RESEARCH PAPER

Connected Fuzzy Majority Domination Number of a Fuzzy Graph

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ABSTRACT
This paper defines the concepts of connected fuzzy majority dominating sets and the connected fuzzy majority domination number of a fuzzy graph G. Characterizations of minimal connected fuzzy majority dominating sets are established along with γ_{CM} – values for certain classes of fuzzy graphs. Furthermore, several bounds for the connected fuzzy majority domination number of G are derived and the relationships between the fuzzy domination parameters $\gamma_M(G)$, $\gamma_{CM}(G)$ and $\gamma_C(G)$ are studied.
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1. Introduction

Graph theory is used to model a wide range of problems. The graph theory deals with the values that are certain. But the real world is full of uncertainties. To address such situations, A. Rosenfeld introduced the "fuzzy graph" in 1975 [2]. In 1962, Ore and Berge introduced the concept of domination in graphs. In 1979, Sampathkumar and Walikar discussed the connected domination in graphs[3]. The domination in fuzzy graphs using effective edges was discussed by Somasundaram and Somasundaram in 1998[4]. In 2007, Nagoorgani and Chandrasekaran investigated domination applying strong arcs in fuzzy graphs [5]. Manjusha and Sunitha introduced the connected domination in fuzzy graphs using strong arcs in 2015[6]. The idea of the connected majority dominating set of a graph was presented by Joseline Manora and Muthukani Vairavel in 2017 [7]. In this paper, the connected fuzzy majority dominating set of a fuzzy graph has been presented. Also, the relationships between the fuzzy domination parameters $\gamma_M(\mathcal{G}), \gamma_{CM}(\mathcal{G})$ and $\gamma_C(\mathcal{G})$ are studied. The connected fuzzy majority domination numbers for certain classes of fuzzy graphs are found. Characterizations of minimal connected fuzzy majority dominating sets are established. Furthermore, several bounds for the connected fuzzy majority domination number of $\mathcal{G}$ are derived. The results are discussed using strong arcs.

2. Preliminaries

Definition 2.1[1]. A fuzzy graph (FG) $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ is a non-empty set $\mathcal{V}$ together with a pair of functions $\sigma : \mathcal{V} \to [0,1]$ and $\mu : \mathcal{V} \times \mathcal{V} \to [0,1]$ such that for all $v_i, v_j \in \mathcal{V}$, $\mu(v_i, v_j) \leq \min\{\sigma(v_i), \sigma(v_j)\}$ where $\sigma(v_i)$ and $\mu(v_i, v_j)$ represent the membership values of the vertex v_i and edge (v_i, v_j) in $\mathcal{G}$.

Definition 2.2[1]. Let $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ be a fuzzy graph. The order p and size q of $\mathcal{G}$ are defined by $p = \sum_{v_i \in \mathcal{V}} \sigma(v_i)$ and $q = \sum_{(v_i, v_j) \in \mathcal{V} \times \mathcal{V}} \mu(v_i, v_j)$.

Definition 2.3[1]. Let $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ be a fuzzy graph and $S \subseteq \mathcal{V}$. Then the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.

Definition 2.4[8]. The strength of the connectedness between two vertices u, v in a fuzzy graph $\mathcal{G}$ is $\mu^{\infty}(u, v) = \sup\{\mu^k(u, v): k = 1, 2, 3, \dots\}$ where $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \mu(u_1, u_2) \dots \wedge \mu(u_{k-1}, v)\}$. An arc (u, v) is said to be a strong arc or strong edge if $\mu(u, v) \geq \mu^{\infty}(u, v)$ and the vertex v is said to be strong neighbor of u. Otherwise, it is called weak arc or weak edge. A vertex is said to be isolated if $\mu(u, v) = 0$, for all $u \neq v$.

Definition 2.5[8]. In a fuzzy graph every arc is a strong arc, then the graph is called strong arc fuzzy

graph. Let u be a node in fuzzy graph $\mathcal{G}$ then $N(u) = \{v: (u,v)\ is\ a\ strong\ arc\}$ is called neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called closed neighborhood of u.

Definition 2.6[8]. Let $\mathcal{G}$ be a fuzzy graph and S be a subset of $\mathcal{V}$. A node v is said to be fuzzy private neighbor of $u \in S$ with respect to S if $N[v] \cap S = \{u\}$.

Definition 2.7[9]. Let $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$be a fuzzy graph with order p and size q. A subset $S \subseteq \mathcal{V}(\mathcal{G})$ of vertices in a fuzzy graph $\mathcal{G}$ is called a fuzzy majority dominating set if at least half of the order of vertices of $\mathcal{V}(\mathcal{G})$ are either in the cardinality of vertices of S or the cardinality of vertices of $N[S]$.

$$(i.e)|N[S]| = \sum_{v_i \in N[S]} \sigma(v_i) \geq \frac{p}{2}$$

Definition 2.8[9]. A fuzzy majority dominating (FMD) set S of $\mathcal{G}$ is minimal if no proper subset of S is fuzzy majority dominating set.

Definition 2.9[9]. The minimum fuzzy cardinality of a minimal fuzzy majority dominating set is called a fuzzy majority domination number. It is denoted by $\gamma_M(\mathcal{G})$ (Lower fuzzy majority domination number).

Definition 2.10[11]. A dominating set D of a fuzzy graph $\mathcal{G}$ is connected dominating set if the induced fuzzy subgraph $\mathcal{H} = \langle D \rangle$ is connected. The minimum cardinality of a connected dominating set of $\mathcal{G}$ is called the connected dominating number of $\mathcal{G}$ and is denoted by $\gamma_C(\mathcal{G})$.

