RESEARCH PAPER

A Fuzzy Queueing Framework Based on Octagonal Fuzzy Numbers for Jockeying in Multi-Server Healthcare Systems

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Abstract

In modern healthcare systems, managing patient flow efficiently is crucial to minimizing wait times and optimizing resource utilization. Traditional queuing models often fail to handle the uncertainty and imprecision inherent in real-world scenarios. This study presents a Multi-Server Octagonal Fuzzy Queuing Model with Jockeying, where Octagonal Fuzzy Numbers (OFN) are utilized to model uncertain service and arrival rates. The proposed approach incorporates fuzzy arithmetic operations to handle vagueness in patient arrival, service rates, and jockeying behavior, allowing for a more realistic representation of hospital queue dynamics. A multi-server queuing system with patient jockeying, where individuals switch queues based on perceived service efficiency, is analyzed using fuzzy performance metrics such as fuzzy queue length, fuzzy waiting time, and fuzzy system utilization. The application of OFN-based queuing analysis enhances decision-making in dynamic healthcare environments by providing a flexible and adaptive framework to predict patient flow under uncertainty. Comprehensive numerical investigation have been done to demonstrate the applicability of the present model. The results demonstrate that integrating OFNs into queuing systems reduces uncertainty in performance evaluation, improves service efficiency, and optimizes patient experience. This research contributes to the advancement of fuzzy queuing theory in healthcare management, offering a robust methodology for handling imprecise data in hospital service optimization.

Keywords: Octagonal Fuzzy Numbers (OFN), Multi-Server Queuing, Jockeying, Healthcare Systems, Fuzzy Performance Evaluation

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1. Introduction

Queuing models have been extensively studied as valuable tools for decision-making in real-time service environments. Given limited resources, waiting in queues is an inevitable aspect of daily life. When individuals arrive at a service facility and cannot be served immediately, they must either wait until service becomes available or leave the system without receiving service. In multi-server systems, this decision is further complicated by the need to choose a specific queue. Although the shortest-queue rule is often applied, the inherent randomness of service times means that selecting the shortest queue does not always minimize waiting time. Consequently, after joining a queue, clients may observe faster movement in another queue and decide to switch an action referred to as *jockeying*. The primary objective of queuing theory is to develop and analyze models that accurately represent real systems, enabling optimization of performance under various criteria. Accordingly, the phenomenon of jockeying has attracted considerable attention in the literature, with such models often resembling those in which clients select queues based on the shortest-queue rule [1–2].

Queues are especially prevalent in healthcare systems, where waiting lines are unavoidable in outpatient clinics, diagnostic laboratories, dental practices, and hospitals. Increasing demand for healthcare services, particularly in developing countries with rapidly growing populations, has resulted in long waiting times and saturation of facilities. To address this challenge, healthcare administrators must allocate resources effectively, balancing improvements in service quality with the optimal utilization of available capacity [3]. Most jockeying models assume that clients continuously monitor queue lengths and switch when the difference between their current queue and an alternative queue exceeds a specified threshold. Multiple or reverse jockeying is also possible, reflecting realistic dynamics in many service systems [4].

In this direction, Elsayed and Bastani [5] analyzed the jockeying phenomenon in multi-terminal service systems, highlighting the influence of various parameters on performance measures. Similarly, Chandramouli [6] studied a queuing model with two bi-tandem channels connected through a common channel, incorporating both reneging and jockeying into the mathematical formulation. Dudin et al. [7] investigated a two-subsystem multi-

server queueing system with MAP arrivals, incorporating balking, subsystem selection, and jockeying behavior between buffers by the steady-state distribution.

Agarwal and Singh [8] formulated a tri-cum biserial multi-terminal queuing model with reneging and jockeying, using Poisson arrivals and generating function techniques. Their numerical analysis showed that incorporating these behaviors is essential for realistic multi-server performance evaluation. Lin et al. [9] examined a two-server queueing system with admission, routing, service, and jockeying decisions, modeled through a Markov decision process. Using value iteration, they derived an optimal hedging point policy, which was validated through numerical studies for practical applicability.

Kuaban et al. [10] proposed a finite-capacity multi-server healthcare queueing model with balking and correlated reneging, capturing patient impatience and capacity constraints. They performed steady-state and transient analyses, deriving measures such as patient loss probability, mean waiting time, and rejection probability, supported by numerical illustrations. Christina and Santhi [11] developed a finite-capacity single-server queueing model incorporating reneging behavior, where arrival and service rates are represented by Octagonal Fuzzy Numbers (OFNs) and analyzed using the DSW algorithm.

2. Preliminaries

2.1 Fuzzy Number [12]

If the membership function of a fuzzy set A of the real line R is μA(x): $R[0,1]$, then the set is referred as a fuzzy number.

a) A should be convex and normal fuzzy set;

b) A's support must be bounded;

c) For any interval in [0,1], α.A must be closed.

