RESEARCH PAPER

Advances in Hexagonal Graph Topology and Its Potential Applications

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Abstract: The application of hexa graph topological spaces spans across various disciplines where the spatial arrangement and connectivity of elements are essential. It offers a different perspective compared to traditional topological spaces based on square or triangular grids, potentially leading to new insights and optimizations in diverse fields.

Keywords: Hexa graph Topology, Hexa Discrete Topology

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Introduction and preliminaries

Hexa graph topology offers versatile applications across various domains, leveraging its geometric properties and connectivity advantages. Whether in spatial analysis, image processing, network optimization, simulation modeling, or other fields, hexagonal grids provide effective tools for representing and analyzing interconnected data structures in a structured and efficient manner. As technology advances, the adoption of hexa graph topology is expected to continue growing, further enhancing our capabilities in understanding and manipulating complex systems and datasets.

Definition 1. *Let G be a undirected graph with six disjoint graph decomposition* $J_h(s)$ *where* $h = 1,2,3,4,5,6$ *of* G. *Consider* $S_i, i = 1,2,3,4,5,6$ *be any six subgraphs of* G , *then we define a lower hexa subgraph and upper hexa subgraph as* $H_-(S_i) = \bigcup_{s\in G}\{s: J_1(s) \subseteq S_1 \vee J_2(s) \subseteq S_2 \vee J_3(s) \subseteq S_3 \vee J_4(s) \subseteq S_4 \vee J_5(s) \subseteq S_5 \vee J_6(s) \subseteq S_6\}$, $H^-(S_i) = \bigcup_{s\in G}\{s: J_1(s) \cap S_1 \neq \phi \wedge J_2(s) \cap S_2 \neq \phi \wedge J_3(s) \cap S_3 \neq \phi \wedge J_4(s) \cap S_4 \neq \phi \wedge J_5(s) \cap S_5 \neq \phi \wedge J_6(s) \cap S_6 \neq \phi\}$.*The boundary hexa subgraph is defined as* $B_N(S) = H^-(S_i) - H_-(S_i)$.

Definition 2. *Let G be the non-empty undirected graph and* $J_h(s), h = 1,2,3,4,5,6$ *be any six disjoint graph decomposition of* G.*Then for any six subgraphs* $S_i, i = 1,2,3,4,5,6$ *of G and* $\zeta_H(S_i)$= *{ϕ,G,H_{-}(S_i), H^{-}(S_i), B_h(S_i)\}. Let* $S_i \subseteq G, \zeta_H(S_i)$ *satisfies the following axioms*

1. $G, V_o \in \zeta_H(S_i)$, *where* G = *full graph with edge set,* V_o = *Null graph*
2. *Any union of elements of* $\zeta_H(S_i)$ *is in* $\zeta_H(S_i)$
3. *The finite intersection of the elements of* $\zeta_H(S_i)$ *is in* $\zeta_H(S_i)$.

Then a pair $\zeta_H(S_i)$ *is known as Hexa graph topology on G with regard to a subgraph* S_i *of* G.*We call the pair* $(G, \zeta_H(S_i))$ *as the Hexa graph topological space on* G.*The elements of Hexa graph topological spaces are regard as Hexa open subgraph of G and the complement of the Hexa open subgraph of G is called an Hexa closed subgraph of* G.

Definition 3. *Let* $S_i, i = 1,2,3,4,5,6$ *be any six subgraphs of* $G, \zeta_H(S_i) = \{\phi, G\}$ *be a collection of the trivial subgraphs of* G, *then* $\zeta_H(S_i)$ *is a Hexa graph topology on G and is said to be the Indiscrete Hexa graph topology on* G.

Definition 4. *Let G be a non-empty undirected graph and for a six subgraph* S_i *of* G. *If* $\zeta_H(S_i)$ *is the Hexa Graph topology on G concerning to* S_i , *then* $B = \{G, \mathrm{H}_-(S_i), \mathrm{B}_h(S_i)\}$ *is the basis for* $\zeta_H(S_i)$.

Definition 5. *Let* $\zeta_H(S_i)$ *be a Hexa graph topological space concerning to six subgraph* S_i *of* G. *The Hexa graph interior of a subgraph P of G is described as the union of all Hexa open subgraph which is an edge induced subgraph of P and it is identified by* $\mathfrak{N}_h Int(P)$. *That is* $\mathfrak{N}_h Int(P)$ *is the*

greatest Hexa open subgraph of P. The Hexa graph closure of P is specific as the intersection of all Hexa closed subgraph which is a supergraph of P and it is identified by $\mathfrak{N}_h Cl(P)$. *Therefore,* $\mathfrak{N}_h Cl(P)$ *is the lowest Hexa closed subgraph of P.*

2 Applications and Future Research Directions

Social Media

Social media has revolutionized online communication, enabling individuals to create and share content at an unprecedented rate. This transformation is reshaping how people interact and disseminate information, driven by its ease of use, speed, and extensive reach. Hexa graph topology can offer an intriguing perspective on how social media networks are structured and how information flows within them. While social media platforms typically rely on graph structures (such as social graphs or interest graphs), hexa graph topology introduces a structured and interconnected pattern that could potentially enhance certain aspects of social media platforms.