Definition 2.11[6]. A connected fuzzy graph $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ is called a fuzzy tree if it has a fuzzy spanning subgraph $F: (\sigma, \nu)$, which is a tree [spanning tree], where for all arcs (x, y) not in F there exists a path from x to y in F whose strength is more than $\mu(x, y)$.

Definition 2.12[6]. A maximum spanning tree of a connected fuzzy graph $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ is a fuzzy spanning subgraph $T: (\sigma, \nu)$, such that T is a tree and for which $\sum_{u \neq v} \nu(u, v)$ is maximum.

Definition 2.13[10]. Let $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ be a fuzzy graph and $S \subseteq \mathcal{V}$. A vertex $u \in S$ is said to be enclave of S if $\mu(u, v) < \sigma(u) \wedge \sigma(v)$ for all $v \in (\mathcal{V} - S)$ that is $N[u] \subseteq S$.

3. Definitions and Relations between $\gamma_M(\mathcal{G}), \gamma_{CM}(\mathcal{G})$ and $\gamma_C(\mathcal{G})$

Definition 3.1. A set $S \subseteq \mathcal{V}(\mathcal{G})$ is a connected fuzzy majority dominating (CFMD) set if the set S is a fuzzy majority dominating set and the subgraph $\langle S \rangle$ induced by S is connected in $\mathcal{G}$.

Definition 3.2. The connected fuzzy majority dominating set S is minimal if no proper subset of S is a connected fuzzy majority dominating set.

Definition 3.3. The minimum cardinality of a minimal connected fuzzy majority dominating set is called the connected fuzzy majority domination number and is denoted by $\gamma_{CM}(\mathcal{G})$.

The maximum cardinality of a minimal connected fuzzy majority dominating set of $\mathcal{G}$ is called the upper connected fuzzy majority domination number of $\mathcal{G}$, denoted by $\Gamma_{CM}(\mathcal{G})$.

Proposition 3.4. *For any connected fuzzy graph $\mathcal{G}$, $\gamma_{CM}(\mathcal{G}) \leq \gamma_C(\mathcal{G})$, where $\gamma_C(\mathcal{G})$ is the connected domination number of $\mathcal{G}$.*

Proof. Since every connected fuzzy dominating set is a connected fuzzy majority dominating set of $\mathcal{G}$, $\gamma_{CM}(\mathcal{G}) \leq \gamma_C(\mathcal{G})$.

Remark 3.5. If $\mathcal{G}$ is a complete fuzzy graph then $\gamma_{CM}(\mathcal{G}) = \gamma_C(\mathcal{G})$.

Example 3.6.

$\mathcal{G}$:

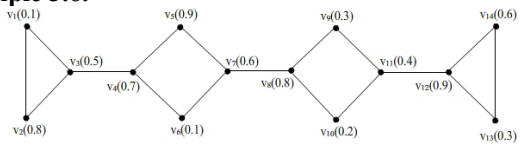


Figure 1.

Consider the strong arc fuzzy graph $\mathcal{G}$ with $p = 7.2$ and $\frac{p}{2} = 3.6$. All the minimal connected fuzzy majority dominating sets of $\mathcal{G}$ are as follows:

$S_1 = \{v_3, v_4, v_6\}, S_2 = \{v_4, v_6, v_7\}, S_3 = \{v_6, v_7, v_8\}, S_4 = \{v_7, v_8, v_{10}, v_{11}\}$,

$S_5 = \{v_8, v_{10}, v_{11}, v_{12}\}, S_6 = \{v_2, v_3, v_4, v_5\}$ with $|S_1| = 1.3, |S_2| = 1.4, |S_3| = 1.5$, $|S_4| = 2.0, |S_5| = 2.3, |S_6| = 2.9$.

Thus $\gamma_{CM}(\mathcal{G}) = |S_1| = 1.3$ and $\Gamma_{CM}(\mathcal{G}) = |S_6| = 2.9$

The minimal connected dominating sets of $\mathcal{G}$ are $D_1 = \{v_3, v_4, v_5, v_7, v_8, v_9, v_{11}, v_{12}\}$, $D_2 = \{v_3, v_4, v_6, v_7, v_8, v_{10}, v_{11}, v_{12}\}$ with $|D_1| = 5.1, |D_2| = 4.2$

Therefore $\gamma_C(\mathcal{G}) = |D_2| = 4.2$. Hence $\gamma_{CM}(\mathcal{G}) < \gamma_C(\mathcal{G})$.

Proposition 3.7. *For any connected fuzzy graph $\mathcal{G}$, $\gamma_M(\mathcal{G}) \leq \gamma_{CM}(\mathcal{G}) \leq \gamma_C(\mathcal{G})$.*

Proof. Since every connected fuzzy majority dominating set is a fuzzy majority dominating set of a fuzzy graph $\mathcal{G}$, $\gamma_M(\mathcal{G}) \leq \gamma_{CM}(\mathcal{G})$. By Proposition (3.4), $\gamma_{CM}(\mathcal{G}) \leq \gamma_C(\mathcal{G})$. Hence $\gamma_M(\mathcal{G}) \leq \gamma_{CM}(\mathcal{G}) \leq \gamma_C(\mathcal{G})$.

Example 3.8.

1. Let $\mathcal{G} = K_\sigma$ be a complete fuzzy graph. Then $\gamma_M(\mathcal{G}) = \gamma_{CM}(\mathcal{G}) = \gamma_C(\mathcal{G})$.
2. Let $\mathcal{G} = P_7$ with $p = 3.5$ and $\frac{p}{2} = 1.75$. The minimum fuzzy majority dominating set is $S_1 = \{v_2, v_4\}$ and $\gamma_M(\mathcal{G}) = 0.4$. The minimum connected fuzzy majority dominating set is $S_2 = \{v_3, v_4\}$ and $\gamma_{CM}(\mathcal{G}) = 1.2$. The minimum connected dominating set is $S_3 = \{v_2, v_3, v_4, v_5, v_6\}$ and $\gamma_C(\mathcal{G}) = 2.6$. Therefore $\gamma_M(\mathcal{G}) < \gamma_{CM}(\mathcal{G}) < \gamma_C(\mathcal{G})$.

