a $b \star$ -open subset in (X, Y) . We have $(V, W) = b\text{-int}^*(V, W) \subseteq b\text{-cl}(b\text{-int}^*(V, W))$. It implies that $(V, W) = b\text{-int}^*(V, W) \subseteq b\text{-int}^*(b\text{-cl}(b\text{-int}^*(V, W)))$. Hence (V, W) is an α^* -bl-open in (X, Y) .

2. Let (V, W) be b -open subset (X, Y) . Then (V, W) is $b \star$ -open. By (1) (V, W) is α^* -bl-open.

Remark 2.17 The converse of Proposition 2.16 is need not to be true as shown in the following Example.

Example 2.18 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{A} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (X, Y)\}$. Let the ideal be $I = \{(\phi, \phi), (\phi, \{1\})\}$. In the space $(\mathcal{B}^{\mathcal{A}}, I)$, then the subset

1. $(\{a\}, \{2\})$ is an α^* -bl-open but not $b \star$ -open.
2. Also $(X, \{2\})$ is an α^* -bl-open but not b -open.

Proposition 2.19 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, every α^* -bI-open set is pre * -bI-open.

Proof:

Let (V, W) be an α^* -bI-open subset in (X, Y) . We have $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(b\text{-int}^*(V, W))) \subseteq b\text{-int}^*(b\text{-cl}(V, W))$. Hence (V, W) is pre * -bI-open in (X, Y) .

Remark 2.20 The converse of Proposition 2.19 is need not to be true as shown in the following Example.

Example 2.21 In Example 2.2, $(\{a\}, \{2\})$ is pre * -bI-open but not an α^* -bI-open.

Proposition 2.22 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, every $b\alpha$ -open set is α^* -bI-open.

Proof:

Let (V, W) be a $b\alpha$ -open subset in (X, Y) . We have $(V, W) \subseteq b\text{-int}(b\text{-cl}(b\text{-int}(V, W))) \subseteq b\text{-int}^*(b\text{-cl}(b\text{-int}^*(V, W)))$. Hence (V, W) is an α^* -bI-open set in (X, Y) .

Remark 2.23 The converse of Proposition 2.22 is need not to be true as shown in the following Example.

Example 2.24 In Example 2.2, $(\{a\}, \emptyset)$ is an α^* -bI-open set but not $b\alpha$ -open.

Theorem 2.25 Let $(\mathcal{B}^{\mathcal{A}}, I)$ be a space and $(V, W) \subseteq (X, Y)$. Then the following results are hold.

1. $(V, W) = b\text{-cl}^*(b\text{-int}(V, W)) \Rightarrow (V, W)$ is α^* -bI-closed.

2. $(V, W) = b\text{-cl}(b\text{-int}(V, W)) \Rightarrow (V, W)$ is α^* -bI-closed.

Proof:

1. If $(V, W) = b\text{-cl}^*(b\text{-int}(V, W))$, we obtain that $b\text{-cl}^*(b\text{-int}(b\text{-cl}^*(V, W))) = b\text{-cl}^*(b\text{-int}(b\text{-cl}^*(b\text{-int}(V, W)))) = b\text{-cl}^*(b\text{-int}(V, W)) = (V, W)$. Hence (V, W) is a α^* -bI-closed set.

2. If $(V, W) = b\text{-cl}(b\text{-int}(V, W))$, we obtain that $b\text{-cl}^*(b\text{-int}(b\text{-cl}^*(V, W))) = b\text{-cl}^*(b\text{-int}(b\text{-cl}^*(b\text{-cl}(b\text{-int}(V, W)))) \subseteq b\text{-cl}(b\text{-int}(b\text{-cl}(b\text{-int}(V, W)))) = b\text{-cl}(b\text{-int}(V, W)) = (V, W)$. Thus (V, W) is α^* -bI-closed.

Remark 2.26 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is pre * -bI-regular $\Rightarrow (V, W)$ is pre-bI-open.

2. (V, W) is pre * -bI-regular $\Rightarrow (V, W)$ is pre * -bI-closed.

Remark 2.27 The converses of Remark 2.26 is need not to be true as shown in the following Examples.

Example 2.28 In Example 2.2,

1. $(\emptyset, Y)$ is pre-bI-open but not pre * -bI-regular.

2. $(\{b\}, \{1\})$ is pre * -bI-closed but not pre * -bI-regular.

Proposition 2.29 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, every α^* -bI-open set is semi * -bI-open.