Figure 1 -Social Media



Network System

The field of networking extensively utilizes topological graph theory and to analyze the application of Topological graph theory in networking, two main approaches are typically considered: the utilization of graph-based representations and the application of network theory. Topological graph based representation offers several advantages, such as providing diverse perspectives, simplifying complex issues, and offering more precise definitions. Network theory offers numerous tools for studying graphs and applying network analysis techniques using graph representations. The terms "graph" and "network" can be considered synonymous both concepts refer to a particular framework in which there are vertices (such as hubs or specks) and edges (such as joins or lines). There exist diverse types of graphs and networks that exhibit a wide range of structural characteristics[8]. These two terms can be distinguished based on their respective utility. In the field of mathematics, the term "graph" is employed, whereas in the field of material science, the term "network" is utilized.Hexa graph topology can be a fascinating concept to explore and potentially apply in the context of network systems, offering unique advantages and considerations compared to traditional network topologies.

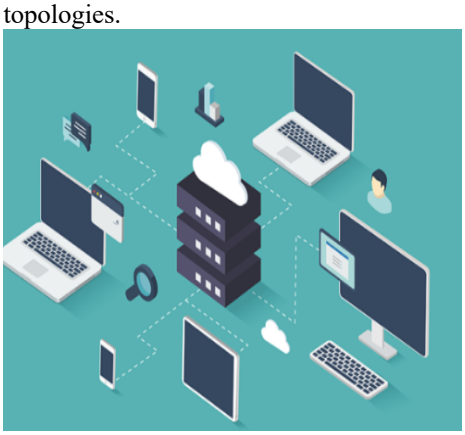


Figure 2 -Network System

Data Structure

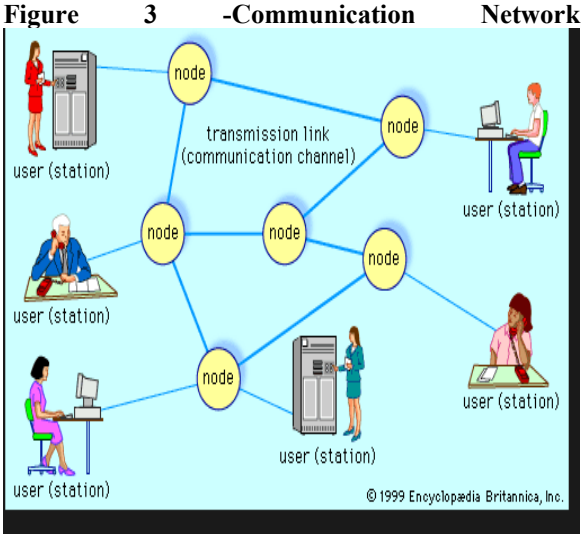
Data can be organized in multiple ways and the formal or mathematical representation of a particular organization of data is referred to as a "data structure". The choice of a data model is contingent upon two considerations: it must possess a sufficient level of complexity in its structure to accurately represent the inherent relationships between data in the real world. The structure should be sufficiently elementary to enable effective data processing when necessary. The graph theoretical concepts effectively address these contemplations. The discretionary association between data can also be represented by a graph and its lattice. Operations conducted on these measurements are additionally beneficial for deducing relationships and data affiliation [19]. This is valuable for comprehending how this data may be stored in memory.Hexa graph topology in data structures offers a versatile and efficient way to represent spatial relationships and connectivity in various applications, including GIS, image processing, network routing, and more. While it introduces some complexity in implementation, its benefits in terms of efficiency, natural spatial representation, and regular structure make it a valuable approach in computational geometry and related fields.