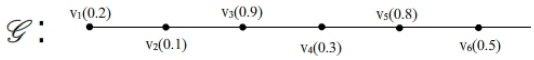


Figure 2.

Proposition 3.9. *Let $v_i \in \mathcal{V}(\mathcal{G})$ be a full degree vertex with $\sigma(v_i) = minimum$. Then $\gamma_C(\mathcal{G}) = \sigma(v_i)$ if and only if $\gamma_{CM}(\mathcal{G}) = \sigma(v_i)$.*

Proof. Let $\gamma_C(\mathcal{G}) = \sigma(v_i)$ and $|N[v_i]| = p$. Then $\gamma_{CM}(\mathcal{G}) = \sigma(v_i)$. The converse is obvious.

Proposition 3.10. *If $\gamma_C(\mathcal{G}) = \min_{v_i \in \mathcal{V}} \sigma(v_i)$ then $\gamma_{CM}(\mathcal{G}) = \min_{v_i \in \mathcal{V}} \sigma(v_i)$. The converse is not true.*

Proof. If $\gamma_C(\mathcal{G}) = \min_{v_i \in \mathcal{V}} \sigma(v_i)$ then $t\,deg_{SN}(v_i) = p$. It implies that $|N[v_i]| = p > \frac{p}{2}$. Therefore $\gamma_{CM}(\mathcal{G}) = \min_{v_i \in \mathcal{V}} \sigma(v_i)$.

Suppose $\gamma_{CM}(\mathcal{G}) = \min_{v_i \in \mathcal{V}} \sigma(v_i)$. Let $S = \{v_i\}$ be the γ_{CM}- set of $\mathcal{G}$ and $|N[S]| \geq \frac{p}{2}$ but $t\,deg_{SN}(v_i) \neq p$. Hence $\gamma_C(\mathcal{G}) \neq \min_{v_i \in \mathcal{V}} \sigma(v_i)$.

The following example shows that the converse of the previous result is not true.

Example 3.11.

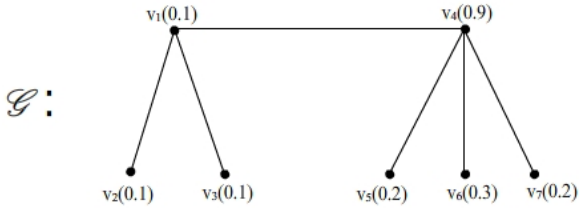


Figure 3.

Consider the strong arc fuzzy graph with $p = 1.9$ and $\frac{p}{2} = 0.95$. The connected fuzzy majority dominating set is $S_1 = \{v_1\}$ and $|N[v_1]| = 1.2 > \frac{p}{2}$. The fuzzy connected dominating set is $S_2 = \{v_1, v_4\}$ and $|N[S_2]| = p$. Therefore S_1 and S_2 are the connected fuzzy majority dominating set and connected fuzzy dominating set of $\mathcal{G}$ respectively. Hence $\gamma_{CM}(\mathcal{G}) = \min_{v_i \in \mathcal{V}} \sigma(v_i) = 0.1$ and $\gamma_C(\mathcal{G}) = 1.0 \neq \min_{v_i \in \mathcal{V}} \sigma(v_i)$.

Observation 3.12. $\gamma(\mathcal{G}) = \gamma_{CM}(\mathcal{G}) = \gamma_C(\mathcal{G}) = \gamma_M(\mathcal{G})$ if and only if the fuzzy graph $\mathcal{G}$ has a vertex v with $\sigma(v) =$ minimum and $t\,deg_{SN}(v) = p$.

Proposition 3.13. *Any connected fuzzy majority dominating set of $\mathcal{G}$ is a fuzzy majority dominating set of $\mathcal{G}$.*

Proof. Since any connected fuzzy majority dominating set S satisfies the condition $|N[S]| \geq \frac{p}{2}$, S is a fuzzy majority dominating set of $\mathcal{G}$.

The converse of the previous result need not be true as shown in the following example.

Example 3.14.

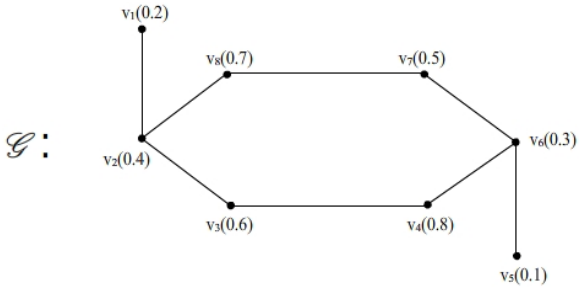


Figure 4.

Consider the strong arc fuzzy graph $\mathcal{G}$ with $p = 3.6$ and $\frac{p}{2} = 1.8$. Here $S = \{v_1, v_6\}$ is a minimal fuzzy majority dominating set, but not a connected fuzzy majority dominating set, since the induced fuzzy subgraph $\langle S \rangle$ is not connected.