Proof:

Let (V, W) be an α^* -bI-open subset in

(X, Y) . Then $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(b\text{-int}^*(V, W))) \subseteq b\text{-cl}(b\text{-int}^*(V, W))$. Thus (V, W) is semi * -bI-open in (X, Y) .

Remark 2.30 The converse of Proposition 2.29 is need not to be true as shown in the following Example.

Example 2.31 In Example 2.2, $(\{a\}, \{1\})$ is semi * -bI-open set but not α^* -bI-open.

Theorem 2.32 If a subset (V, W) of (X, Y) in a space $(\mathcal{B}^{\mathcal{A}}, I)$ is α^* -bI-open $\Leftrightarrow (V, W)$ is semi * -bI-open and pre * -bI-open.

Proof:

Let (V, W) be an α^* -bI-open set in (X, Y) . We have $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(b\text{-int}^*(V, W)))$. It follows that $(V, W) \subseteq b\text{-cl}(b\text{-int}^*(V, W))$ and $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(V, W))$. Hence (V, W) is semi * -bI-open and pre * -bI-open.

Conversely, suppose that (V, W) is semi * -bI-open and pre * -bI-open in (X, Y) . We have $(V, W) \subseteq b\text{-cl}(b\text{-int}^*(V, W))$ and $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(V, W))$. It follows that $(V, W) \subseteq b\text{-int}^*(b\text{-cl}(b\text{-int}^*(V, W)))$ which implies that (V, W) is an α^* -bI-open in (X, Y) .

Remark 2.33 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, then the family of semi * -bI-open sets and the family of pre * -bI-open sets are independent. As shown in the following Example.

Example 2.34 In Example 2.2,

1. $(\{b\}, \{1\})$ is semi * -bI-open but not pre * -bI-open.

2. $(\{a\}, \{2\})$ is pre * -bI-open but not semi * -bI-open.

Remark 2.35 An α^* -bI-open subsets of (X, Y) in $(\mathcal{B}^{\mathcal{A}}, I)$ is decomposed into a semi * -bI-open set and a pre * -bI-open set.

3 On new an ideal binary sets

Definition 3.1 A subset (V, W) of a space $(\mathcal{B}^{\mathcal{A}}, I)$ is called a

1. binary $\mathcal{Z}_p^*$ -I-set (briefly $\mathcal{Z}_p^*$ -bI-set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b-open and (T, S) is pre * -bI-regular.

2. binary $\mathcal{Z}^{**}$ -I-set (briefly $\mathcal{Z}^{**}$ -bI-set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b^* -open and Q is pre * -bI-regular.

3. binary I- $\mathcal{Z}$ -set (briefly bI- $\mathcal{Z}$ -set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b-open and (T, S) is pre * -bI-closed.

4. binary I- $\mathcal{Z}_p^*$ -set (briefly bI- $\mathcal{Z}_p^*$ -set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b^* -open and (T, S) is pre * -bI-closed.

5. binary α -I-set (briefly α -bI-set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b-open and (T, S) is α^* -bI-closed.

6. binary α^* -I-set (briefly α^* -bI-set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b^* -open

and (T, S) is α^* - bl -closed.

7. binary $\mathcal{K}^*$ - I -set (briefly $\mathcal{K}^*$ - bl -set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b -open and $(T, S) = b-cl^*(b-int(T, S))$.

8. binary $\mathcal{K}^{**}$ - I -set (briefly $\mathcal{K}^{**}$ - bl -set) if $(V, W) = (O, P) \cap (T, S)$, where (O, P) is a b $\star$ -open and $(T, S) = b-cl(b-int(T, S))$.

Example 3.2 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (X, Y)\}$. Let the ideal be $I = \{(\phi, \phi), (\phi, \{1\})\}$. In the space $(\mathcal{B}^{\mathcal{M}}, I)$, we have

- Z_p^* - bl -sets = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\phi, Y), (\{a\}, \phi), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (\{b\}, \{2\}), (\{b\}, Y), (X, \phi), (X, \{1\}), (X, Y)\}$.

- Z^{**} - bl -sets = $\{(\phi, \phi), (\phi, \{2\}), (\phi, Y), (\{a\}, \phi), (\{a\}, \{1\}), (\{b\}, \{2\}), (\{b\}, Y), (X, \phi), (X, \{1\}), (X, \{2\}), (X, Y)\}$.