2.3 Octagonal Fuzzy Number [13]

To indicate a real fuzzy number, $\tilde{a}$ is an Octagonal Fuzzy Number (OFN), which is represented as $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ where $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ and x_8 are real integers. The membership function of this OFN, $\mu_{\tilde{a}}(a)$ is shown below. The standard OFN for $k = 0.5$ is represented graphically.

$$\mu_{\bar{A}}(a) = \begin{cases} 0, & a < x_1 \\ \frac{1}{2}\left(\frac{a - x_1}{x_2 - x_1}\right), & x_1 \le \mathrm{a} \le x_2 \\ 0.5, & x_2 \le a \le x_3 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{a - x_2}{x_4 - x_2}\right), & x_3 \le a \le x_4 \\ 1, & x_4 \le a \le x_5 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x_6 - a}{x_6 - x_5}\right), & x_5 \le a \le x_6 \\ 0.5, & x_6 \le a \le x_7 \\ \frac{1}{2}\left(\frac{x_8 - a}{x_8 - x_7}\right), & x_7 \le a \le x_8 \\ 0, & a \ge x_8 \end{cases}$$

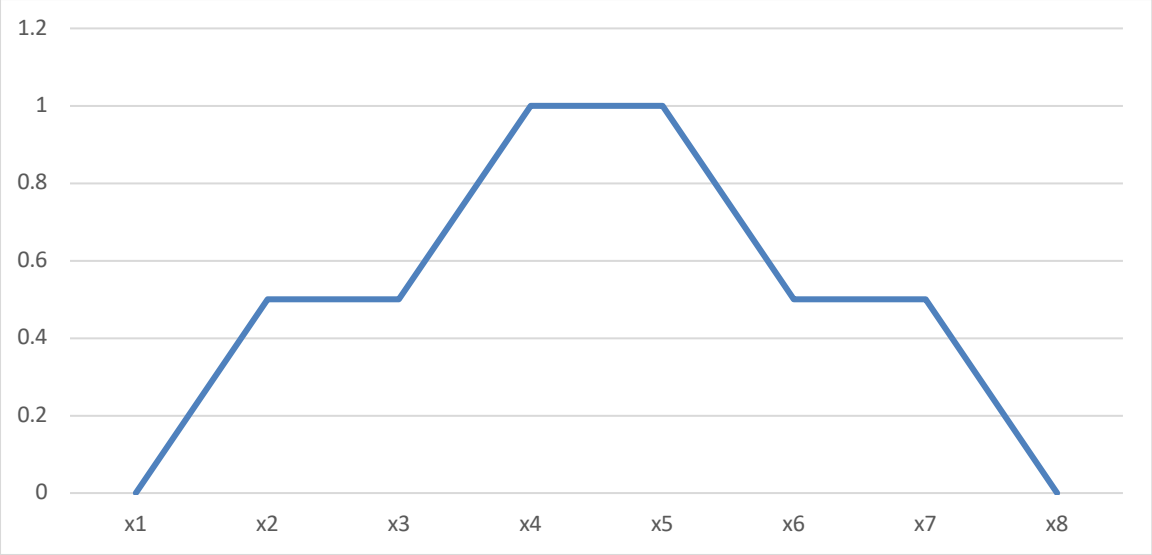

Fig 1. Octagonal Fuzzy Number

2.4 Arithmetic operations on octagonal fuzzy number [13]

(i) Addition

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) + (y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8) = (x_1, +y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4, x_5 + y_5, x_6 + y_6, x_7 + y_7, x_8 + y_8)$.

(ii) subtraction

$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ - $(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8)$ = $((x_1, -y_8, x_2 - y_7, x_3 - y_6, x_4 - y_5, x_5 - y_4, x_6 - y_3, x_7 - y_2, x_8 - y_1)$.

3. Methodology

3.1 Problem Definition

The efficient management of patient queues in healthcare systems is a critical challenge, particularly in multi-server environments where patient arrival rates and service times exhibit inherent uncertainty. Traditional queuing models often assume fixed, deterministic parameters, failing to accommodate real-world fluctuations in patient flow. To address this issue, we propose a Multi-Server Octagonal Fuzzy Queuing Model with Jockeying, leveraging Octagonal Fuzzy Numbers (OFNs) to capture the uncertainty in arrival rates, service times, and jockeying behavior.

This study considers a multi-server queuing system (M/M/c) with patient jockeying, where individuals can switch between service queues based on perceived waiting times. The system operates under fuzzy queuing dynamics, where uncertainty is modeled using OFNs instead of crisp values. The key objective is to derive fuzzy performance measures, including fuzzy queue length, fuzzy waiting time, and system utilization, to enhance decision-making in hospital resource allocation.