Theorem 3.15. *Let $\mathcal{G}$ be a connected fuzzy graph having no full degree vertex. Then $\gamma_{CM}(\mathcal{G}) \leq \frac{\gamma_C(\mathcal{G})}{2}$.*

Proof. Let S be a minimum connected fuzzy dominating set of $\mathcal{G}$. Then $|N[S]| = |\mathcal{V}(\mathcal{G})| = p$. Let $S = S_1 \cup S_2$ such that $|S_1| \geq \frac{\gamma_C(\mathcal{G})}{2}$ and $|S_2| \leq \frac{\gamma_C(\mathcal{G})}{2}$ ..... (1)

Suppose $N[S] = \{N[S_1] - N[S_2]\} \cup N[S_2]$. Then $|N[S]| = \{|N[S_1]| - |N[S_2]|\} \cup |N[S_2]|$. Since $|N[S]| = p, \{|N[S_1]| - |N[S_2]|\} \geq \frac{p}{2}$ or $N[S_2] \geq \frac{p}{2}$.

Case i. If $\{|N[S_1]| - |N[S_2]|\} \geq \frac{p}{2}$ then $|N[S_1]| \geq \frac{p}{2}$. Since $\{N[S_1] - N[S_2]\} \subseteq N[S_1]$, S_1 is a connected fuzzy majority dominating set but not a minimal connected fuzzy majority dominating set of $\mathcal{G}$.

Case ii. If $\{|N[S_1]| - |N[S_2]|\} < \frac{p}{2}$ then S_1 is not a connected fuzzy majority dominating set of $\mathcal{G}$.

Therefore $|N[S_2]| \geq \frac{p}{2}$ and $\langle S_2 \rangle$ is connected in $\mathcal{G}$. Hence $\gamma_{CM}(\mathcal{G}) = |S_2| \leq \frac{\gamma_C(\mathcal{G})}{2}$, by (1). Hence $\gamma_{CM}(\mathcal{G}) \leq \frac{\gamma_C(\mathcal{G})}{2}$.

4. Connected Fuzzy Majority Domination Number for Classes of Fuzzy Graphs

Proposition 4.1. *Let $\mathcal{G} = K_\sigma$ be a complete fuzzy graph. Then $\gamma_{CM}(\mathcal{G}) = \wedge\{\sigma(v_i) : v_i \in \mathcal{V}\}$.*

Proof. Since $\mathcal{G}$ is a complete fuzzy graph, all edges are strong and each vertex is adjacent to all other vertices. Hence $S = \{v_i\}$ is a connected fuzzy majority dominating set for each $v_i \in \mathcal{V}$. Hence the result follows.

Proposition 4.2. *Let $\mathcal{G} = K_{m,n}, m \leq n$ be a complete bipartite fuzzy graph and $D \subseteq \mathcal{V}$ be the set of all fuzzy majority dominating vertices. Then $\gamma_{CM}(\mathcal{G}) = \wedge\{\sigma(v_i) : v_i \in D\}$.*

Proof. Let $D \subseteq \mathcal{V}_1$ and $D \subseteq \mathcal{V}_2$ such that $t\,deg_{SN}(v_i) \geq \frac{p}{2}$, for all $v_i \in D$. Then each vertex of D forms a minimal connected fuzzy majority

dominating set of $\mathcal{G}$. Thus, the minimum membership value of vertex v_i forms a $\gamma_{CM}-$ set for $\mathcal{G}$. Hence $\gamma_{CM}(\mathcal{G}) = \wedge\{\sigma(v_i): v_i \in D\}$.

Proposition 4.3. *Let $\mathcal{G} = C_n, n \geq 3$ be a fuzzy cycle. Then $\gamma_{CM}(\mathcal{G}) \geq \frac{p}{2} - |N(S)|$, where S is a minimum connected fuzzy majority dominating set.*

Proof. Let $\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \dots v_n\}, n \geq 3$ be the vertex set of $\mathcal{G}$. Let $S \subseteq \mathcal{V}(\mathcal{G})$ and $S = \{v_1, v_2, \dots v_m\}$ where $m \leq \left\lceil \frac{n}{2} \right\rceil$ with $d(v_i, v_{i+1}) = 1, i = 1, 2, \dots, (m-1)$ be a minimum connected fuzzy majority dominating set. Then $|N[S]| = |N(S)| + |S| \geq \frac{p}{2}$. It implies that $|S| \geq \frac{p}{2} - |N(S)|$. Since $d(v_i, v_{i+1}) = 1$, the induced fuzzy subgraph $\langle S \rangle$ is connected. Therefore S is a $\gamma_{CM}-$ set of $\mathcal{G}$ and $\gamma_{CM}(\mathcal{G}) = |S| \geq \frac{p}{2} - |N(S)|$.

Corollary 4.4. *Let $\mathcal{G} = P_n, n \geq 2$ be a fuzzy path. Then $\gamma_{CM}(\mathcal{G}) \geq \frac{p}{2} - |N(S)|$, where S is a minimum connected fuzzy majority dominating set of $\mathcal{G}$.*

Proposition 4.5. *If a fuzzy cycle $\mathcal{G} = C_n, n \geq 3$ is expressed in a two way, arranging the membership values in an ascending or descending order and random order then $[\gamma_{CM}(\mathcal{G})]_{Random} \geq [\gamma_{CM}(\mathcal{G})]_{Ascending\ or\ Descending\ order}$.*

Proof. Let $\mathcal{G} = C_n, n \geq 3$ be a fuzzy cycle. Assume $\mathcal{G}_1$, the cycle with membership values arranged in an ascending or descending order and $\mathcal{G}_2$, the cycle with membership values in a random way.

In $\mathcal{G}_1$, the vertices with maximum σ- value has neighbours with high σ- value. Hence these vertices v_i become fuzzy majority dominating vertices. Thus $S = \{v_i\}$ form a connected fuzzy majority dominating set. But in $\mathcal{G}_2$, the maximum σ- value vertices may not appear consecutively. Hence the possibility for a vertex to be a fuzzy majority dominating vertex is very less. Thus $S = \{v_1, v_2, \dots, v_m\}, m \geq 2$ form a connected fuzzy majority dominating set. Therefore $[\gamma_{CM}(\mathcal{G})]_{Random} \geq [\gamma_{CM}(\mathcal{G})]_{Ascending\ or\ Descending\ order}$.