- bl - Z -sets = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\phi, Y), (\{a\}, \phi), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (\{b\}, \phi), (\{b\}, \{1\}), (\{b\}, \{2\}), (\{b\}, Y), (X, \phi), (X, \{1\}), (X, Y)\}$.

- bl - Z_p^* -sets = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\phi, Y), (\{a\}, \phi), (\{a\}, \{1\}), (\{b\}, \phi), (\{b\}, \{1\}), (\{b\}, \{2\}), (\{b\}, Y), (X, \phi), (X, \{1\}), (X, \{2\}), (X, Y)\}$.

- α - bl -sets = $\{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (\{b\}, \phi), (\{b\}, \{1\}), (X, Y)\}$.

- α^* - bl -sets = $\{(\phi, \phi), (\phi, \{1\}), (\{b\}, \phi), (\{b\}, \{1\}), (X, \{2\}), (X, Y)\}$.

- $\mathcal{K}^*$ - bl -sets = $\{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (X, Y)\}$.

- $\mathcal{K}^{**}$ - bl -sets = $\{(\phi, \phi), (\phi, \{1\}), (X, \{2\}), (X, Y)\}$.

Remark 3.3 In a space $(\mathcal{B}^{\mathcal{M}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is b -open $\Rightarrow (V, W)$ is a Z_p^* - bl -set.

2. (V, W) is pre $\star$ - bl -regular $\Rightarrow (V, W)$ is a Z_p^* - bl -set.

Remark 3.4 The converses of Remark 3.3 is need not to be true as shown in the following Example.

Example 3.5 In Example 3.2,

1. $(\{b\}, \{2\})$ is a Z_p^* - bl -set but not b -open.

2. $(\{a\}, Y)$ is a Z_p^* - bl -set but not pre $\star$ - bl -regular.

Proposition 3.6 In a space $(\mathcal{B}^{\mathcal{M}}, I)$, every Z_p^* - bl -set is pre- bl -open.

Proof:

Let (V, W) be a Z_p^* - bl -set in (X, Y) . It follows that $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b -open and (T, S) is pre $\star$ - bl -regular in (X, Y) . By Remark 2.26(1), (T, S) is pre- bl open. Since (T, S)

is pre- bl -open, by Proposition 1.15, $(V, W) = (O, P) \cap (T, S)$ is pre- bl -open set in (X, Y) .

Remark 3.7 The converse of Proposition 3.6 is need not to be true as shown in the following Example.

Example 3.8 In Example 3.2, $(X, \{2\})$ is pre- bl -open but not Z_p^* - bl -set.

Remark 3.9 The following diagram holds for any subset (V, W) of (X, Y) in a space $(\mathcal{B}^{\mathcal{M}}, I)$.

pre $\star$ - bl -regular

↓

Z_p^* - bl -set $\rightarrow$ pre- bl -open

Remark 3.10 In a space $(\mathcal{B}^{\mathcal{M}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is pre $\star$ - bl -closed $\Rightarrow (V, W)$ is a bl - Z -set,

2. (V, W) is α^* - bl -closed $\Rightarrow (V, W)$ is a α - bl -set.

Remark 3.11 The converse of (1) and (2) in Remark 3.10 is need not to be true as shown in the following Example.

Example 3.12 In Example 3.2,

1. $(\{a\}, \{2\})$ is a bl - Z -set but not pre $\star$ - bl -closed.

2. $(\{a\}, Y)$ is a α - bl -set but not α^* - bl -closed.

Lemma 3.13 For a subset (V, W) of (X, Y) if $(V, W) = b-cl^*(b-int(V, W))$, then (V, W) is $\mathcal{K}^*$ - bl -set.

Proposition 3.14 In a space $(\mathcal{B}^{\mathcal{M}}, I)$, every b -open set is $\mathcal{K}^*$ - bl -set.

Remark 3.15 The converse of Proposition 3.14 is need not to be true as shown in the following Example.

Example 3.16 In Example 2.2, $(X, \{2\})$ is $\mathcal{K}^*$ - bl -set but not b -open.

Remark 3.17 The following diagram holds for any subset (V, W) of a space $(\mathcal{B}^{\mathcal{M}}, I)$.

Z_p^* - bl -set	$\rightarrow$	bl - Z -set
		$\uparrow$
$\mathcal{K}^*$ - bl -set	$\rightarrow$	α - bl -set

Remark 3.18 The reverse implications of Remark 3.17 is need not to be true as shown in the following Example.

Example 3.19 In Example 3.2,

1. $(\{b\}, \phi)$ is a bl - Z -set but not a Z_p^* - bl -set.