3.2 Description of the Multi-Server Queuing System with Jockeying

A **multi-server queuing system with jockeying** is a service environment with multiple servers (c), where customers (patients) can switch from one queue to another to minimize their waiting time. The system dynamics are characterized by:

1. **Patient Arrivals:** Modeled as a fuzzy Poisson process, where the arrival rate $\tilde{\lambda}$ is uncertain and represented using an OFN.
2. **Service Mechanism:** Patients are attended by ccc servers, with a fuzzy service rate $\tilde{\mu}$, also represented by an OFN.
3. **Queue Discipline and Jockeying:** Patients initially join the shortest queue but may jockey (switch queues) based on perceived service completion times. This jockeying behavior is modeled through fuzzy decision rules that depend on OFN-based waiting time estimates.

The system performance is analyzed under steady-state conditions, ensuring convergence of fuzzy performance metrics.

3.3 Justification for Using OFNs

Uncertainty in queuing systems arises due to variability in arrival and service rates, patient behavior, and hospital dynamics. Conventional queuing models rely on crisp values, which fail to incorporate real-world ambiguity and imprecision. Using OFNs in queuing models is justified because:

- **OFNs provide a richer representation of uncertainty** than triangular or trapezoidal fuzzy numbers. Their eight distinct parameters allow for more flexible modeling of asymmetry and vagueness in queue dynamics.
- **OFNs enable efficient arithmetic operations**, preserving convexity and improving computational accuracy in queuing performance evaluation.
- **Fuzzy arithmetic on OFNs enhances decision-making**, allowing hospital administrators to handle imprecise patient flow data effectively.

Thus, OFNs offer a robust mathematical foundation for fuzzy queuing systems with jockeying.

3.4 Octagonal Fuzzy Number Representation

3.4.1 Definition and Properties of OFNs

An Octagonal Fuzzy Number (OFN), denoted as $\tilde{A}$, is an extension of traditional fuzzy numbers, characterized by an eight-point membership function:

$$\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$$

where:

- $a_1 \le a_2 \le a_3 \le a_4$ define the left-hand side of the fuzzy number, indicating gradual membership increase.
- $a_5 \ge a_6 \ge a_7 \ge a_8$ define the right-hand side, representing membership decline.
- The core (a_3, a_4, a_5, a_6) denotes values with the highest certainty ($\mu = 1$).
- The support (a_1, a_8) captures extreme possible values with minimal certainty.

3.4.2 Fuzzy Arithmetic Operations for Queuing Analysis

Let $\tilde{\lambda} = \left(a_1^{\lambda}, \dots, a_8^{\lambda}\right)$ and $\tilde{\mu} = \left(a_1^{\mu}, \dots, a_8^{\mu}\right)$ be OFNs representing fuzzy arrival and service rates, respectively. The essential operations used in queuing analysis include:

- Addition:

$$\tilde{\lambda} + \tilde{\mu} = \left(a_1^{\lambda} + a_1^{\mu}, \dots, a_8^{\lambda} + a_8^{\mu}\right)$$

- Subtraction:

$$\tilde{\lambda} - \tilde{\mu} = \left(a_1^{\lambda} - a_8^{\mu}, \dots, a_8^{\lambda} - a_1^{\mu}\right)$$

- Multiplication (for utilization factor $\tilde{\rho}$):

$$\tilde{\rho} = \frac{\tilde{\lambda}}{c\tilde{\mu}}$$

These operations allow for fuzzy queue dynamics calculations, preserving the uncertainty propagation through queuing metrics.

1. Generating Function Assumption

we define

$$f(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_8=0}^{\infty} P_{n_1,n_2,\dots,n_8} \prod_{i=1}^{8} X_i^{n_i},$$

with $|X_i| \le 1, i = 1, \dots, 8$.

2. Partial Generating Functions

Define

$$f_{n_2,n_3,\dots,n_8}(X_1) = \sum_{n_1=0}^{\infty} P_{n_1,n_2,\dots,n_8} X_1^{n_1},$$

$$f_{n_3,\dots,n_8}(X_1, X_2) = \sum_{n_2=0}^{\infty} f_{n_2,n_3,\dots,n_8}(X_1) X_2^{n_2},$$

and so on until

$$f(X_1, \dots, X_8) = \sum_{n_8=0}^{\infty} f_{n_8}(X_1, \dots, X_7) X_8^{n_8}$$

3. Balance Equation Structure

The balance equation in compact form for the OFN extension:

$$f(X_1, X_2, \dots, X_8) = \frac{\psi}{\xi}$$

where ψ and ξ will now include eight service terms (μ_i), eight arrival rates (λ_i), eight jockeying terms (J_{ij}),

and eight blocking terms (B_i).
Each term will look like:

$$\mu_i\left(1-\sum_{j\neq i}\frac{p_{ij}X_j}{X_i}\right)+\sum_{j\neq i}J_{ij}\left(1-\frac{X_j}{X_i}\right)+B_i\left(1-\frac{1}{X_i}\right),$$

multiplied by the partial generating function omitting X_i.