Communication Network

Topological graph theoretical concepts are employed in various computer applications such as data mining,

image segmentation, clustering, image recognition, networking, and so forth. The application of topological graph theory can be employed to represent and analyze communication networks. A communications network refers to a collection of terminals, connections, and hubs that facilitate telecommunications between the users of these terminals. Each terminal within the network must possess a unique location in order to ensure that messages or connections are appropriately directed to their intended recipients. The collection of addresses within a network is commonly referred to as the location space. Every communication network consists of three essential components: terminals (the points where the network begins and ends), processors (which facilitate data transmission control functions), and transmission channels (which facilitate the transmission of data). The communication network intends to facilitate the transmission of data bundles among various devices such as computers, phones, processors, and other gadgets. The term "parcel" refers to a specific quantity of data, typically consisting of either 256 bytes or 4096 bytes. The parcels undergo transmission from the source to the destination through various switches. Communication networks can be analyzed using various mathematical structures, which also enable us to study different representations based on congestion, switch size, and switch count. Graphs play a crucial role in the representation of communication networks. In the context of graphs, vertices typically represent terminals or processors, while edges represent transmission channels such as wires or filaments[20]. The pathway through which data is transmitted. In this manner, a data bundle traverses the network from an information terminal, through a network of switches connected by coordinated edges, to an output terminal.transmitted. In this manner, a data bundle traverses the network from an information terminal, through a network of switches connected by coordinated edges, to an output terminal.Hexa graph topology can be a compelling model for designing and optimizing communication networks, offering unique advantages in terms of connectivity, resilience, and efficiency.Hexa graph topology, with its structured and interconnected node relationships in a hexagonal lattice pattern, can be an intriguing concept to explore in the context of data structures. While traditional data structures like arrays, linked lists, trees, and graphs are more commonly used, hexa graph topology offers unique advantages and challenges that can be beneficial in specific scenarios.

Figure 3 -Communication Network

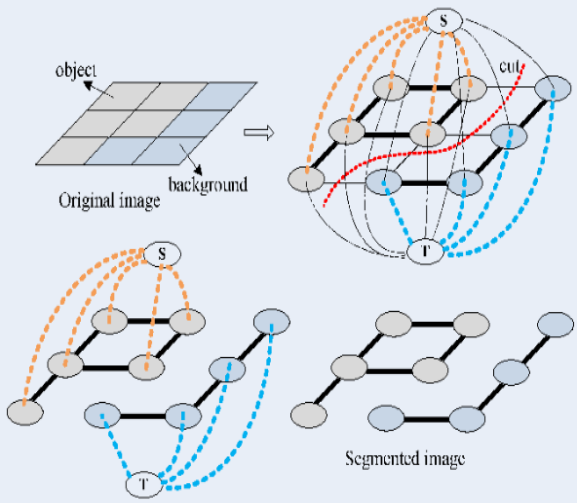


Graph Coloring

Topological graph coloring is a widely employed technique in various areas of computer science, including data mining, image segmentation, clustering, image recognition, and networking. For instance, a tree can be represented as a data structure consisting of vertices and edges. Additionally, the visualization of network typologies can be achieved through the application of topological graph theory principles. The concept of graph coloring is also employed in asset allocation and booking, serving as a crucial idea in this context. Similarly, in the field of topological graph theory, methods such as paths, walks, and circuits find extensive applications in various domains including the traveling salesperson problem, concepts related to database structures, and resource networking. This phenomenon stimulates the development of novel algorithms and hypotheses that can be effectively employed in extensive applications. The concept of graph coloring holds great significance in the field of topological graph theory and finds extensive application in various ongoing computer science endeavors. The proper coloring of a graph refers to the assignment of colors to its vertices and edges in a manner that minimizes the number of colors used, while ensuring that no two vertices share the same color. The fundamental quantity denoting the number of colors required to properly color a graph is referred to as the chromatic number, while the graph itself is appropriately termed a shaded graph. The problem of vertex coloring is widely acknowledged as one of the most prominent issues in graph coloring. The problem at hand involves finding a solution for coloring the vertices of a graph using a given set of m colors, such that no two adjacent vertices are assigned the same color. Other graph coloring problems, such as edge coloring (where no vertex is incident to two edges of the same

color) and face coloring (also known as geographical map coloring), can be transformed into vertex coloring[21]. The chromatic number of a graph, denoted as the minimum number of colors required to color its vertices such that no adjacent vertices share the same color, is a fundamental concept in topological graph theory.Hexa graph topology can provide a unique perspective and potential optimizations for graph coloring problems, which are fundamental in various fields such as scheduling, register allocation in compilers, frequency assignment in wireless networks, and more.