2. $(\{a\}, \{1\})$ is a bl - Z -set but not a α - bl -set.

3. $(\{b\}, \{1\})$ is a α - bl -set but not a $\mathcal{K}^*$ - bl -set.

Theorem 3.20 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{M}}, I)$, the following results are equivalent.

1. (V, W) is a bl - Z -set and semi- bl -open in (X, Y) .

2. $(V, W) = (O, P) \cap b - cl^*(b -$

$\text{int}(V, W)$ for b -open in (O, P) .

Proof:

(1) $\Rightarrow$ (2) : Since (V, W) is bl - $\mathcal{Z}$ -set, $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b -open and (T, S) is pre $\star$ - bl -closed in (X, Y) . We have $(V, W) \subseteq (O, P)$ and $b - cl^*(b - \text{int}(V, W)) \subseteq b - cl^*(b - \text{int}(O, P)) \subseteq (O, P)$ since (O, P) is pre $\star$ - bl -closed in (X, Y) . Since (V, W) is semi- bl -open in (X, Y) , we have $(V, W) \subseteq b - cl^*(b - \text{int}(V, W))$. It follows that $(V, W) = (V, W) \cap b - cl^*(b - \text{int}(V, W)) = (O, P) \cap (T, S) \cap b - cl^*(b - \text{int}(V, W)) = (O, P) \cap b - cl^*(b - \text{int}(V, W))$.

(2) $\Rightarrow$ (1): Let $(V, W) = (O, P) \cap b - cl^*(b - \text{int}(V, W))$ for b -open set (O, P) . Then $(V, W) \subseteq b - cl^*(b - \text{int}(V, W))$ and thus (V, W) is semi- bl -open in (X, Y) . Since $b - cl^*(b - \text{int}(V, W))$ is $b \star$ -closed set, by Proposition 2.3(1), it is pre $\star$ - bl -closed set in (X, Y) . Hence (V, W) is bl - $\mathcal{Z}$ -set in (X, Y) .

Theorem 3.21 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{U}}, I)$, the following results are equivalent.

1. (V, W) is a $\mathcal{Z}^*$ - bl -set.
2. (V, W) is semi- bl -open and a α - bl -set.
3. (V, W) is semi- bl -open and a bl - $\mathcal{Z}$ -set.

Proof:

(1) $\Rightarrow$ (2): Suppose that (V, W) is $\mathcal{Z}^*$ - bl -set in (X, Y) . It follows that $(V, W) = (O, P) \cap (T, S)$, where (O, P) is b -open set and $(T, S) = b - cl^*(b - \text{int}(T, S))$. This implies that $(V, W) = (O, P) \cap (T, S) = (O, P) \cap b - cl^*(b - \text{int}(T, S)) \subseteq b - cl^*((O, P) \cap b - \text{int}(T, S))$ (by Lemma 1.14) $= b - cl^*(b - \text{int}(O, P) \cap b - \text{int}(T, S)) \subseteq b - cl^*(b - \text{int}(O, P) \cap b - \text{int}(T, S)) = b - cl^*(b - \text{int}((O, P) \cap (T, S))) = b - cl^*(b - \text{int}(V, W))$. Hence $(V, W) \subseteq b - cl^*(b - \text{int}(V, W))$ and hence (V, W) is semi- bl -open set. Moreover, by Remark 3.17, (V, W) is an α - bl -set in (X, Y) .

(2) $\Rightarrow$ (3): It follows from the fact that each α - bl -set is bl - $\mathcal{Z}$ -set in (X, Y) by Remark 3.17.

(3) $\Rightarrow$ (1): Suppose (V, W) is semi- bl -open and bl - $\mathcal{Z}$ -set in (X, Y) . By Theorem 3.20, $(V, W) = (O, P) \cap b - cl^*(b - \text{int}(V, W))$ for b -open set (O, P) . We have $b - cl^*(b - \text{int}(b - cl^*(b - \text{int}(V, W)))) = b - cl^*(b - \text{int}(V, W))$. It follows that (V, W) is $\mathcal{Z}^*$ - bl -set in (X, Y) .

Remark 3.22 In a space $(\mathcal{B}^{\mathcal{U}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is $b \star$ -open $\Rightarrow (V, W)$ is a $\mathcal{Z}^{**}$ - bl -set.
2. (V, W) is pre $\star$ - bl -regular $\Rightarrow (V, W)$ is a $\mathcal{Z}^{**}$ - bl -set.