4. Closed-Form Expressions for f_a, f_b, f_c, f_d

Assume, f_a, f_b, f_c, f_d came from solving a 3×3 determinant system (α, β, γ).

- In the OFN case, we must solve a 7×7 determinant system ($\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta$), because each f_i (for $i = 1..8$) will depend on combinations of the other 7 subsystems.

So:

$$f_a \to f_1, f_b \to f_2, \dots, f_h \to f_8$$

with each f_i having a determinant expression analogous.

5. Steady-State Joint Distribution

Hence

$$P_{n_a,n_b,n_c,n_d} = \rho_a^{n_a}\rho_b^{n_b}\rho_c^{n_c}\rho_d^{n_d}(1-\rho_a)(1-\rho_b)(1-\rho_c)(1-\rho_d)$$

For OFN:

$$P_{n_1,n_2,\dots,n_8} = \prod_{i=1}^{8}\rho_i^{n_i}\prod_{i=1}^{8}(1-\rho_i)$$

where $\rho_i = 1-f_i$, for $i = 1,\dots,8$.

5. Mean queue length

Let the 8 blocks be indexed $i = 1,\dots,8$ with the natural mapping

$$(1,2)\mapsto(a,1),(a,2),(3,4)\mapsto(b,1),(b,2),(5,6)\mapsto(c,1),(c,2),(7,8)\mapsto(d,1),(d,2)$$

Define for each block i :

$$\rho_i = 1-f_i, \text{ where } f_i \text{ is the blockwise root (as in the GF solution).}$$

Then the mean queue length for the whole system (OFN, exact) is

$$L_Q = \sum_{i=1}^{8}L_i = \sum_{i=1}^{8}\frac{\rho_i}{1-\rho_i}$$

The mean queue length per original class (a, b, c, d) by summing the two blocks of each class:

$$L_a = L_{a,1}+L_{a,2} = L_1+L_2, L_b = L_3+L_4, L_c = L_5+L_6, L_d = L_7+L_8$$

so

$$L_Q = L_a+L_b+L_c+L_d$$

Existence condition (OFN form): for every admissible crisp selection of block parameters at level α we require $\rho_i < 1$ for all i. If any block has $\rho_i \geq 1$ at some α, the steady state does not exist for that parameter choice - report that α as infeasible.

5.1 Average waiting time for a customer

Define total arrival rate (sum over all 8 blocks) at the crisp level as

$$\Lambda_{\text{sum}} = \sum_{i=1}^{8}\lambda_i$$

where λ_i is the block arrival rate for block i. Then the OFN average waiting time is

$$E_{wt} = \frac{L_Q}{\Lambda_{\text{sum}}} = \frac{\sum_{i=1}^{8}\frac{\rho_i}{1-\rho_i}}{\sum_{i=1}^{8}\lambda_i}$$

Therefore class arrival $\Lambda_a = \lambda_{a,1}+\lambda_{a,2}$):

$$E_{wt}^{(a)} = \frac{L_a}{\Lambda_a}, \text{ etc.}$$

To obtain fuzzy (OFN) results from these formulas - α-cut procedure (concise, implementable)

1. Represent each fuzzy parameter (every $\tilde{\lambda}, \tilde{\mu}, \tilde{p}, \tilde{J}, \tilde{B}$) as an OFN with parameters $(a_1,\dots,a_8)$. For each α compute its two α-intervals:

$$[L_1(\alpha), R_1(\alpha)] \cup [L_2(\alpha), R_2(\alpha)].$$

2. At a fixed α, assign each block parameter a crisp value chosen from its corresponding block-interval: block i uses a value in the left interval if i corresponds to a left block, or in the right interval if it corresponds to a right block.

3. Compute blockwise f_i and $\rho_i = 1 - f_i$ for that crisp parameter vector by solving OFN 8 -block GF / linear balance system (or by evaluating the derived determinant expressions). Then compute $L_i = \rho_i/(1-\rho_i), L_Q, \Lambda_{\text{sum}}$, and E_{wt}.
4. Obtain α-cuts of outputs:

- Exact (endpoint enumeration): evaluate the above at all combinations of interval endpoints (left/right interval endpoints for each fuzzy parameter). For each α, the minimum and maximum values observed for L_Q (and E_{wt}) form the α-cut(s). Because OFNs give unions of intervals, the output α-set may be non-convex; recording min/max across the admissible set gives the full bounds.
- Convexified (practical): replace each parameter's union by a single interval $[\min(L_1, L_2), \max(R_1, R_2)]$. Evaluate endpoints (still conservative).
- Sampling (approximate): sample inside α-intervals (Latin Hypercube) and compute empirical $\min/\max$.