Theorem 4.6. *Let $\mathcal{G} = FDS(r, s), r \leq s$ be a fuzzy double star with bi-centres u and v having $p_1 = |N[u]|$ and $p_2 = |N[v]|$. Then*

$$\gamma_{CM}(\mathcal{G}) = \begin{cases} \wedge\{\sigma(u), \sigma(v)\}, & \text{if } p_1 \geq \frac{p}{2} \text{ and } p_2 \geq \frac{p}{2} \\ \sigma(u), & \text{if } p_1 \geq \frac{p}{2} \text{ and } p_2 < \frac{p}{2} \\ \sigma(v), & \text{if } p_1 < \frac{p}{2} \text{ and } p_2 \geq \frac{p}{2} \\ \min_{w_i \in D} \sigma(w_i), & \text{if } D \text{ contains fuzzy majority dominating pendants of } \mathcal{G} \end{cases}$$

Proof. Let $\mathcal{G}$ be a fuzzy double star with $\mathcal{V}(\mathcal{G}) = \{u, u_1, u_2, \dots, u_r, v, v_1, v_2, \dots, v_s\}$ having $p_1 = |N[u]|$, $p_2 = |N[v]|$ and $p = p_1 + p_2 - [\sigma(u) + \sigma(v)]$.

Case i. If $p_1 \geq \frac{p}{2}$ and $p_2 \geq \frac{p}{2}$ then $|N[u]| \geq \frac{p}{2}$ and $|N[v]| \geq \frac{p}{2}$. This implies that $S_1 = \{u\}$ and $S_2 = \{v\}$ are connected fuzzy majority dominating sets and all other fuzzy pendants are not fuzzy majority dominating vertices. Therefore $\gamma_{CM}(\mathcal{G}) = \wedge \{\sigma(u), \sigma(v)\}$.

Case ii. If $p_1 \geq \frac{p}{2}$ and $p_2 < \frac{p}{2}$ then $|N[u]| \geq \frac{p}{2}$ and $|N[v]| < \frac{p}{2}$. Hence $S = \{u\}$ is a connected fuzzy majority dominating set. Therefore $\gamma_{CM}(\mathcal{G}) = \sigma(u)$.

Case iii. If $p_1 < \frac{p}{2}$ and $p_2 \geq \frac{p}{2}$ then $S = \{v\}$ is a connected fuzzy majority dominating set. Hence $\gamma_{CM}(\mathcal{G}) = \sigma(v)$.

Case iv. Let $D = \{w_1, \dots, w_k\}$ be the collected fuzzy majority dominating pendant vertices among v_i's and u_i's. Each vertex $w_i \in D$ forms a connected fuzzy majority dominating set and $\min\{\sigma(w_1), \dots, \sigma(w_k)\}$ gives a connected fuzzy majority dominating number for $\mathcal{G}$. Hence $\gamma_{CM}(\mathcal{G}) = \min_{w_i \in D} \sigma(w_i)$.

Theorem 4.7. (Characterization) *Let S be a connected fuzzy majority dominating set of a fuzzy graph $\mathcal{G}$. Then S is minimal if and only if for every $v \in S$, either the condition (i) or the condition (ii) holds.*

i. *$|N[S]| = \frac{p}{2}$, then either v is an enclave of S or $pn[v, S] \cap (\mathcal{V} - S) \neq \emptyset$ and $\langle S \rangle$ is connected.*
ii. *$|N[S]| > \frac{p}{2}$, then $|pn[v, S]| > |N[S]| - \frac{p}{2}$ and $\langle S \rangle$ is connected.*

Proof. Assume $S \subseteq \mathcal{V}(\mathcal{G})$ is a minimal connected fuzzy majority dominating set of a fuzzy graph $\mathcal{G}$. Then $|N[S]| \geq \frac{p}{2}$ and the induced fuzzy subgraph $\langle S \rangle$ is connected in $\mathcal{G}$. Let $|N[S]| = \frac{p}{2}$. For every vertex $v \in S$, suppose the condition (i) is not true, then the vertex v is neither an enclave of S nor has a private neighbour in $(\mathcal{V} - S)$. That is $pn[v, S] \cap (\mathcal{V} - S) = \emptyset$ and $\langle S \rangle$ is not connected. We know that $pn[v, S] = |N[S]| - |N[S - v]|$, for any $v \in S$. Then $|N[S - v]| = |N[S]| = \frac{p}{2}$. This implies that the set $S - \{v\}$ is a fuzzy majority dominating set, which is a contradiction. If $\langle S \rangle$ is not connected, then S is not a minimal connected fuzzy majority dominating set for $\mathcal{G}$. Hence the condition (i) is true.

Let $|N[S]| > \frac{p}{2}$. Since S is a minimal connected fuzzy majority dominating set, $S - \{v\}$ is not a connected fuzzy majority dominating set for $\mathcal{G}$. This implies that $|N[S - v]| < \frac{p}{2}$. But $|N[S - v]| = |N[S]| - |pn[v, S]| < \frac{p}{2}$. Therefore $|pn[v, S]| > |N[S]| - \frac{p}{2}$, for every $v \in S$. Also, the set $S \subseteq \mathcal{V}(\mathcal{G})$ consists of vertices with distance $d(v_i, v_j) = 1$, for any $v_i, v_j \in S$ and $i \neq j$. Hence the induced fuzzy subgraph $\langle S \rangle$ is connected in $\mathcal{G}$. Thus, the condition (ii) holds, for every $v \in S$.