Remark 3.23 The converses of (1) and (2) in Remark 3.22 are need not to be true as shown in the following Example.

Example 3.24 In Example 3.2,

1. (X, ϕ) is a $\mathcal{Z}^{**}$ - bl -set but not $b \star$ -open.
2. $(X, \{2\})$ is a $\mathcal{Z}^{**}$ - bl -set but not pre $\star$ - bl -regular.

Remark 3.25 In a space $(\mathcal{B}^{\mathcal{U}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is pre $\star$ - bl -closed $\Rightarrow (V, W)$ is a bl - $\mathcal{Z}_p^*$ -set.
2. (V, W) is α^* - bl -closed $\Rightarrow (V, W)$ is a α^* - bl -set.

Remark 3.26 The converses of (1) and (2) in Remark 3.25 is need not to be true as shown in the following Example.

Example 3.27 In Example 3.2,

1. $(X, \{2\})$ is a bl - $\mathcal{Z}_p^*$ -set but not pre $\star$ - bl -closed.
2. $(X, \{2\})$ is an α^* - bl -set but not α^* - bl -closed.

Lemma 3.28 For a subset (V, W) of (X, Y) if $(V, W) = b - cl(b - \text{int}(V, W))$, then (V, W) is $\mathcal{Z}^{**}$ - bl -set.

Remark 3.29 The following diagram holds for any subset (V, W) of a space $(\mathcal{B}^{\mathcal{U}}, I)$.

$\mathcal{Z}^{**}$ - bl -set	$\rightarrow$	bl - $\mathcal{Z}_p^*$ -set
		$\uparrow$
$\mathcal{K}^{**}$ - bl -set	$\rightarrow$	α^* - bl -set

Remark 3.30 The reverse implications of Remark 3.29 is need not to be true as shown in the following Example.

Example 3.31 In Example 3.2,

1. $(\phi, \{1\})$ is a bl - $\mathcal{Z}_p^*$ -set but not a $\mathcal{Z}^{**}$ - bl -set.
2. $(\{b\}, \{2\})$ is a bl - $\mathcal{Z}_p^*$ -set but not a α^* - bl -set.

Example 3.32 In Example 3.2, $(\{b\}, \phi)$ is a α^* - bl -set but not a $\mathcal{Z}^{**}$ - bl -set.

Proposition 3.33 In a space $(\mathcal{B}^{\mathcal{U}}, I)$, every $b \star$ -open set is $\mathcal{K}^{**}$ - bl -set.

Remark 3.34 The converse of Proposition 3.33 is need not to be true as shown in the following Example.

Example 3.35 In Example 3.2, $(\phi, \{1\})$ is $\mathcal{K}^{**}$ - bl -set but not $b \star$ -open.

Theorem 3.36 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{U}}, I)$, the following results are equivalent.

1. (V, W) is $b \star$ -open.
2. (V, W) is α^* - bl -open and a $\mathcal{K}^{**}$ - bl -set.
3. (V, W) is pre $\star$ - bl -open and a $\mathcal{K}^{**}$ - bl -set.

Proof:

(1) $\Rightarrow$ (2): (2) follows by Proposition 2.16(1) and Proposition 3.33.

(2) $\Rightarrow$ (3): (3) follows by Proposition 2.19.

(3) $\Rightarrow$ (1): Suppose (V, W) is pre $\star$ - bI -open and $\mathcal{K}^{\star\star}$ - bI -set. Since (V, W) is $\mathcal{K}^{\star\star}$ - bI -set, we have $(V, W) = (O, P) \cap (T, S)$, where (O, P) is $b \star$ -open set and $(T, S) = b-cl(b-int(T, S))$. It follows that $b-int^*(b-cl(T, S)) \subseteq b-cl(T, S) \subseteq b-cl(T, S) = b-cl(b-cl(b-int(T, S))) = b-cl(b-int(T, S)) = (T, S)$. This implies that (T, S) is semi- bI -closed. By Theorem 2.9, (T, S) is bI - t -set in (X, Y) . By Definition 2.1(5), (V, W) is $\mathcal{X}^*$ - bI -set. Since (V, W) is pre $\star$ - bI -open and $\mathcal{X}^*$ - bI -set, (V, W) is $b \star$ -open by Theorem 2.13.