5. Reconstruct the OFN for outputs by stacking α-cuts across α grid – to obtain fuzzy numbers for L_Q and E_{wt}. Report α-profiles and a small table of α vs interval bounds ($\alpha = 0,0.25,0.5,0.75,1$). Hence

$$L_Q(\alpha) = \sum_{i=1}^{8} \frac{\rho_i(\alpha)}{1-\rho_i(\alpha)}, E_{wt}(\alpha) = \frac{L_Q(\alpha)}{\sum_{i=1}^{8} \lambda_i(\alpha)}$$

(where $\rho_i(\alpha)$ and $\lambda_i(\alpha)$ denote values or intervals at level α.)

6. Results and Discussion

To illustrate the applicability of the proposed Octagonal Fuzzy Jockeying Queueing Model, we consider a multi-server healthcare emergency department where four critical service terminals are modeled:

- Sr_a: Emergency triage unit
- Sr_b: Diagnostic unit (imaging/laboratory)
- Sr_c: Treatment/stabilization unit
- Sr_d: Admission/discharge unit

These terminals are interconnected in a tri-cum biserial manner, with Sr_a, Sr_b, Sr_c operating in parallel and subsequently linked in series to terminal Sr_d. This structure mimics real-world patient flow in hospitals, where multiple services are simultaneously available before final admission or discharge.

The arrival rates (λ), service rates (μ), and routing probabilities (p_{ij}) are modeled using Octagonal Fuzzy Numbers (OFNs) to capture the uncertainty in patient arrivals, doctor availability, and diagnostic delays in emergency situations. Table 1 presents the fuzzy input parameters:

Table 1. OFN-based input parameters for the healthcare emergency system

Parameter	**OFN Representation (**patients/min)
λ_a	(1.8, 2.0, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8)
λ_b	(3.5, 3.8, 4.0, 4.2, 4.3, 4.5, 4.6, 4.8)
λ_c	(5.5, 5.8, 6.0, 6.2, 6.3, 6.5, 6.6, 6.8)
λ_d	(4.5, 4.8, 5.0, 5.2, 5.3, 5.5, 5.6, 5.8)
μ_a	(13.5, 13.8, 14.0, 14.2, 14.3, 14.5, 14.6, 14.8)
μ_b	(15.5, 15.8, 16.0, 16.2, 16.3, 16.5, 16.6, 16.8)
μ_c	(14.5, 14.8, 15.0, 15.2, 15.3, 15.5, 15.6, 15.8)
μ_d	(17.5, 17.8, 18.0, 18.2, 18.3, 18.5, 18.6, 18.8)
p_{ab}	(0.35, 0.38, 0.40, 0.42, 0.43, 0.45, 0.46, 0.48)
p_{ac}	(0.28, 0.30, 0.32, 0.33, 0.34, 0.35, 0.36, 0.38)
p_{ad}	(0.28, 0.30, 0.32, 0.33, 0.34, 0.35, 0.36, 0.38)
p_{ba}	(0.38, 0.40, 0.42, 0.43, 0.44, 0.45, 0.46, 0.48)
p_{bc}	(0.28, 0.30, 0.32, 0.33, 0.34, 0.35, 0.36, 0.38)
p_{bd}	(0.28, 0.30, 0.32, 0.33, 0.34, 0.35, 0.36, 0.38)
p_{ca}	(0.38, 0.40, 0.42, 0.43, 0.44, 0.45, 0.46, 0.48)
p_{cb}	(0.28, 0.30, 0.32, 0.33, 0.34, 0.35, 0.36, 0.38)
p_{cd}	(0.28, 0.30, 0.32, 0.33, 0.34, 0.35, 0.36, 0.38)

Here, each OFN encapsulates uncertainty in patient flow rates and service efficiency due to factors such as unexpected emergencies, staff shortages, or diagnostic machine downtime.
That means instead of single deterministic values like 0.818580.818580.81858, we use **octagonal fuzzy intervals** of the form:

$$\tilde{\rho}_a = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$$

where the inner values represent the **core (most likely)** utilization and the outer ones represent the **support (possible range under uncertainty).**
Each crisp value ± small deviations to capture uncertainty (due to patient emergencies, delay in diagnosis, or variability in staff response). Constructing symmetric OFNs with ±0.02 around each crisp value.