Conversely, let S be a connected fuzzy majority dominating set and conditions (i) or (ii) holds for

every $v \in S$. Suppose S is not minimal. Then either $|N[S-v]| \geq \frac{p}{2}$, for any $v \in S$ or the induced fuzzy subgraph $\langle S \rangle$ is not connected.

If $|N[S-v]| > \frac{p}{2}$ then $|N[S-v]| = |N[S]| - |pn[v,S]| > \frac{p}{2}$ .Therefore, $|pn[v,S]| < |N[S]| - \frac{p}{2}$ and $\langle S \rangle$ is not connected which is a contradiction to the condition (ii).

If $|N[S-v]| = \frac{p}{2}$ then $|N[S-v]| = |N[S]| - |pn[v,S]| = \frac{p}{2}$. This implies that $|pn[v,S]| = |N[S]| + \frac{p}{2} > \frac{p}{2} + \frac{p}{2} = p$ which is absurd result and contradiction. Hence S is a minimal connected fuzzy majority dominating set of $\mathcal{G}$.

5. Bounds of $\gamma_{CM}(\mathcal{G})$

Proposition 5.1. *For any connected fuzzy graph $\mathcal{G}$, $\wedge\{\sigma(v): v \in \mathcal{V}\} \leq \gamma_{CM}(\mathcal{G}) < \frac{p}{2}$.*

Proof. Let S be a minimum connected fuzzy majority dominating set of $\mathcal{G}$. If $\mathcal{G}$ has a vertex v with $\sigma(v) =$ minimum and $|N[v]| \geq \frac{p}{2}$ then $S = \{v\}$ forms a connected fuzzy majority dominating set and $\gamma_{CM}(\mathcal{G}) = \sigma(v) = \wedge\{\sigma(v): v \in \mathcal{V}(\mathcal{G})\}$.

Suppose $\mathcal{G}$ does not contain such a vertex v. Choose a set $S = \{v_1, v_2, \ldots v_m\}, m < n$ with $m \geq 2$ such that $|N[S]| \geq \frac{p}{2}$ and the induced fuzzy subgraph $\langle S \rangle$ is connected. Then $|S| = \sigma(v_1) + \cdots + \sigma(v_m)$ and $|S| > m\,\sigma(v_i)$, where $\sigma(v_i) =$ minimum. Thus S is a γ_{CM}- set and $\gamma_{CM}(\mathcal{G}) = |S| > m\,\sigma(v_i) > \wedge\{\sigma(v): v \in \mathcal{V}\}$.

Since $\mathcal{G}$ is connected, for any connected fuzzy majority dominating set S, $|N(S)| = \sum_{v_i \in N(S)} \sigma(v_i) > 0$. Since $|N[S]| \geq \frac{p}{2}, |S| + |N(S)| \geq \frac{p}{2}$. This implies that $|N(S)| \geq \frac{p}{2} - |S|$. Since $|N(S)| > 0, \frac{p}{2} - |S| > 0$. This implies that $|S| < \frac{p}{2}$. Therefore S is a γ_{CM}- set of $\mathcal{G}$ and $\gamma_{CM}(\mathcal{G}) < \frac{p}{2}$. Hence the result.

Theorem 5.2. *Let $\mathcal{G} = (\mathcal{V}, \sigma, \mu)$ be any connected fuzzy graph and $\mathcal{H} = (\mathcal{V}, \eta, \tau)$ be any maximum spanning tree of $\mathcal{G}$. Then every connected fuzzy majority dominating set of $\mathcal{H}$ is also a connected fuzzy majority dominating set of $\mathcal{G}$ and $\gamma_{CM}(\mathcal{H}) \geq \gamma_{CM}(\mathcal{G})$.*

Proof. Let S be a minimum connected fuzzy majority dominating set of $\mathcal{H}$. Since $\mathcal{H}$ is a maximum spanning tree of $\mathcal{G}$, $\sigma(v) = \eta(v)$.

Case i. Suppose the vertices in S has almost same adjacent vertices both in $\mathcal{H}$ and $\mathcal{G}$. Then the vertices of S dominates the same vertices in $\mathcal{G}$. Therefore $|N[S]| \geq \frac{p}{2}$ and the induced fuzzy subgraph $\langle S \rangle$ is connected in both $\mathcal{H}$ and $\mathcal{G}$. Thus S is a minimum connected fuzzy majority dominating set of $\mathcal{G}$. Hence $\gamma_{CM}(\mathcal{H}) = \gamma_{CM}(\mathcal{G})$.

Case ii. Suppose the vertices in S has less adjacent vertices in $\mathcal{H}$ than in $\mathcal{G}$. Then for each $v_i \in S$, $[\,t\,deg_{SN}(v_i)]_{\mathcal{H}} < [\,t\,deg_{SN}(v_i)]_{\mathcal{G}}$ that implies $[|N[v_i]|]_{\mathcal{H}} < [|N[v_i]|]_{\mathcal{G}}$. Therefore $\gamma_{CM}(\mathcal{H}) > \gamma_{CM}(\mathcal{G})$.

Proposition 5.3. *For any connected fuzzy graph $\mathcal{G}, \gamma_{CM}(\mathcal{G}) = \sigma(v)$ if and only if the fuzzy graph $\mathcal{G}$ has a fuzzy majority dominating vertex v with $\sigma(v) =$ minimum.*

Proof. Let $\gamma_{CM}(\mathcal{G}) = \sigma(v)$ if and only if S is a γ_{CM}-set with only one vertex v with $\sigma(v) =$ minimum and the fuzzy graph $\mathcal{G}$ has a vertex v with $\sigma(v) =$ minimum such that the total strong neighborhood degree $t\,deg_{SN}(v) \geq \frac{p}{2}, (i.e)$ if and only if $\mathcal{G}$ has a fuzzy majority dominating vertex $v \in \mathcal{V}(\mathcal{G})$ with $\sigma(v) =$ minimum.