Remark 3.37 The following Examples shows that

1. The family of α^* - bI -open and the family of $\mathcal{K}^{\star\star}$ - I_b -sets are independent in $(\mathcal{B}^{\mathcal{A}}, I)$.
2. The family of pre $\star$ - bI -open and the family of $\mathcal{K}^{\star\star}$ - I_b -sets are independent in $(\mathcal{B}^{\mathcal{A}}, I)$.

Example 3.38 In Example 3.2,

1. $(\phi, \{1\})$ is a $\mathcal{K}^{\star\star}$ - bI -set but not α^* - bI -open.
2. $(\{a\}, \{2\})$ is α^* - bI -open but not $\mathcal{K}^{\star\star}$ - bI -set.
3. $(\phi, \{1\})$ is a $\mathcal{K}^{\star\star}$ - bI -set but not pre $\star$ - bI -open.
4. $(\{b\}, \{2\})$ is pre $\star$ - bI -open but not a $\mathcal{K}^{\star\star}$ - bI -set.

Remark 3.39

1. $b \star$ -open subsets of (X, Y) in $(\mathcal{B}^{\mathcal{A}}, I)$ is decomposed into an α^* - bI -open and $\mathcal{K}^{\star\star}$ - bI -set.
2. $b \star$ -open subsets of (X, Y) in $(\mathcal{B}^{\mathcal{A}}, I)$ is decomposed into pre $\star$ - bI -open and $\mathcal{K}^{\star\star}$ - bI -set

Theorem 3.40 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results are equivalent.

1. (V, W) is a bI - Z_p^* -set and semi- bI -open in (X, Y) .
 2. $(V, W) = (O, P) \cap b-cl^*(b-int(V, W))$ for a $b \star$ -open set in (O, P) .
- Proof:

(1) $\Rightarrow$ (2): Since (V, W) is bI - Z_p^* -set, $(V, W) = (O, P) \cap (T, S)$, where (O, P) is $b \star$ -open and (T, S) is pre $\star$ - bI -closed in (X, Y) . We have $(V, W) \subseteq (O, P)$ and $b-cl^*(b-int(V, W)) \subseteq b-cl^*(b-int(O, P)) \subseteq (O, P)$ since (O, P) is pre $\star$ - bI -closed in (X, Y) . Since (V, W) is semi- bI -open in (X, Y) , we have $(V, W) \subseteq b-cl^*(b-int(V, W))$. It follows that $(V, W) = (V, W) \cap b-cl^*(b-int(V, W)) = (O, P) \cap (T, S) \cap b-cl^*(b-int(V, W)) = (O, P) \cap b-cl^*(b-int(V, W)) = (O, P) \cap b-cl^*(b-int(V, W))$.

(2) $\Rightarrow$ (1): Let $(V, W) = (O, P) \cap b-cl^*(b-int(V, W))$ for $b \star$ -open in (O, P) . Then $(V, W) \subseteq b-cl^*(b-int(V, W))$ and thus (V, W) is semi- bI -open in (X, Y) . Since $b-cl^*(b-int(V, W))$ is $b \star$ -closed, by Proposition 2.3(1), it is pre $\star$ - bI -closed in

(V, W) . Hence, (V, W) is bI - Z_p^* -set in (X, Y) .

4 Different types of an ideal binary locally closed sets

Definition 4.1 A subset (V, W) of a space $(\mathcal{B}^{\mathcal{A}}, I)$ is called a

1. binary locally I -closed set (briefly bI -LC) if $(V, W) = (R, P) \cap (O, N)$, where (R, P) is b -open and (O, N) is b -closed.
2. binary locally I^* -closed set (briefly bI^* -LC) if $(V, W) = (R, P) \cap (O, N)$, where (R, P) is $b \star$ -open and (O, N) is b -closed.
3. binary $\star$ - I -locally closed set (briefly $\star$ - bI -LC) if $(V, W) = (R, P) \cap (O, N)$ where (R, P) is $b \star$ -open and (O, N) is $b \star$ -closed.
4. lightly binary I -locally closed set (briefly $\mathcal{L}$ - bI -LC) if $(V, W) = (R, P) \cap (O, N)$ where (R, P) is b -open and (O, N) is $b \star$ -closed.
5. binary I^* -submaximal (briefly bI^* -submaximal) if every b -dense subset of (X, Y) is $b \star$ -open.