Figure 4 -Image segmentation



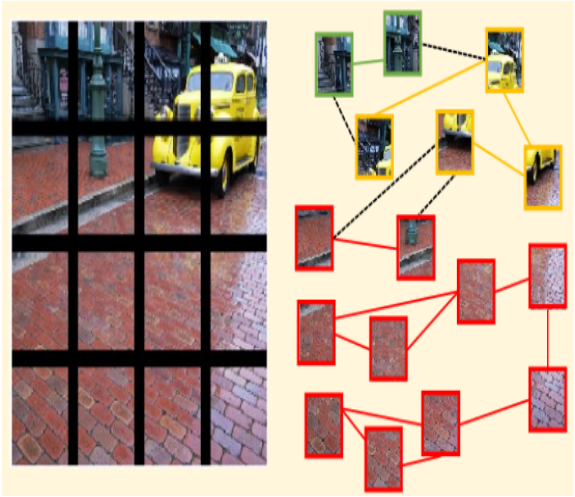
Operating system

A topological graph can be defined as a data structure that consists of a finite collection of sets, referred to as edges and vertices, which are organized in a specific manner. The field of operating systems benefits from the use of topological graph to address various pragmatic issues, such as job planning and resource allocation problems. The concept of topological graph coloring can be utilized in the context of job scheduling problems involving CPUs[24]. Each job is represented as a vertex in the graph, and an edge is established between two jobs that cannot be executed simultaneously. Furthermore, a one-to-one correspondence exists between feasible graph colorings and valid job schedules.Hexa graph topology principles can be conceptually applied in operating systems to enhance resource management, scheduling algorithms, and overall system performance.

Image Processing

Image analysis is the process through which information is extracted from images and the analysis of images is primarily conducted using advanced techniques in the field of digital image processing. The utilization of a graph theoretic methodology has the potential to enhance the performance of image processing techniques. Hexa graph topology offers promising avenues for enhancing image processing techniques by leveraging its natural spatial representation, efficient neighbor relationships, and potential for improved compression and analysis algorithms. While adoption may require overcoming implementation challenges, the benefits in accuracy, efficiency, and visual appeal make hexagonal grids a compelling option for certain types of image processing tasks.

Figure 5 -Image Processing



Data Base Designing

Topological graph databases are commonly employed in database planning for the purpose of graph representation. The utilization of graph database involves the representation and storage of data through the use of nodes, edges, and properties in a graphical format. The topological graph structure plays a crucial role in database planning as it enables efficient execution processes by leveraging various functionalities and properties inherent to graph structures. Topological graph databases are often more efficient for collaborative datasets that directly align with the architecture of object-oriented applications.Hexa graph topology in database designing offers innovative possibilities for representing and managing spatial data, network structures, and connectivity patterns. While it introduces challenges in implementation and maintenance, the benefits in spatial representation, query efficiency, and storage optimization make it a compelling choice for applications where spatial relationships are central to the database design.

Website Designing Hexa graph topology offers a fresh and innovative approach to website designing, leveraging its aesthetic appeal, responsive capabilities, and potential for enhancing user interaction and engagement. While implementation challenges exist, the creative possibilities and benefits in visual design,

navigation clarity, and content presentation make hexagonal grids a compelling choice for designers looking to create distinctive and engaging web experiences.

Hexa Discrete Topology

In discrete topology, hexa graph topology refers to the use of hexagonal grids or networks to study and analyze topological properties in a discrete space. Hexa graph topology provides a structured and geometrically meaningful approach to discrete topology, enabling the study and analysis of topological properties in a wide range of applications. By leveraging the regularity and connectivity of hexagonal grids, researchers and practitioners can develop sophisticated models and algorithms that enhance our understanding of discrete spaces and their topological characteristics. As computational tools and methodologies evolve, hexa graph topology is likely to continue playing a significant role in advancing the fields of discrete mathematics, computational geometry, and simulation modeling.

Future Research Directions

The methods suggested in this work can be extended and utilized in the following research areas to get a better solution for many real life situations:

1. Extending the concept of a hexa graph topological space using lattices and ideals involves incorporating lattice theory and order theory into the framework of hexa graphs.
2. Investigating hexa graph topology in the context of image processing involves utilizing the hexa graph structure to enhance and clarify images through various filters, noise reduction techniques, and edge adjustments.
3. Analyzing the hexa graph topology in the context of a bipartite graph or a spanning tree with factorization involves understanding how the hexa graph structure can represent and manipulate these graph types.
4. Extending hexa graph topology methods to neutrosophic environments involves adapting the framework to accommodate the principles and uncertainties inherent in neutrosophy
5. converting hexa graph topology to hexa discrete topology involves configuring how nodes are connected using both adjacent and non-adjacent edges. This approach enhances the representation and applicability of hexa graph structures in diverse fields requiring spatial analysis, network modeling, and optimization.
6. Characterize and analyse the domination parameters on the Hexa graph topology.

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  Authour 3 -Review and give some idea to improve the Quality of the Article
  Authour 4,5 - Review and Editing the Manuscript
- Compliance with Ethical Standards: This article does not contain any studies with human participants or animals performed by any of the authors.
- Conflict of interest/Competing interests: Author 1 declares that she has no conflict of interest.
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