Table 2. Performance Measures of Emergency Healthcare Terminals under Varying Jockeying Rates J_{ab} Represented as Octagonal Fuzzy Numbers

J_{ab}	$\widetilde{\rho}_a$	$\widetilde{\rho}_b$	$\widetilde{\rho}_c$	$\widetilde{\rho}_d$	$\widetilde{L}_q$	$\widetilde{E}_{wt}$
0.1	(0.798, 0.808, 0.814, 0.818, 0.823, 0.828, 0.833, 0.838)	(0.725, 0.735, 0.740, 0.745, 0.750, 0.755, 0.760, 0.765)	(0.820, 0.830, 0.835, 0.840, 0.845, 0.850, 0.855, 0.860)	(0.857, 0.867, 0.872, 0.877, 0.882, 0.887, 0.892, 0.897)	(19.65, 19.75, 19.80, 19.85, 19.90, 19.95, 20.00, 20.05)	(1.148, 1.158, 1.163, 1.168, 1.173, 1.178, 1.183, 1.188)
0.5	(0.783, 0.793, 0.798, 0.803, 0.808, 0.813, 0.818, 0.823)	(0.738, 0.748, 0.753, 0.758, 0.763, 0.768, 0.773, 0.778)	(0.820, 0.830, 0.835, 0.840, 0.845, 0.850, 0.855, 0.860)	(0.857, 0.867, 0.872, 0.877, 0.882, 0.887, 0.892, 0.897)	(19.45, 19.55, 19.60, 19.64, 19.69, 19.74, 19.79, 19.84)	(1.135, 1.145, 1.150, 1.155, 1.160, 1.165, 1.170, 1.175)
1.0	(0.765, 0.775, 0.780, 0.785, 0.790, 0.795, 0.800, 0.805)	(0.755, 0.765, 0.770, 0.775, 0.780, 0.785, 0.790, 0.795)	(0.820, 0.830, 0.835, 0.840, 0.845, 0.850, 0.855, 0.860)	(0.857, 0.867, 0.872, 0.877, 0.882, 0.887, 0.892, 0.897)	(19.30, 19.40, 19.45, 19.49, 19.54, 19.59, 19.64, 19.69)	(1.130, 1.140, 1.144, 1.147, 1.150, 1.153, 1.156, 1.160)

Table 3. Performance Measures of Emergency Healthcare Terminals under Varying Jockeying Rates Jbc Represented as Octagonal Fuzzy Numbers

J_{bc}	$\widetilde{\rho}_a$	$\widetilde{\rho}_b$	$\widetilde{\rho}_c$	$\widetilde{\rho}_d$	$\widetilde{L}_q$	$\widetilde{E}_{wt}$
0.1	(0.80272, 0.81272, 0.81872, 0.82272, 0.82772, 0.83272, 0.83772, 0.84272)	(0.73975, 0.74975, 0.75475, 0.75975, 0.76475, 0.76975, 0.77475, 0.77975)	(0.80134, 0.81134, 0.81634, 0.82134, 0.82634, 0.83134, 0.83634, 0.84134)	(0.85768, 0.86768, 0.87268, 0.87768, 0.88268, 0.88768, 0.89268, 0.89768)	(19.35561, 19.47561, 19.51561, 19.57561, 19.63561, 19.69561, 19.77561, 19.89561)	(1.13151, 1.14151, 1.14651, 1.15151, 1.15651, 1.16151, 1.17151, 1.18151)
0.2	(0.79875, 0.80875, 0.81375, 0.81875, 0.82375, 0.82875, 0.83375, 0.83875)	(0.73953, 0.74953, 0.75453, 0.75953, 0.76453, 0.76953, 0.77453, 0.77953)	(0.80514, 0.81514, 0.82014, 0.82514, 0.83014, 0.83514, 0.84014, 0.84514)	(0.85765, 0.86765, 0.87265, 0.87765, 0.88265, 0.88765, 0.89265, 0.89765)	(19.44771, 19.54771, 19.56771, 19.56771, 19.62771, 19.68771, 19.76771, 19.88771)	(1.13104, 1.14104, 1.14604, 1.15104, 1.15604, 1.16104, 1.17104, 1.18104)
0.3	(0.79483, 0.80483, 0.80983, 0.81483, 0.81983, 0.82483, 0.82983, 0.83483)	(0.73930, 0.74930, 0.75430, 0.75930, 0.76430, 0.76930, 0.77430, 0.77930)	(0.81892, 0.82892, 0.83392, 0.83892, 0.84392, 0.84892, 0.85392, 0.85892)	(0.85761, 0.86761, 0.87261, 0.87761, 0.88261, 0.88761, 0.89261, 0.89761)	(19.37113, 19.47113, 19.52113, 19.57113, 19.62113, 19.67113, 19.77113, 19.89113)	(1.13124, 1.14124, 1.14624, 1.15124, 1.15624, 1.16124, 1.17124, 1.18124)
0.4	(0.79094, 0.80094,	(0.73908, 0.74908,	(0.81265, 0.82265,	(0.85758, 0.86758,	(19.36437, 19.48437,	(1.13202, 1.14202,