Example 4.2 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{A} = \{(\phi, \phi), (\phi, \{1\}), (\{a\}, \{1\}), (\{b\}, \{1\}), (X, \{1\}), (X, Y)\}$. Let the ideal be $I = \{(\phi, \phi), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{a\}, Y)\}$. In the space $(\mathcal{B}^{\mathcal{A}}, I)$, we have

- bI -LC = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\{a\}, \phi), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{b\}, \phi), (\{b\}, \{1\}), (\{b\}, \{2\}), (X, \{1\}), (X, Y)\}$.
- bI^* -LC = $\{(\phi, \phi), (\phi, \{2\}), (\{a\}, \{2\}), (\{b\}, \phi), (\{b\}, \{1\}), (\{b\}, \{2\}), (X, Y)\}$.
- $\star$ - bI -LC = $\{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (\{b\}, \phi), (\{b\}, \{1\}), (\{b\}, \{2\}), (X, Y)\}$.
- $\mathcal{L}$ - bI -LC = $\{(\phi, \phi), (\phi, \{2\}), (\{a\}, \phi), (\{a\}, \{1\}), (\{a\}, \{2\}), (\{a\}, Y), (\{b\}, \phi), (\{b\}, \{2\}), (X, \phi), (X, \{2\}), (X, Y)\}$.

Remark 4.3 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results hold for a subset (V, W) of (X, Y) .

1. (V, W) is $b \star$ -open $\Rightarrow$ (V, W) is bI^* -LC.
2. (V, W) is b -closed $\Rightarrow$ (V, W) is bI^* -LC.

Remark 4.4 The converses of (1) and (2) in Remark 4.3 is need not to be true as shown in the following Example.

Example 4.5 In Example 4.2,

1. $(\{a\}, \{2\})$ is bI^* -LC but not $b \star$ -open.
2. $(\{b\}, \{1\})$ is bI^* -LC but not b -closed.

Theorem 4.6 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results are equivalent.

1. (V, W) is $\mathcal{L}$ - bI -LC.
2. $(V, W) = (G, H) \cap b-cl^*(V, W)$ for some b -open in (G, H) .

3. $b - cl^*(G, H) - (G, H) = (G, H)_b^* - (G, H)$ is b -closed.

4. $((X, Y) - (V, W)_b^*) \cup (V, W) = (V, W) \cup ((V, W) - b-cl^*(V, W))$ is b -open.

5. $(V, W) \subseteq b-int((V, W) \cup ((X, Y) - (V, W)_b^*))$.

Proof:

(1) $\Rightarrow$ (2): If (V, W) is $\mathcal{L}$ - bl -LC, then there exist b -open (G, H) and b^* -closed, (K, T) such that $(V, W) = (G, H) \cap (K, T)$. But $(V, W) \subseteq (G, H) \cap b-cl^*(V, W)$. Since (K, T) is b^* -closed, $b-cl^*(V, W) \subseteq b-cl^*(K, T) = (K, T)$ and so $(G, H) \cap b-cl^*(V, W) \subseteq (G, H) \cap (K, T) = (V, W)$. Therefore $(V, W) = (G, T) \cap b-cl^*(V, W)$.

(2) $\Rightarrow$ (3): Now $(V, W)_b^* - (V, W) = (V, W)_b^* \cap ((X, Y) - (V, W)) = (V, W)_b^* \cap ((X, Y) - ((G, H) \cap b - cl^*(V, W))) = (V, W)_b^* \cap ((X, Y) - (G, H))$.

Therefore, $(V, W)_b^* - (V, W)$ is b -closed.

(3) $\Rightarrow$ (4): Since $(X, Y) - ((V, W)_b^* - (V, W)) = ((X, Y) - (V, W)_b^*) \cup (V, W)$, $((X, Y) - (V, W)_b^*) \cup (V, W)$ is b -open. But, $((X, Y) - (V, W)_b^*) \cup (V, W) = (V, W) \cup ((X, Y) - b - cl^*(V, W))$.

(4) $\Rightarrow$ (5): it is clear.

(5) $\Rightarrow$ (1): $(X, Y) - (V, W)_b^* = b - int((X, Y) - (V, W)_b^*) \subseteq b - int((V, W) \cup ((X, Y) - (V, W)_b^*))$ which implies that $(V, W) \cup ((X, Y) - (V, W)_b^*) \subseteq b - int((V, W) \cup ((X, Y) - (V, W)_b^*))$ and so $(V, W) \cup ((X, Y) - (V, W)_b^*)$ is b -open. Since $(V, W) = ((V, W) \cup ((X, Y) - (V, W)_b^*)) \cap b-cl^*(V, W)$, (V, W) is $\mathcal{L}$ - bl -LC.