Theorem 5.4. *Let $\mathcal{G}$ be a connected fuzzy graph with $t\,deg_{SN}(v_i) < \frac{p}{2}$, for every $v_i \in \mathcal{V}(\mathcal{G})$. Then $\gamma_{CM}(\mathcal{G}) > \wedge\{\sigma(v_i): v_i \in \mathcal{V}\}$.*

Proof. Since $t\,deg_{SN}(v_i) < \frac{p}{2}$, for every $v_i \in \mathcal{V}(\mathcal{G})$, the fuzzy graph $\mathcal{G}$ does not have a fuzzy majority dominating vertex. Therefore, any connected fuzzy majority dominating set contains atleast two vertices of $\mathcal{G}$. Then the γ_{CM}- set S contains atleast two vertices with $\sigma(v_i)$ and $\sigma(v_j)$. Therefore $\gamma_{CM}(\mathcal{G}) > \wedge\{\sigma(v_i): v_i \in \mathcal{V}\}$.

The following example shows that if $\gamma_{CM}(\mathcal{G}) > \wedge\{\sigma(v_i): v_i \in \mathcal{V}\}$ then there may exist some vertices $v_i \in \mathcal{V}(\mathcal{G})$ such that $t\,deg_{SN}(v_i) \geq \frac{p}{2}$.

Example 5.5.

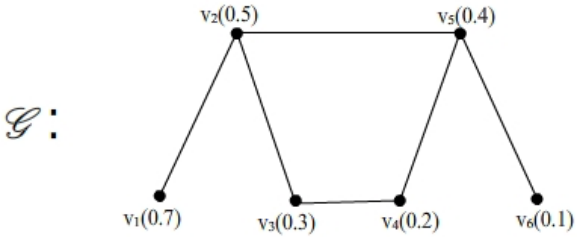


Figure 2.

Consider the fuzzy graph $\mathcal{G}$ having all strong edges with $p = 2.2$ and $\frac{p}{2} = 1.1$. Here γ_{CM}- set is $S = \{v_5\}$ and $\gamma_{CM}(\mathcal{G}) = 0.4 > 0.1 = \wedge\{\sigma(v_i) : v_i \in \mathcal{V}\}$. But $t\,deg_{SN}(v_1) = 1.2 > \frac{p}{2}$, $t\,deg_{SN}(v_2) = 1.9 > \frac{p}{2}$ and $t\,deg_{SN}(v_5) = 1.2 > \frac{p}{2}$. Therefore, the vertices v_1, v_2, v_5 satisfy the condition $t\,deg_{SN}(v_i) \geq \frac{p}{2}$.

Theorem 5.6. *Let $\mathcal{G}$ be a connected fuzzy graph with $|\mathcal{V}(\mathcal{G})| \geq 2$. If S is a minimal connected fuzzy majority dominating set of $\mathcal{G}$ then there may or may not exist atleast one minimal connected fuzzy majority dominating set of $\mathcal{G}$ in $(\mathcal{V} - S)$.*

Proof. Since S is a connected fuzzy majority dominating set of $\mathcal{G}$, $|N[S]| \geq \frac{p}{2} \ldots\ldots\ldots\ldots (a)$

and the induced fuzzy subgraph $\langle S\rangle$ of S is connected in $\mathcal{G}$ ……(b) Then $(\mathcal{V}-S)$ is the set of remaining vertices of $\mathcal{G}$.

Case i. Subcase i. If $|N[\mathcal{V}-S]| \geq \frac{p}{2}$ and the induced fuzzy subgraph $\langle \mathcal{V}-S\rangle$ is connected in $\mathcal{G}$ then $(\mathcal{V}-S)$ itself is a minimal connected fuzzy majority dominating set.

Subcase ii. Let S' be a subset of $(\mathcal{V}-S)$ such that $|N[S']| \geq \frac{p}{2}$ and the induced fuzzy subgraph $\langle S'\rangle$ is connected. Then S' is a minimal connected fuzzy majority dominating set in $(\mathcal{V}-S)$.

Subcase iii. Suppose there exists an edge (v_i, v_j) in $(\mathcal{V}-S)$ such that $[\sigma(v_i)+\sigma(v_j)] \geq \frac{p}{2}$. Then the sets $S_1=\{v_i\}$ and $S_2=\{v_j\}$ form a minimal connected fuzzy majority dominating set of $\mathcal{G}$ in $(\mathcal{V}-S)$.

Case ii. Subcase i. If $|N[\mathcal{V}-S]| < \frac{p}{2}$ then $(\mathcal{V}-S)$ could not be a connected fuzzy majority dominating set of $\mathcal{G}$.

Subcase ii. Suppose there is no $S' \subseteq (\mathcal{V}-S)$ satisfying the two conditions (a) and (b) then there does not exist a connected fuzzy majority dominating set of $\mathcal{G}$ in $(\mathcal{V}-S)$.

Proposition 5.7. *Let S be a γ_{CM} – set of a connected fuzzy graph $\mathcal{G}$. Then the set $(\mathcal{V}-S)$ may or may not contain a γ_{CM} – set.*

Proof. Let S be a $\gamma_{CM}-$ set of $\mathcal{G}$. Then $\gamma_{CM}(\mathcal{G}) = |S| = l = \sum_{v_i \in S}\sigma(v_i)$ = minimum.