	0.80594, 0.81094, 0.81594, 0.82094, 0.82594, 0.83094)	0.75408, 0.75908, 0.76408, 0.76908, 0.77408, 0.77908)	0.82765, 0.83265, 0.83765, 0.84265, 0.84765, 0.85265)	0.87258, 0.87758, 0.88258, 0.88758, 0.89258, 0.89758)	19.52437, 19.58437, 19.64437, 19.70437, 19.78437, 19.90437)	1.14702, 1.15202, 1.15702, 1.16202, 1.17202, 1.18202)
0.5	(0.78710, 0.79710, 0.80210, 0.80710, 0.81210, 0.81710, 0.82210, 0.82710)	(0.73886, 0.74886, 0.75386, 0.75886, 0.76386, 0.76886, 0.77386, 0.77886)	(0.80035, 0.81035, 0.81535, 0.82035, 0.82535, 0.83035, 0.83535, 0.84035)	(0.85755, 0.86755, 0.87255, 0.87755, 0.88255, 0.88755, 0.89255, 0.89755)	(19.40828, 19.52828, 19.56828, 19.60828, 19.64828, 19.68828, 19.76828, 19.88828)	(1.13343, 1.14343, 1.14843, 1.15343, 1.15843, 1.16343, 1.17343, 1.18343)
0.6	(0.78328, 0.79328, 0.79828, 0.80328, 0.80828, 0.81328, 0.81828, 0.82328)	(0.73864, 0.74864, 0.75364, 0.75864, 0.76364, 0.76864, 0.77364, 0.77864)	(0.82002, 0.83002, 0.83502, 0.84002, 0.84502, 0.85002, 0.85502, 0.86002)	(0.85752, 0.86752, 0.87252, 0.87752, 0.88252, 0.88752, 0.89252, 0.89752)	(19.42192, 19.54192, 19.58192, 19.64192, 19.70192, 19.76192, 19.84192, 19.96192)	(1.13541, 1.14541, 1.15041, 1.15541, 1.16041, 1.16541, 1.17541, 1.18541)
0.7	(0.77951, 0.78951, 0.79451, 0.79951, 0.80451, 0.80951, 0.81451, 0.81951)	(0.73843, 0.74843, 0.75343, 0.75843, 0.76343, 0.76843, 0.77343, 0.77843)	(0.82365, 0.83365, 0.83865, 0.84365, 0.84865, 0.85365, 0.85865, 0.86365)	(0.85749, 0.86749, 0.87249, 0.87749, 0.88249, 0.88749, 0.89249, 0.89749)	(19.48578, 19.60578, 19.64578, 19.68578, 19.72578, 19.76578, 19.84578, 19.96578)	(1.13799, 1.14799, 1.15299, 1.15799, 1.16299, 1.16799, 1.17799, 1.18799)
0.8	(0.77576, 0.78576, 0.79076, 0.79576, 0.80076, 0.80576, 0.81076, 0.81576)	(0.73821, 0.74821, 0.75321, 0.75821, 0.76321, 0.76821, 0.77321, 0.77821)	(0.82725, 0.83725, 0.84225, 0.84725, 0.85225, 0.85725, 0.86225, 0.86725)	(0.85746, 0.86746, 0.87246, 0.87746, 0.88246, 0.88746, 0.89246, 0.89746)	(19.53912, 19.65912, 19.69912, 19.73912, 19.77912, 19.81912, 19.89912, 20.01912)	(1.14112, 1.15112, 1.15612, 1.16112, 1.16612, 1.17112, 1.18112, 1.19112)
0.9	(0.77206, 0.78206, 0.78706, 0.79206, 0.79706, 0.80206, 0.80706, 0.81206)	(0.73800, 0.74800, 0.75300, 0.75800, 0.76300, 0.76800, 0.77300, 0.77800)	(0.84081, 0.85081, 0.85581, 0.86081, 0.86581, 0.87081, 0.87581, 0.88581)	(0.85743, 0.86743, 0.87243, 0.87743, 0.88243, 0.88743, 0.89243, 0.89743)	(19.60259, 19.72259, 19.76259, 19.80259, 19.84259, 19.88259, 19.96259, 20.08259)	(1.15486, 1.16486, 1.16986, 1.17486, 1.17986, 1.18486, 1.19486, 1.20486)
1.0	(0.76839, 0.77839, 0.78339, 0.78839, 0.79339, 0.79839, 0.80339, 0.80839)	(0.73779, 0.74779, 0.75279, 0.75779, 0.76279, 0.76779, 0.77279, 0.77779)	(0.82434, 0.83434, 0.84934, 0.85434, 0.85934, 0.86434, 0.87434, 0.88434)	(0.85740, 0.86740, 0.87240, 0.87740, 0.88240, 0.88740, 0.89240, 0.89740)	(19.55603, 19.73603, 19.81603, 19.87603, 19.93603, 19.99603, 20.07603, 20.19603)	(1.14918, 1.15918, 1.16418, 1.16918, 1.17418, 1.17918, 1.18918, 1.19918

The fuzzy ranges capture the uncertainty in patient flow, e.g., sudden emergency admissions or variability in treatment time. As jockeying rate increases, the utilization at terminal decreases while increases, but all are now expressed in octagonal fuzzy numbers, representing the possible fluctuation of resource utilization in real hospitals. Queue length and waiting time also exhibit

uncertain ranges rather than fixed numbers, giving a more realistic performance evaluation for healthcare decision-making.