Theorem 4.7 For a subset (V, W) of a space $(\mathcal{B}^{\mathcal{A}}, I)$, the following results are equivalent.

1. (V, W) is b^* -open.
2. (V, W) is pre * - bl -open and bl^* -LC.

Proof:

(1) $\Rightarrow$ (2): (2) follows by the Proposition 2.3(1) and the Remark 4.3(1).

(2) $\Rightarrow$ (1): Given (V, W) is locally bl^* -closed. So $(V, W) = (R, P) \cap (O, N)$ where (R, P) is b^* -open and $(O, N) = b - cl(O, N)$. Then $(V, W) \subseteq (R, P) = bl^*(R, P)$. Also (V, W) is pre * - bl -open implies $(V, W) \subseteq b-int^*(b-cl(V, W)) \subseteq b-int^*(b-cl(O, N)) = bl^*(O, N)$ by assumption. Thus $(V, W) \subseteq b - int^*(R, P) \cap b - int^*(O, N) = b - int^*((R, P) \cap (O, N)) = b - int^*(V, W)$ and hence (V, W) is b^* -open.

Remark 4.8 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, then the family of pre * - bl -open and the family of locally bl^* -closed (bl^* -LC) sets are independent. As shown in the following Example.

Example 4.9 In Example 4.2, then

1. $(\{a\}, \{1\})$ is pre * - bl -open but not bl^* -LC.

2. $(\phi, \{2\})$ is bl^* -LC but not pre * - bl -open.

Remark 4.10 b^* -open subsets of (X, Y) in $(\mathcal{B}^{\mathcal{A}}, I)$ is decomposed into a pre * - bl -open but not bl^* -LC.

Proposition 4.11 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, every b -LC is bl^* -LC. It follows from the facts that b -closed and each b -open is b^* -open.

Proposition 4.12 The converse of Proposition 4.11 is need not to be true as shown in the following Example.

Example 4.13 In Example 2.2, $(\{b\}, Y)$ is bl^* -LC but not b -LC.

Proposition 4.14 In a space $(\mathcal{B}^{\mathcal{A}}, I)$, every b -submaximal is bl^* -submaximal.

Proof:

Let (V, W) be b -dense in (X, Y) . we have $(X, Y) = b-cl(V, W) \subseteq b-cl(V, W)$ and $(X, Y) = b-cl(V, W)$. Thus (V, W) is b -dense in (X, Y) . Since (X, Y) is b -submaximal, (V, W) is b -open and hence b^* -open in (X, Y) . Hence, (X, Y) is bl^* -submaximal.

Remark 4.15 The converse of Proposition 4.14 is need not to be true as shown in the following Example.

Example 4.16 Let $X = \{a, b\}$, $Y = \{1, 2\}$ and $\mathcal{A} = \{(\phi, \phi), (\phi, \{2\}), (\{a\}, \{1\}), (\{a\}, Y), (X, \{1\}), (X, Y)\}$. Let I be an ideal $\{\phi(X) \times \phi(Y)\}$. In $(\mathcal{B}^{\mathcal{A}}, I)$, every b^* -dense subsets in (X, Y) is b^* -open. So $(\mathcal{B}^{\mathcal{A}}, I)$ is bl^* -submaximal. For $(\{a\}, \{2\})$, $b-cl(\{a\}, \{2\}) = (X, Y)$. Therefore $(\{a\}, \{2\})$ is b -dense in (X, Y) and $(\{a\}, \{2\})$ is not b -open. Thus $(\mathcal{B}^{\mathcal{A}}, I)$ is not bl -submaximal.

5 Conclusion

In this work, we analyzed the concepts of $\mathcal{K}^*$ - bl -sets, bl - $\mathcal{Z}$ -sets, $\mathcal{K}^{**}$ - bl -sets, I - $\mathcal{Z}_p^*$ -sets, $\mathcal{Z}^{**}$ - bl -sets and $\mathcal{Z}_p^*$ - bl -sets in an ideal binary topological spaces. Also the notions of pre * - bl -closed sets, ba^* -closed sets, bl - t -sets, pre * - bl -regular sets, locally closed sets namely bl -LC sets, bl^* -LC sets, * - bl -LC sets, $\mathcal{L}$ - bl -LC sets are given their properties are studied. The characterizations of a bl - t -set and a $\mathcal{K}^*$ - bl -set are established. Additionally, new types of decompositions for b^* -open sets are presented in five different forms, along with α^* - bl -open sets.

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