Case i. Suppose there exists a vertex v_i in $(\mathcal{V}-S)$ such that $t\,deg_{SN}(v_i) \geq \frac{p}{2}$ and $\sigma(v_i) = l$. Then $S' = \{v_i\}$ forms another $\gamma_{CM}-$ set in $(\mathcal{V}-S)$.

Case ii. Let $S' \subseteq (\mathcal{V}-S)$ be a connected fuzzy majority dominating set of $\mathcal{G}$. If $|S'| = l$, then S' is the another $\gamma_{CM}-$ set in $(\mathcal{V}-S)$.

Case iii. Suppose $S_1', S_2', \ldots, S_r'$ are the connected fuzzy majority dominating sets of $\mathcal{G}$ in $(\mathcal{V}-S)$ with $|S_1'| \geq |S_2'| \geq \ldots \geq |S_r'| > l$. Then the set $(\mathcal{V}-S)$ does not contain a $\gamma_{CM}-$ set of $\mathcal{G}$.

Example 5.8.

Consider the complete fuzzy graph K_5 with $\sigma(v_i)$. In $\mathcal{G}$, $\gamma_{CM}-$ set$=\{v_1\} \Longrightarrow \gamma_{CM}(\mathcal{G}) = 0.1 = l$. All other vertices of K_5 are connected fuzzy majority dominating sets, but $\sigma(v_i) \neq l, i = 2,3,4,5$. Therefore in $(\mathcal{V}-S)$, there is no $\gamma_{CM}-$ set.

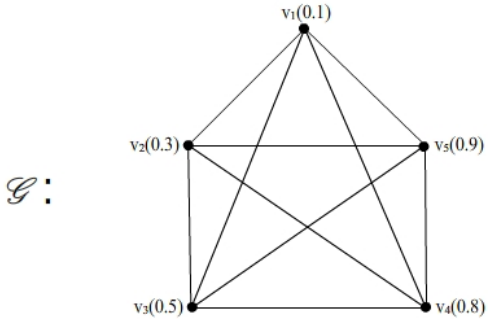


Figure 5.

Theorem 5.9. *Every connected fuzzy graph $\mathcal{G}$ has a connected fuzzy majority dominating set. But the converse is not true.*

Proof. Let $S=\{v_1, v_2, \ldots, v_k\} \subseteq \mathcal{V}(\mathcal{G}), k < n$. Then it satisfies the condition $|N[S]| \geq \frac{p}{2}$ and the induced subgraph $\langle S\rangle$ is connected. Thus S is the connected fuzzy majority dominating set of $\mathcal{G}$. The converse is not true.

The following example illustrates the concept.

Example 5.10.

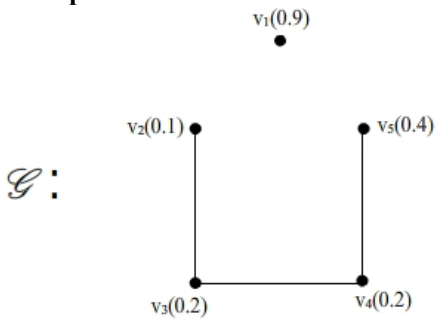


Figure 6.

Any fuzzy graph $\mathcal{G}$ with a connected fuzzy majority dominating set need not be connected. Consider the disconnected fuzzy graph $\mathcal{G}$ with $p = 1.8$ and $\frac{p}{2} = 0.9$. Here $S=\{v_3, v_4\}$ is a connected fuzzy majority dominating set, since $|N[S]| = 0.9 = \frac{p}{2}$ and $\langle S\rangle$ is connected. Hence $\gamma_{CM}(\mathcal{G}) = 0.4$. Thus, the disconnected fuzzy graph $\mathcal{G}$ has a connected fuzzy majority dominating set.

6. Applications of Connected Fuzzy Majority Dominating Sets

6.1. Public Health Awareness Network: A health department often arranges a public health awareness campaigns to inform people about vaccination, disease prevention, sanitation, nutrition and any other health-related issues. The health department may have limited time, personnel and financial resources that have been allotted for this purpose. Having only little sources they cannot communicate directly to every individual in a community. In this situation, the concept of connected fuzzy majority dominating set helps to identify a small connected group of people

who can disseminate the information to the majority of people in the community.

6.2. Disaster Preparedness Network: There are some place**s** which often face natural calamities. For example, if a city is affected by flood during the rainy season, it is necessary to prepare some prevention measures to meet the situation. The flood management requires proper preparation, rescue arrangements, essential supplies, communication facilities and rehabilitation support. These arrangements cannot be done in all the houses of the city. In this scenario, the city can be represented as a fuzzy graph connecting to each street. The junctions are denoted as the vertices of the fuzzy graph. By using the concept, connected fuzzy majority dominating set, the connected network is selected which covers more than half of the city. The needed arrangements are kept ready in these places, so that the rescue team can reach fast in the affected places.

Thus, the connected fuzzy majority dominating set provides a mathematical approach in selecting a connected group of emergency team who reach out to the majority of the people. It helps in planning communication strategies, coordinating disaster preparedness activities and making efficient use of limited resources.

7. Conclusion

The concept of domination in graphs is highly developed with significant theoretical advancements and numerous practical applications. Various domination parameters have been extensively studied by researchers. In this paper, we have introduced the concept of connected fuzzy majority domination by some theoretical results and also studied the relationship between $\gamma_M(\mathcal{G}), \gamma_{CM}(\mathcal{G})$ and $\gamma_C(\mathcal{G})$. We have also found the connected fuzzy majority domination number for some classes of fuzzy graphs. Characterization theorem is established and some bounds are given. Future works will present research on connected fuzzy majority domination in various types of fuzzy graphs.

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