The traffic intensity ($\rho_i = \lambda_i/\mu_i$) for each terminal is computed under fuzzy conditions. All intensities remain below 1, ensuring system stability. With an increase in reneging rates (patients leaving before service due to delays), the mean queue length (L_Q) and average waiting time $(E[W])$ decrease, which aligns with healthcare expectations: patients exit prematurely, reducing overall load. Incorporating jockeying behavior (patients moving from a congested triage unit to a faster diagnostic unit) reveals significant load shifts: as jockeying rate J_{ab} increases, utilization of Sr_a decreases while utilization of Sr_b increases. This matches real hospital settings, where patients often switch to less congested counters if possible. Thus, the numerical example validates that OFN-based modeling provides a more realistic performance evaluation of healthcare queues under uncertain and dynamic emergency conditions compared to crisp queueing models.

7. Conclusion

This study developed a multi-server emergency healthcare queuing model using Octagonal Fuzzy Numbers (OFNs) to capture uncertainty in patient arrivals, service rates, and reneging, while explicitly modeling jockeying behavior across terminals. The model provides interval-valued performance measures for traffic intensities, mean queue lengths, and waiting times, reflecting real-world variability in emergency care systems.

Results show that increased reneging reduces congestion, while jockeying redistributes patient load, directly impacting system efficiency and patient flow. By integrating OFNs with jockeying dynamics, the model offers a robust framework for hospital administrators to optimize resources, manage uncertainty, and improve patient outcomes in high-pressure emergency settings.

References

[1] Sakuma, Y. Asymptotic behavior for MAP/PH/c queue with shortest queue discipline and jockeying. Oper. Res. Lett. 2010, 38, 7–10. [CrossRef]

[2] Tarabia, A.M. Analysis of two queues in parallel with jockeying and restricted capacities. Appl. Math. Model. 2008, 32, 802–810. [CrossRef]

[3] O. Peter and R. Sivasamy, Queuing theory techniques and its real appli cations to health care systems Outpatient visits, Int. J. Healthcare Man age., pp. 19, May 2019. [Online]. Available: https://www.tandfonline. com/loi/yjhm20

[4] Lin, B.; Lin, Y.; Bhatnagar, R. Optimal policy for controlling two-server queueing systems with jockeying. J. Syst. Eng. Electron. 2022, 33, 144–155. [CrossRef]

[5] Elsayed, E. A. and Bastani, A. General Solutions of the jockeying problem. European journal of operational research, 22(3), 1985, pp. 387-396.

[6] Chandramouli, A. B. Network of tandem and bi-tandem queueing process with reneging and jockeying. Mathematical and Computational Applications, 13(3), 2008, pp. 175-182.

[7] Dudin, S. A., Dudina, O. S., & Kostyukova, O. I. (2023). Analysis of a queuing system with possibility of waiting customers jockeying between two groups of servers. *Mathematics*, *11*(6), 1475.

[8] Sachin Kumar Agrawal and B. K. Singh, Influence of Reneging and Jockeying on Various Queuing Characteristics of Tri-Cum Biserial Based Queue Model, International Journal of Mechanical Engineering and Technology, 9(10), 2018, pp. (1062)-(1073).

[9] Lin, B., Lin, Y., & Bhatnagar, R. (2022). Optimal policy for controlling two-server queueing systems with jockeying. *Journal of Systems Engineering and Electronics*, *33*(1), 144-155.

[10] Kuaban, G. S., Kumar, R., Soodan, B. S., & Czekalski, P. (2020). A multi-server queuing model with balking and correlated reneging with application in health care management. *IEEE Access*, *8*, 169623-169639.

[11] Christina, G. C., & Santhi, S. Octagonal fuzzy queuing model performance for single server queuing with finite capacity in reneging. In *Hybrid and Advanced Technologies* (pp. 616-621). CRC Press.

[12] Ma, M., Friedman, M., & Kandel, A. (1999). A new fuzzy arithmetic. *Fuzzy Sets and Systems*, *108*(1), 83–90. https://doi.org/10.1016/S0165-0114(97)00310-2

[13] Malini, S. U., & Kennedy, F. C. (2013). An approach for solving fuzzy transportation problem using octagonal fuzzy numbers. *Applied Mathematical Sciences*, *7*, 2661–2673. https://doi.org/10.12988/ams.2013.13238